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A Transfer Function-Autoregressive Noise Model of Naira Exchange Rates for Us Dollar and Swiss Franc

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ABSTRACT

This work aims at modelling the Naira exchange rates for US Dollars X_t and Swiss Franc Y_t using a transfer function (TF) technique. Data for the two currencies were obtained from the Central Bank of Nigeria (for a period of 53 years). After obtaining stationarity of the two series using appropriate transformation, the input series x_t was pre-whitened to remove the spurious correlation effect. The output series y_t was also pre-whitened and the cross correlation between the pre-whitened input (α_t) and output (β_t) was examined. From the behavior of the cross correlation, rational polynomial representation $\frac{\omega(B)}{\delta(B)}$ of the dynamic transfer function was obtained. The estimated noise $(\hat{n}_t)$ was found to be auto correlated. Thus, the noise was modelled separately using Box and Jenkins autoregressive (AR) method. This provided the missing component of the TF model which was used to fit the overall model. The resulted model underwent a diagnostic check and was found to be appropriate. Hence forecast was generated.

INTRODUCTION

Basically, no country is completely economically independent. Every country must depend on one or several countries for her economic growth. All nations are inter-dependent, because they have limited resources and have to trade with each other to satisfy their wants. This implies that the analysis of economic variables such as gross domestic product (GDP), inflation, exchange rates etc. are of paramount importance to any nation's growth. Unfortunately in most African countries, less attention is paid to the study and monitoring of the rise and falls of these essential and sensitive variables. A volatile or inappropriate exchange rate has been a major hindrance to the growth of many African countries which Ghana is inclusive Appiah and Adetunde [1].

Exchange rates vary largely according to extent and nature of each country's exposure to trade and global financial markets. In some stable economies, exchange rates of currencies compete favorably while the developing economies appear to be influenced heavily by the developed ones. This is because, the less developed countries receives less attention concerning exchange rate compared to the developed or industrialized economies [2].

The currencies of many sub-Saharan African countries faced depreciations with respect to US Dollar at the onset of the global financial crisis. According to Ltaifa, et al. [3], five countries: Ghana, Kenya, Nigeria, Uganda and Zambia depreciated by at least 20% between June 2008 and March 2009; while Tanzania and Rwanda depreciated by10% or less during same period.

According to Jeffrey [4], Argentina being the victim of the worst market financial crisis in 2001; experienced problems in the late 1990's and the situation became severe because of its link with a particular currency, the US Dollar. The Dollar gained value over other major currencies beginning in the mid-1995, and caused the market for Argentina's important agricultural export products (wheat, meat and soya beans) to decline sharply. Thus, the decline in the prices of these commodities expressed in Dollar was particularly dramatic. All these put together, led to sharp increase in the ratio of debt to exports.

Some earlier studies suggest that instability in exchange rate has the potential to affect a country's economic performance. In Nigeria, the value of Naira has depreciated significantly compared with other currencies due to poor economic management, political system and other unknown factors. However, a close observation of the Naira exchange rate for US Dollar and Swiss Franc shows that the two currencies are highly related. Thus, studying the behavior and relationship of these two currencies in relation to Naira can provide useful information for the economy of the two Countries (USA and Switzer Land) and Nigeria at large. This work therefore, intends to build a model that establishes the relationship between the two currencies (US Dollar and Swiss Francc) in relation to Nigeria's currency (Naira). The model is called the transfer function model.

Transfer function model (TFM) is a statistical model describing the relationship between an output variable and one or more input variables [5]. TFM can be a single equation or multiple equation model, which the latter could be referred to as a simultaneous transfer function (STF) model [6-8]. Some authors will prefer to distinguish the two models as single-input and multiple-input transfer function models. In most applications, linear equation is used to describe the relationship resulting from the distributed-lag model.

Elkhtem, et al. [9] considered the nature of crude oil as a mixture of hydrocarbons with different boiling temperatures. Control became essential for the fractionation column to keep products at the limitations. The paper revealed the identification of transfer function for relevant different developed control strategy and relevant transfer function were identified using MATLAB Software.

A Work on forecasting foreign exchange by Znaczko [10] investigated a transfer function model with multiple variables. The study revealed that the best forecast of foreign exchange rates depends on current and past prices.

Kannan and Farook [11] fitted a transfer function model to global warming entities such as atmospheric temperature, and atmospheric CO_2 emissions. A strong relationship was demonstrated between annual atmospheric CO_2 emissions and present/ past atmospheric temperature.

Practically, the output is not a deterministic function of the input. Transfer function model is quite different from the ARIMA model. The latter is a univariate time series model while the transfer function model is a multivariate time series model. The ARIMA model relates the series only to its past, but beyond this indication, the transfer function model will relate the series to other time series [12]. Due to the close relationship with the regression models, the transfer function models are also referred to as dynamic regression models [13]. The transfer function models can use more than one explanatory variable, but the explanatory variables must be linearly independent of each other [14].

