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A POSSIBLE APPLICATION OF QUANTUM ALGORITHM FOR MARKET PRICE PREDICTION

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Abstract: Quantum computation exploits the inherent parallelism due to the principle of superposition of quantum states and hence it has the potential to increase the solution speed for many computational problems. In recent years Grover's search algorithm, Sore's factorization algorithm and Deutsch's algorithm have proven to solve some of the problems exponentially faster than their classical counter parts. The study of financial market is highly sensitive, uncertain and complex in nature. However the traditional classical computers have limited computational capacity with respect to complexity of the problem. In this paper a possible application of Deutsch's algorithm for market price prediction has been discussed.

Key Words: Quantum computing, Quantum algorithm, Computational Finance

INTRODUCTION

The study of financial market is highly sensitive, uncertain and unpredictable and depends upon many external factors. There is no well defined analytical procedure to predict the financial market. Since there is no well-defined mathematical or analytical solution for financial market predictions, people generally apply numerical methods and computer simulation to get better results. The introduction of computer science to solve the financial problems is known as computational finance, where computer simulation tries to find out the most approximate result of a given problem. But the traditional classical computers have limited computational capacity with respect to complexity of the problem. Hence the processing time depends upon the speed of the processor and the speed of processor depends upon the number of instructions executed in each clock period.

Therefore if we want to increase the proceeding capacity of the classical computer we have to add more and more computational components to the physical system which ultimately increases the hard ware components of a system. According to the prediction of Moore's Law, if we will increase the number of transistors in a fixed size integrated circuit, a day will come the size of transistors will tend to atomic. Hence, naturally we are coming to a point where quantum theory is entering to the field of computation known as quantum computation. Quantum computation exploits the inherent parallelism due to the principle of superposition of quantum states and hence it has the potential to increase the solution speed for many computational problems. In recent years for some cases the quantum algorithms, like Grover's algorithm for unstructured data base search [4, 5, 13], Sore's factorization algorithm [11] and Deutsch's algorithm, solve the problems exponentially faster than their classical counter parts. Hence it is worth to introduce quantum algorithms in context of computational finance. So introducing quantum concepts in financial domain may lead better approximation to make a

good decision in a stipulated time period. In this paper a theoretical method has been discussed that predicts the market price of a financial instrument using Deutsch algorithm. The organization of the paper is as follows: section-2 introduces quantum concepts in computation, section-3 working principle of quantum algorithm, section-4 definition of Deutsch's algorithm, section-5 Statement of the financial problem and solution, section-6 Conclusion and references.

QUANTUM CONCEPTS IN COMPUTATION

A bit is either 0 or 1, i.e. {0, 1}. An n-bit string is the sequence of n-bits and is an element of {0, 1}ⁿ. Arithmetic addition between two bits is denoted by \oplus : $0 \oplus 0 = 1 \oplus 1 = 0$ and $0 \oplus 1 = 1 \oplus 0 = 1$. Which is nothing but the addition modulo 2. Like the conventional bit a quantum bit(qubit) is a two dimensional quantum system denoted by $|0\rangle$ and $|1\rangle$.

The state of several qubits is separated by the tensor product of individual qubits. If the first qubit is in state $|0\rangle$ and second qubit is in state $|1\rangle$, then we can write the joint system of two qubits as $|0\rangle\otimes|1\rangle$. The qubits can exist in the mixture of both the states $|0\rangle$ and $|1\rangle$ at the same time. This is possible due to the superposition principle of quantum mechanics [1,6,4]. Mathematically a qubit is represented as a vector $|\Psi\rangle$ in a two dimensional complex vector space .The qubit $|\Psi\rangle$ can be written in general form as $|\Psi\rangle = \alpha|p\rangle + \beta|q\rangle$, where the complex coefficients α and β satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$.Hence $|\Psi\rangle$ is the superposition of the basis states $|p\rangle$ and $|q\rangle$ and can be represented in an infinite number of ways simply by varying values of the coefficients α and β subject to the normalization condition.

