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A Suzuki Type Unique Common Coupled Fixed Point Theorem in Metric Spaces

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Abstract: In this article, we study a unique common coupled fixed point theorem of Suzuki type for Jungck type mappings in metric spaces. Our result generalize and modify several comparable results in the literature.

Keywords: Coupled fixed point, metric space, weakly compatible maps.

Mathematics Subject Classification: 54H25, 47H10.

I. Introduction And Preliminaries

Banach contraction principle plays a very important role in nonlinear analysis and has many generalizations.Recently Suzuki [31] proved generalized versions of both Banach's and Edelstein's basic results and thus initiated a lot of work in this direction, for example refer [3,4,5,8,13,18-21,24-28,31,32] and the references in them .

In 2006, Bhaskar and Lakshmikantham [29]introduced the notion of a coupled fixed point in partially ordered metric spaces,also discussed some problems of the uniqueness of a coupled fixed point and applied their results to the problems of the existence and uniqueness of a solution for the periodic boundary value problems. Later several authors obtained coupled fixed point theorems in various spaces,for example refer [1,2,6,7,9-12,14-17,22,23,29,30,33-36] and the references in them.

The aim of this paper is to combine the ideas of coupled fixed points and Suzuki type fixed point theorems to obtain a unique common coupled fixed point theorem for Jungck type mappings in a metric space.

First we give the following theorem of Suzuki [31].

Theorem 1.1. Let (X, d) be a complete metric space and let T be a mapping on X, define a non-increasing function θ from $[0,1)$ into $(\frac{1}{2}, 1]$ by

$$\theta(r) = \begin{cases} 1 & \text{if } 0 \le r < \frac{\sqrt{5}-1}{2} \\ \frac{1-r}{r^2} & \text{if } \frac{\sqrt{5}-1}{2} \le r < \frac{1}{\sqrt{2}} \\ \frac{1}{1+r} & \text{if } \frac{1}{\sqrt{2}} \le r < 1 \end{cases}$$

assume that $r \in [0, 1)$, such that $\theta(r)\, d(x,Tx) \le d(x, y)$ implies $d(Tx,Ty) \le r d(x, y)$ for all $x, y \in X$, then there exists a unique fixed point z of T. Moreover, $\lim_{n\to\infty} T^n x = z$ for all $x \in X$.

Now we give some known definitions which are used to prove our main result.

Definition 1.2 (See [29]) Let X be a nonempty set. An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if $x = F(x, y)$ and $y = F(y, x)$.

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Definition 1.3 (See [17]) Let X be a nonempty set. An element $(x, y) \in X \times X$ is called

(i) a coupled coincidence point of $F : X \times X \to X$ and $f : X \to X$ if $fx = F(x, y)$ and $fy = F(y, x)$.

(ii) a common coupled fixed point of $F : X \times X \to X$ and $f : X \to X$ if $x = fx = F(x, y)$ and $y = fy = F(y, x)$.

Definition 1.4 (See [17]) Let X be a nonempty set and $F : X \times X \to X$ and $f : X \to X$. The pair (F, f) is said to be W-weakly compatible if $f\left(F\ (x, y)\right) = F(fx, fy)$ whenever $fx = F(x, y)$ and $fy = F(y, x)$ for some $(x, y) \in X \times X$.

Now we prove our main result.

II. MAIN RESULT

Theorem 2.1. Let (X, d) be a metric space and $F : X \times X \to X$ and $f : X \to X$ be mappings satisfying the following:

(2.1.1) If there exists a constant $\theta \in [0,1)$ such that

$$\eta(\theta)d(fx, F(x, y) \le \max \{d(f\,x,fu),d(fy,fv),d(fx,F(x,y)),d(fy.F(y,x))\} \text{ implies}$$

$$d\left(F(x, y), F(u,v)\right) \le \theta \max \begin{Bmatrix} d(fx, fu), d(fy, fv), d(fx, F(x, y), d(fy, F(y,x), \\ d(fu, F(u,v), d(fv, F(v,u),d(fu, F(x, y), d(fv, F(y,x) \end{Bmatrix}$$

for all $x, y, u, v \in X$, where $\eta : [0,1) \to [\frac{1}{2}, 1)$ defined by $\eta(\theta) = \dfrac{1}{1+\theta}$ is a strictly decreasing function,

(2.1.2) $F\left(X \times X\right) \subseteq f\left(X\right)$ and $f(X)$ is complete,

(2.1.3) F and f are W-compatible.