Transfer function approach as a method of data analysis, has been in use for some decades. Virtually every field of study has demonstrated the importance and relevance of the transfer function approach especially in its ability to enhance forecast based on relating different series. In a stable economic trend, such model can enable a researcher or planner to predict favorable chances an indigenous economy stands amidst higher ones in the global market.

In time series analysis, the Box-Jenkins method describes the transfer function model as a model that predicts future values of a time series (output series) from past values of same series and one or more related series (input series).

In this work, however, we intend to apply the Box-Jenkins method and the pre-whitening approach to build a transfer function-AR noise model of Naira exchange rates for the US Dollar and Swiss Franc. In our method, the leading indicator is identified and the noise component is modeled separately and added to the dynamic relationship between the input and output series. The resulting noise is found to satisfy its basic assumptions [$\varepsilon_t \sim \mathrm{NIID}(0, \acute{a}^2)$] thus showing that the model is adequate.

METHODOLOGY

Let X_t and Y_t be two time series.

Stationarity

The time series X_t is said to be stationary if the statistical properties are essentially constant through time. In other words, $E(X_t) = \mu$ and $Var(X_t) = \sigma^2$. In this work, stationarity is obtain by differencing. That is, $x_t = X_t - X_{t-1}$.

Pre-Whitening

Pre-whitening approach involves finding autoregressive integrated moving average (ARIMA) model for X_t that yields white noise residuals while the functional relationship between the X_t and Y_t is still maintained.

For this work, an ARIMA (p,d,o) model is identified and we have

$$x_t = \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + \alpha_t$$

Rewriting the equation, the residuals or the pre-whitened input $\hat{\alpha}_t$ is obtained as

$$\hat{\alpha}_t = x_t - \{\phi_1 x_{t-1} +\}$$

The y_t is substituted for x_t to get the filtered equation for the pre-whitened output as:

$\hat{\beta}_t = y_t - \{\phi_1 y_{t-1} + \ldots + \phi_p y_{t-p}\}$

By pre-whitening, cross correlation accurately reflects the structure of the impulse function [15]. Here, α and β represent the pre-whitened input and output series respectively.

Cross Correlation

The cross correlation for lag k is given as

$$r(xy) = \frac{C_{xy}}{S_x S_y}$$

$$C_{xy} = \frac{1}{n}\sum_{t=1}^{n-k}(x_t - \bar{x})(y_{t+k} - \bar{y}) \quad k = 0, 1, 2 \ldots$$

Wher $\bar{x}$ and $\bar{y}$ are the sample means of x_t and y_t, s_t and s_y are the sample standard deviations respectively.

Transfer Function Model

The dynamic relationship between input series Y_t and output series X_t is usually approximated by a linear transfer function

$$y_t = v_0 x_t + v_1 x_{t-1} + \ldots + n_t,$$

$$= v(B) x_t + n_t$$

Where $v_j^{'}s$ are the transfer function weights and n_t is a noise term. Estimates of the transfer function weights can be calculated from

$$\hat{v}_j = \frac{r_{\hat{a}\hat{a}}(j) s_{\hat{a}}}{s_\alpha}, \qquad j = 0, \ldots, k \;; \tag{3.13}$$

Where s_α and s_β are the estimated standard deviations of the pre-whitened input and the output data respectively. $r_{\alpha\beta}$ is the estimated cross correlation between the pre-whitened input and output. Insignificant weights are rounded off to zero. B denotes the backshift operator such that $B^m X_t = X_{t-m,}$

The function v (B) determines the impact of input X_t on y_t; and

$$v(B) = \frac{\omega(B)}{\delta(B)} B^b$$

Thus, y_t can be represented as

$$y_t = \frac{\omega(B) B^b}{\delta(B)} x_t + n_t$$

$$= \frac{\omega(B)}{\delta(B)} x_{t-b} + n_t \tag{3.15}$$

According to Box and Jenkins [16], the parameters in v (B) are estimated by the equation

$$\delta(B) v(B) = \omega(B)$$

Explicitly, it can be expressed as,

$$\left(1 - \delta_1 B - \ldots - \delta_r B^r\right)\left(v_0 + v_1 B + v_2 B^2 + \ldots\right) = \left(\omega_0 - \omega_1 B - \ldots - \omega_s B^s\right) B^b$$

The parameters r, b, s, represent the order of the numerator polynomial , the delay parameter that indicates the time lag until input affects the output also called dead-time or delay time, and order of the denominator polynomial respectively .