Two types of operations under go in a quantum system: measurement and quantum state transformations. In classical computing set of universal gates are used for computational purpose, where as in quantum computation most algorithms follow a sequence of quantum state transformations

followed by measurement. Actual quantum computation processes are very different from that of classical counterpart. In classical computer we give input data through the input devices .The input signal is stored in computer memory ,then fed into the microprocessor and the result is stored in memory before it is displayed in the screen .Thus the information travels around the circuit .In contrast , information in quantum computation is first stored in a register and then external fields , such as oscillating magnetic fields , electric fields or laser beams are applied to produce gate operations on the register. These external fields are designed so that they produce desired gate operation, i.e. unitary matrix acting on a particular set of qubits .Hence information sits in the register and they are updated each time the gate operation acts on the register. In order to understand the manipulation of quantum registers we must know how quantum systems evolve. For better understanding refer [8-11].

Working Principle Of Quantum Computer:

Consider a classical register with 3 bits, then possible outcome will be $2^3=8$ (it would be possible to use this register to represent any one of the numbers from 0 to 7 at any instance of time).If we will consider a register of 3 qubits, then the register can represent all the numbers from 0 to 7 simultaneously. A single processor having qubit registers will be able to perform calculations using all possible values of the input registers simultaneously. This phenomenon is called quantum parallelism. Quantum parallelism is possible due to the superposition principle of quantum states. Unlike classical bits qubits can exist simultaneously as 0 and 1, with probability for each state given by numerical coefficients. The basic components of a quantum computer [3, 8, 12] are as follows:

- A register or a set of registers.
- A unitary matrix, which is used to execute the quantum algorithms.
- Measurements to extract information.

WORKING PRINCIPLE OF QUANTUM ALGORITHM

The idea to implement quantum mechanics for algorithmic tasks was introduced by Feynman [1, 2].The most important quantum algorithms discovered to date all perform tasks for which there are no classical equivalents .Deutsch's algorithm [3] is designed to solve the problem of identifying whether a binary function is constant or balanced. Its running time is $O(n)$ while classical method requires $O(2^n)$. Simon's algorithm [11] is designed for finding the periodicity in a 2-1 binary function that is guaranteed to possess a periodic element. Here exponential speed up is also achieved. Another famous algorithm called Grover's algorithm is meant for searching an unsorted database in time $O(\sqrt{N})$, where classical search algorithm has running time $O(N)$. This is an example of a real world problem for which quantum algorithm provides the performance that is classically impossible. Finally the most important quantum algorithm is Shor's algorithm [6] for prime factorization. This algorithm finds the prime factors of very large numbers in polynomial time for which the best classical algorithm requires exponential time. The basic steps used in the quantum algorithms [2, 5, 11] are as follows:

- Initialize the quantum registers.
- Put the registers in superposition of states.
- Evolve the registers using unitary operators.
- Measure the states to get the result.

Quantum Oracle:

Quantum oracle is similar to that of a classical black box function. Classically a black-box function is a box that evaluates an unknown function f . The input is a n -bit string x and the output is given by m -bit string $f(x)$.According to quantum mechanics such a box can only exist if it is reversible. To create a reversible box, the input (x) is output together with $f(x)$ and the black box looks like as shown in the figure(1). In order to make the box reversible, an additional m -bit input y is added and the output of the result is $f(x) \oplus y$, where \oplus denotes bit wise addition modulo 2 operator .If $y = 0$ then the output is simply $f(x)$.



Figure-1 Quantum Black Box

DEFINITION OF DEUTSCH'S ALGORITHM

A function $f\{0,1\} \rightarrow \{0,1\}$ is balanced if $f(0) \neq f(1)$ i.e. one to one and constant if $f(0) = f(1)$.The job is to determine whether f is constant or balanced [12].

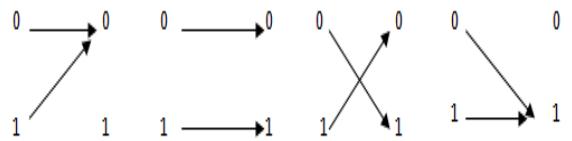


Figure-2

Figue-2 shows the possible outcome of balanced and constant nature of function. Classically it would take two evaluations of the function to tell whether it is one or the other. Quantum mechanically, one can answer this question in a single evaluation only. The reason for this is that quantum mechanically one can pack 0 and 1 into x at the same time. Note that for a classical algorithm to solve this problem with a success probability more than one half one has to query the black box twice, for both $f(0)$ and $f(1)$.Obviously, the worst-case time complexity of the best classical algorithm will take $2^n + 1$ time with different inputs.