Then F and f have a unique common coupled fixed point.

Proof. Let $x_0, y_0 \in X$. Then from (2.1.2) there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $fx_{n+1} = F\left(x_n, y_n\right)$ and $fy_{n+1} = F\left(y_n, x_n\right)$ for all $n = 0, 1, 2, 3 \cdots$

Case(i): Assume that $fx_n \ne fx_{n+1}$ or $fy_n \ne fy_{n+1}$ for all n. --- (1)

Since

$$\eta(\theta)d(fx_0, F(x_0, y_0)) = \eta(\theta)d(fx_0, fx_1) \le d(fx_0, fx_1)$$
$$\le \max\{d(fx_0, fx_1), d(fy_0, fy_1), d(fx_0, F(x_0, y_0)), d(fy_0, F(y_0, x_0))\},$$

by (2.1.1), we have

$$d(fx_1, fx_2) = d(F(x_0, y_0), F(x_1, y_1))$$
$$\le \theta \max \begin{Bmatrix} d(fx_0, fx_1), d(fy_0, fy_1), d(fx_0, fx_1), d(fy_0\ , fy_1), \\ d(fx_1, fx_2), d(fy_1\ , fy_2), d(fx_1, fx_1), d(fy_1\ , fy_1) \end{Bmatrix} \text{ ---------------------------- (2)}$$
$$\le \theta \max\{d(fx_0, fx_1), d(fy_0, fy_1), d(fx_1, fx_2), d(fy_1\ , fy_2)\}$$

Since

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$$\eta(\theta)d(fy_0, F(y_0,x_0)) = \eta(\theta)d(fy_0, fy_1) \le d(fy_0, fy_1)$$
$$\le \max\{d(fx_0, fx_1), d(fy_0, fy_1), d(fx_0, F(x_0,y_0)), d(fy_0, F(y_0,x_0))\},$$

by (2.1.1), we have

$$d(fy_1, fy_2) = d(F(y_0,x_0), F(y_1,x_1))$$
$$\le \theta \max\begin{Bmatrix} d(fy_0, fy_1), d(fx_0, fx_1), d(fy_0, fy_1), d(fx_0, fx_1), \\ d(fy_1, fy_2), d(fx_1, fx_2), d(fy_1, fy_1), d(fx_1, fx_1) \end{Bmatrix} \text{------------------------------- (3)}$$
$$\le \theta \max\{d(fx_0, fx_1), d(fy_0, fy_1), d(fx_1, fx_2), d(fy_1, fy_2)\}$$

Now from (2) and (3), we have

$$\max\{d(fx_1, fx_2), d(fy_1, fy_2)\} \le \theta \max\{d(fx_0, fx_1), d(fy_0, fy_1), d(fx_1, fx_2), d(fy_1, fy_2)\} \text{ ------------ (4)}$$

If $\max\{d(fx_0, fx_1), d(fy_0, fy_1)\} \le \max\{d(fx_1, fx_2), d(fy_1, fy_2)\}$ then from (4), we have $fx_1 = fx_2$ and $fy_1 = fy_2$. It is a contradiction to (1).

Hence from (4), we have $\max\{d(fx_1, fx_2), d(fy_1, fy_2)\} \le \theta \max\{d(fx_0, fx_1), d(fy_0, fy_1)\}$.

Continuing in this way, we get

$$\max\{d(fx_n, fx_{n+1}), d(fy_n, fy_{n+1})\} \le \theta \max\{d(fx_{n-1}, fx_n), d(fy_{n-1}, fy_n)\}$$
$$\le \theta^2 \max\{d(fx_{n-2}, fx_{n-1}), d(fy_{n-2}, fy_{n-1})\}$$
$$\vdots$$
$$\le \theta^n \max\{d(fx_0, fx_1), d(fy_0, fy_1)\}.$$

Thus $d(fx_n, fx_{n+1}) \le \theta^n \max\{d(fx_0, fx_1), d(fy_0, fy_1)\}$ *and*

$$d(fy_n, fy_{n+1}) \le \theta^n \max\{d(fx_0, fx_1), d(fy_0, fy_1)\}$$

For $m > n$, consider

$$d(fx_m, fx_n) \le d(fx_n, fx_{n+1}) + d(fx_{n+1}, fx_{n+2}) + \ldots + d(fx_{m-1}, fx_m)$$
$$\le (\theta^n + \theta^{n+1} + \ldots + \theta^{m-1}) \max\{d(fx_0, fx_1), d(fy_0, fy_1)\}$$
$$\le \tfrac{\theta^n}{1-\theta} \max\{d(fx_0, fx_1), d(fy_0, fy_1)\}$$
$$\to 0 \quad as \quad n \to \infty,\ m \to \infty.$$

Hence $\{fx_n\}$ is a Cauchy sequence in $f(X)$. Similarly we can show that $\{fy_n\}$ is a Cauchy sequence in $f(X)$.