DEUTSCH'S ALGORITHM FOR MARKET PRICE PREDICTION STATEMENT OF A FINANCIAL PROBLEM

Let us consider a method or function $f(x, N)$ that can predict whether the market price of a given financial instrument will increase or decrease after certain time period (let us take after one day).The output of the function is binary (0 or 1). Where x is a binary variable (either 0 or 1) and N is a variable that includes relevant market data (which can be taken constant for a given market day, so we will write $f(x, N) = f(x)$).The working principle of the function is as follows:

- a. If $f(0) = f(1)$ then market price of the financial instrument will increase tomorrow.
- b. If $f(0) \neq f(1)$ then market price of the financial instrument will decrease tomorrow.

If one tries to solve this problem using a traditional computer then it will work as follows:

- a. Put $x = 0$ and $x = 1$ in $f(x)$
- b. Compare, if $f(0) = f(1)$ then market price of the instrument will increase tomorrow.
- c. If $f(0) \neq f(1)$ then market price of the instrument will decrease tomorrow.

The problem in this method is that the function $f(x)$ is very complex in nature and the calculation of $f(x)$ for the value 0 and 1 will take a major time. Suppose we engage a single super-computer to calculate first $f(0)$ then $f(1)$ and compare the results to get a conclusion. It may take more than one day to do this task. So we are not able to forecast the market price of the instrument for tomorrow. Hence our financial problem reduced to a computational problem and its solution depends upon the computing power of the processor and the efficiency of the algorithm.

SOLUTION APPROCH

The problem lies in calculating both $f(0)$ and $f(1)$ in order to compare their values. If we can calculate the joint property of $f(0)$ and $f(1)$ simultaneously then our problem will be solved. Quantum computer is able to answer about the joint properties as it works on the principle of superposition.

Let us consider an operator U that transforms a two qubit register as follows:

$$U: |x\rangle|y\rangle \rightarrow |x\rangle|y\oplus f(x)\rangle$$

(1)

Where \oplus is the XOR operator.

Apply Hadamard operator to the state $|0\rangle$ and apply U operator, we get:

$$\begin{aligned} U: \frac{|0\rangle + |1\rangle}{\sqrt{2}} \oplus \frac{|0\rangle - |1\rangle}{\sqrt{2}} &\rightarrow \frac{|0\rangle}{\sqrt{2}} \otimes \frac{(-1)^{f(0)}(|0\rangle - |1\rangle)}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}} \otimes \frac{(-1)^{f(1)}(|0\rangle - |1\rangle)}{\sqrt{2}} \\ &= \left[\frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}} \right] \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \end{aligned}$$

If we apply Hadamard operator to the first qubit again, we get

$$U: \frac{|0\rangle + |1\rangle}{\sqrt{2}} \oplus \frac{|0\rangle - |1\rangle}{\sqrt{2}} \rightarrow \left[\frac{((-1)^{f(0)} + (-1)^{f(1)})|0\rangle + ((-1)^{f(0)} - (-1)^{f(1)})|1\rangle}{2} \right] \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \quad (4)$$

Equation (4) provides the superposition of states $f(0)$ and $f(1)$. We will get two possible outcome of $f(0)$ and two possible outcome of $f(1)$. If we measure first qubit we get:

$|0\rangle$ with probability $\left| \frac{(-1)^{f(0)} + (-1)^{f(1)}}{2} \right|^2$ and $|1\rangle$ with probability $\left| \frac{(-1)^{f(0)} - (-1)^{f(1)}}{2} \right|^2$

Hence if $f(0) = f(1)$ we get the state $|0\rangle$ with probability 1. Hence market price of the instrument will increase tomorrow.

Initialize the states $|x\rangle|y\rangle$ with inputs $|0\rangle$ and $|1\rangle$ respectively. Applying Hadamard gate in the second qubit we will get equal superposition of states $|0\rangle$ and $|1\rangle$.

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

After applying Hadamard gate the equation (1) becomes:

$$U: |x\rangle \oplus \frac{|0\rangle - |1\rangle}{\sqrt{2}} \rightarrow |x\rangle \oplus \frac{|0\oplus f(x)\rangle - |1\oplus f(x)\rangle}{\sqrt{2}}$$

(2)

Now put the possible values of $f(x)$ in the right side of equation (2)

Putting $f(x) = 0$ in the right side of equation (2) it becomes

$$|x\rangle \oplus \frac{|0\oplus 0\rangle - |1\oplus 0\rangle}{\sqrt{2}} = |x\rangle \oplus \frac{(-1)^0(|0\rangle - |1\rangle)}{\sqrt{2}}$$

Similarly, putting $f(x) = 1$ in the right side of equation (2) it becomes

$$|x\rangle \oplus \frac{|0\oplus 1\rangle - |1\oplus 1\rangle}{\sqrt{2}} = |x\rangle \oplus \frac{(-1)^1(|0\rangle - |1\rangle)}{\sqrt{2}}$$

Hence, equation (2) can be written as follows:

$$\begin{aligned} |x\rangle \oplus \frac{|0\oplus f(x)\rangle - |1\oplus f(x)\rangle}{\sqrt{2}} \\ = |x\rangle \oplus \frac{(-1)^{f(x)}(|0\rangle - |1\rangle)}{\sqrt{2}} \end{aligned}$$

(3)

Equation (3) is the result of Deutsch's algorithm: The operator U allows us to change the sign of the second qubit. Now if we allow the 1st qubit to be in equal super position of $|0\rangle$ and $|1\rangle$, we will be able to use the fact that U only needs to evaluate the function once to give properties of $f(0)$ and $f(1)$.

And if $f(0) \neq f(1)$ we get state $|1\rangle$ with probability 1. Hence market price of the instrument will decrease tomorrow.

Therefore, using Deutsch's algorithm, we only need to evaluate the function $f(x)$ once and get the result that allows us to determine with certainty if the value of $f(x)$ is equal for the two inputs or if $f(x)$ is not equal.

CONCLUSION

In this paper we have discussed working principle of quantum computer, quantum algorithm and a possible application of Deutsch's algorithm to financial market price

prediction. Here we have discussed an example that will predict the market price of a financial instrument evaluating the function only once using Deutsch's algorithm. Thus it is computationally much faster than the classical approach. This algorithm solves our problem efficiently but the drawback is in the implementation of the algorithm using quantum computer.

REFERENCE

- [1]. Aharonov, D.: Quantum computation. In D. Stauffer (Ed.), Annual Reviews of Computational Physics VI. Singapore: World Scientific, (1999)
- [2]. Grover, L.: Quantum Computing. The Sciences, July/August, 24-30, (1999)
- [3]. Lloyd, S.: Quantum-Mechanical Computers. Scientific American, 273 (4), 140-145,(1995)
- [4]. Rieffel, E. and Polak, W.: An Introduction to Quantum Computing for Non- Physicists. ACM Computing Surveys (CSUR) Volume 32, Issue 3, Pages: 300 – 335, (September 2000)
- [5]. Grover, L. K.:A fast quantum mechanical algorithm for database search. Proceedings of the 28th Annual Symposium on the Theory of Computing (STOC), p. 212-219, (1996)
- [6]. DiVincenzo, D. P.: Two-bit gates are universal for quantum computation. Phys. Rev. A, 51(2), p. 1015-1022, (1995)
- [7]. Nakahara, M., Tetsuo, O.: Quantum Computing, From Linear Algebra to Physical Realizations, CRC Pres, Boca Raton (2008)
- [8]. Nielsen, M.A, Chuang, I.L.: Quantum computation and Quantum Information Chuang. Cambridge University press (2002)
- [9]. Benioff, P. A.: Quantum Mechanical Hamiltonian Model of Turing Machine. J Stat. Phy, rev., vo1.29 (3), pp.515-546, (1982)
- [10]. Knights, M.: The Art of Quantum Computing. Engineering & Technology, 2 (1), p. 30, (2007)
- [11]. Shor, P.: Algorithms for Quantum Computation: Discrete Logarithms and Factoring. Proceedings of the 35th Annual Symposium on Foundations of Computer Science, p. 124-134, (1994)
- [12]. Deutsch, D.: Quantum theory, the Church-Turing principle and the universal quantum computer. Proc. R. Soc. Lond. A, 400:97, (1999)
- [13]. Shaktikanta Nayak, Sitakanta Nayak, J.P.Singh, A Simple Explanation of Search Technique in Quantum Framework, Journal of Global Research in Computer science.Vol-3, No-10,(2012)