Since $f(X)$ is complete, there exist $p, q, z_1, z_2 \in X$ such that $fx_n \to p = fz_1$ and $fy_n \to q = fz_2$.

Since $fx_n \to p$ and $fy_n \to q$, we may assume that $fx_n \ne p$ and $fy_n \ne q$ for infinitely many n.

Claim: $\max\{d(fz_1, F(x,y)), d(fz_2, F(y,x))\} \le \theta \max\{d(fz_1, fx), d(fz_2, fy), d(fx, F(x,y)), d(fy, F(y,x))\}$

for all $x, y \in X$ with $fx \ne fz_1$ and $fy \ne fz_2$.

Let $x, y \in X$ with $fx \ne fz_1$ and $fy \ne fz_2$.

Then there exists a positive integer n_0 such that for $n \ge n_0$ we have $d(fz_1, fx_n) \le \frac{1}{3} d(fz_1, fx)$ and $d(fz_2, fy_n) \le \frac{1}{3} d(fz_2, fy)$.

Now for $n \ge n_0$,

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$$\begin{aligned}\eta(\theta)d(fx_n,F(x_n,y_n)) &\le d(fx_n,F(x_n,y_n))\\ &= d(fx_n,fx_{n+1})\\ &\le d(fx_n,fz_1)+d(fz_1,fx_{n+1})\\ &\le \frac{1}{3}d(fx,fz_1)+\frac{1}{3}d(fz_1,fx)\\ &= d(fx,fz_1)-\frac{1}{3}d(fx,fz_1)\\ &\le d(fz_1,fx)-d(fx_n,fz_1)\\ &\le d(fx,fx_n)\\ &\le \max\{d(fx_n,fx),d(fy_n,fy),d(fx_n,F(x_n,y_n)),d(fy_n,F(y_n,x_n))\}.\end{aligned}$$

From (2.1.1), we have

$$d\big(F(x_n,y_n),F(x,y)\big)\le\theta\max\left\{\begin{matrix}d(fx_n,fx),d(fy_n,fy),d(fx_n,fx_{n+1}),\ d(fy_n,fy_{n+1}),\\ d(fx,F(x,y)),d(fy,F(y,x)),d(fx,fx_{n+1}),d(fy,fy_{n+1})\end{matrix}\right\}$$

Letting $n\to\infty$, we get

$$d(fz_1,F(x,y))\le\theta\max\{d(fz_1,fx),d(fz_2,fy),d(fx,F(x,y)),d(fy,F(y,x))\}.$$

Similarly we can show that

$$d(fz_2,F(y,x))\le\theta\max\{d(fz_2,fy),d(fz_1,fx),d(fx,F(x,y)),d(fy,F(y,x))\}.$$

Thus

$$\max\{d(fz_1,F(x,y)),d(fz_2,F(y,x))\}\le\theta\max\left\{\begin{matrix}d(fz_1,fx),d(fz_2,fy)\\ d(fx,F(x,y)),d(fy,F(y,x))\end{matrix}\right\} \text{ ------------------- (5)}$$

Hence the claim. Now consider

$$\begin{aligned}d(fx,F(x,y)) &\le d(fx,fz_1)+d(fz_1,F(x,y))\\ &\le d(fx,fz_1)+\theta\max\{d(fz_1,fx),d(fz_2,fy),d(fx,F(x,y)),d(fy,F(y,x))\}\ \text{ from (5)}\\ &\le(1+\theta)\max\{d(fz_1,fx),d(fz_2,fy),d(fx,F(x,y)),d(fy,F(y,x))\}.\end{aligned}$$

Thus

$$\eta(\theta)d(fx,F(x,y))\le\max\{d(fx,fz_1),d(fy,fz_2),d(fx,F(x,y),d(fy,F(y,x))\}.$$

Hence from (2.1.1), we have

$$d\big(F(x,y),F(z_1,z_2)\big)\le\theta\max\left\{\begin{matrix}d(fx,fz_1),d(fy,fz_2),d(fx,F(x,y)),d(fy,F(y,x)),\\ d(fz_1,F(z_1,z_2)),d(fz_2,F(z_2,z_1)),d(fz_1,F(x,y)),d(fz_2,F(y,x))\end{matrix}\right\} \text{ ---- (6)}$$

Now

$$\begin{aligned}d(fz_1,F(z_1,z_2)) &= \lim_{n\to\infty}d(fx_{n+1},F(z_1,z_2))\\ &= \lim_{n\to\infty}d(F(x_n,y_n),F(z_1,z_2))\\ &\le \lim_{n\to\infty}\theta\max\left\{\begin{matrix}d(fx,fz_1),d(fy,fz_2),d(fx,F(x,y)),d(fy,F(y,x)),\\ d(fz_1,F(z_1,z_2)),d(fz_2,F(z_2,z_1)),d(fz_1,F(x,y)),d(fz_2,F(y,x))\end{matrix}\right\},\ \text{from (6)}\\ &= \theta\max\{d(fz_1,F(z_1,z_2)),d(fz_2,F(z_2,z_1))\}.\end{aligned}$$

Similarly, we can have

$$d(fz_2,F(z_2,z_1))\le\theta\max\{d(fz_1,F(z_1,z_2)),d(fz_2,F(z_2,z_1))\}.$$

Thus $\max\{d(fz_1,F(z_1,z_2)),d(fz_2,F(z_2,z_1))\}\le\theta\max\{d(fz_1,F(z_1,z_2)),d(fz_2,F(z_2,z_1))\}$

so that $fz_1=F(z_1,z_2)$ and $fz_2=F(z_2,z_1)$.

Thus (z_1,z_2) is a coupled coincidence point of F and f. Since the pair (F,f) is W-compatible, we have

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$$fp = f^2 z_1 = f(F(z_1,z_2) = F(fz_1,fz_2) = F(p,q) \text{ ------------------------ } (7)$$
$$fq = f^2 z_2 = f(F(z_2,z_1) = F(fz_2,fz_1) = F(q,p) \text{ ------------------------ } (8)$$

Now

$$\eta(\theta)d(fp,F(p,q)) = 0 \le \max\{d(fz_1,fp), d(fz_2,fq), d(fp,F(p,q)), d(fq,F(q,p))\}.$$

Hence from (2.1.1) we have

$$d(fp,fz_1) = d(F(p,q),F(z_1,z_2))$$
$$\le \theta \max\left\{\begin{matrix} d(fp,fz_1), d(fq,fz_2), d(fp,F(p,q)), d(fq,F(q,p)), \\ d(fz_1,F(z_1,z_2)), d(fz_2,F(z_2,z_1)), d(fz_1,F(p,q)), d(fz_2,F(q,p)) \end{matrix}\right\}$$
$$= \theta \max\{d(fp,fz_1), d(fq,fz_2)\}.$$

Similarly, we have

$$d(fq,fz_2) \le \theta \max\{d(fp,fz_1), d(fq,fz_2)\}.$$

Thus

$$\max\{d(fp,fz_1), d(fq,fz_2)\} \le \theta \max\{d(fp,fz_1), d(fq,fz_2)\}.$$

Hence $p = fz_1 = fp$ and $q = fz_2 = fq$.

Now from (7) and (8), we have (p,q) is a common coupled fixed point of f and F.

Suppose (p',q') is another common coupled fixed point of F and f. Now consider

$$\eta(\theta)d(p,F(p,q)) = 0 \le \max\{d(fp,fp'), d(fq,fq'), d(fp,F(p,q)), d(fq,F(q,p))\}.$$

By (2.1.1), we have

$$d(p,p') = d(F(p,q),F(p',q')) \le \theta \max\{d(p,p'), d(q,q')\}.$$

Similarly, we can show that $d(q,q') \le \theta \max\{d(p,p'), d(q,q')\}$.

Thus $\max\{d(p,p'), d(q,q')\} \le \theta \max\{d(p,p'), d(q,q')\}$.

Hence $p = p'$ and $q = q'$.

Thus (p,q) is the unique common coupled fixed point of F and f.

Case(ii): If $fx_n = fx_{n+1}$ and $fy_n = fy_{n+1}$ for some n then $fx_n = F(x_n,y_n)$ and $fy_n = F(y_n,x_n)$ so that (x_n,y_n) is a coupled coincidence point of F and f. Now proceeding as in case (i) with $fx_n = p$ and $fy_n = q$, we can show that (p,q) is the unique common coupled fixed point of F and f.

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