The Autoregressive (AR)-Noise Model

In the model identification, v (B) parameters are obtained and fitted; and the noise term n_t is added. In this concept, however, it is believed that the noise n_t could be serially correlated; thus violating its assumption. The noise is then modeled separately using Box and Jenkins method as:

$$\phi(B) n_t = \varepsilon_t \Rightarrow n_t = \frac{\varepsilon_t}{\phi(B)}$$

Where,

$$\phi(B) = \left(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p\right)$$

and p is the autoregressive order of n_t and ϕ_i's are the autoregressive parameters.

Thus the resulting model is:

$y_t = v(B)x_t + \frac{\varepsilon_t}{\phi(B)}$ and ε_t is found to be serially uncorrelated as expected.

Data Analysis and Results

The analysis of the data available for this thesis was carried out using Minitab software. Data used was secondary data with fifty three observations each (1960-2013). We intend to build a transfer function -AR noise model which could be used for future forecast of the naira exchange rate for the two currencies. Let $\mathbf{X_t}$ denotes the Naira exchange rate of US Dollar and $\mathbf{Y_t}$, the Naira exchange rate of Swiss Franc

In the language of our study, $\mathbf{X_t}$ is the input series and $\mathbf{Y_t}$ the output series.

Raw Data Plot

Figure 1 (appendix) demonstrates the combined raw data plot for $\mathbf{X_t}$ and $\mathbf{Y_t}$.

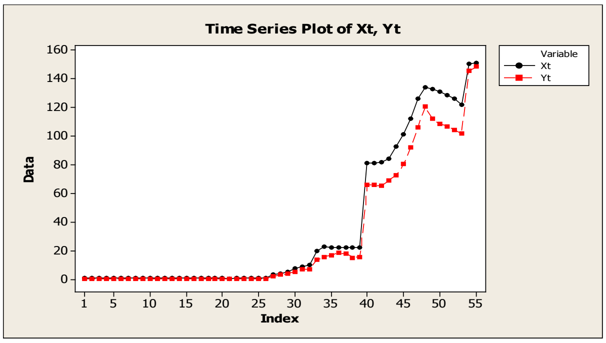

Figure 1. Raw data plot of X_t and Y_t.

From these plots, it is observed that the series are not stationary. This demanded for an appropriate transformation to make them stationary.

Correlation of the Two Series

The correlation between $\mathbf{X_t}$ and $\mathbf{Y_t}$ gives 0.996. This value indicates a strong positive relationship between the two series. This is also evident in **Figure 1.**

Differencing

Xt and **Yt** were both differenced once to obtain stationarity.

Model Identification for the Input Series X_t

An ARIMA (1,1,0) was identified for the $\mathbf{X_t}$ resulting in the model:

$$(1-0.086B)(1-B)X_t = \alpha_t$$

Pre-Whitening of X_t and Y_t

From the above model, the pre-whitened X_t is:

$$\hat{\alpha}_t = X_t - X_{t-1} - 0.0862X_{t-1} + 0.086X_{t-2}$$

and that of Y_t

$$\hat{\beta}_t = Y_t - Y_{t-1} - 0.0862Y_{t-1} + 0.086Y_{t-2}$$

Cross Correlation Function for α_t and β_t

The cross correlation of the pre-whitened series $r_{\alpha\beta}$ are seen in **Figure 2** below. Since the $r_{\alpha\beta}$ at lag 0 and 1 are not statistically significant, the weights v_0 and v_1 are systematically equal to zero. Thus from the figure; b=2, s=1 and r=2. In addition, since $r_{\alpha\beta}(-k)$ are not statistically different from zero; it means the present values of y_t are related to the past values of X_t and not otherwise. Hence, X_t is the leading indicator. The four transfer function significant weights are shown in **Table 1** below with their respective cross correlations and lags.

Identification of the Transfer Function Parameters

We had earlier expressed in methodology that:. $\delta(B)v(B) = \omega(B)$

That is, $(1-\delta_1 B - \ldots - \delta_r B^r)(v_0 + v_1 B + v_2 B^2 + \ldots) = (\omega_0 - \omega_1 B - \ldots - \omega_s B^s)B^b$

Substituting b=2, s=1 and r=2, we have

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
-17  -0.017                          X
-16  -0.028                         XX
-15  -0.058                         XX
-14  -0.040                         XX
-13   0.029                          XXX
-12  -0.071                       XXXX
-11  -0.044                         XX
-10  -0.013                          X
 -9  -0.039                        XX
 -8  -0.079                       XXX
 -7   0.157                         XXXXX
 -6   0.233                         XXXXXX
 -5   0.228                         XXXXX
 -4  -0.086                       XXX
 -3  -0.049                        XX
 -2  -0.024                        XX
 -1  -0.071                       XXX
  0  -0.067                       XXX
  1   0.023                         XX
  2   0.837                         XXXXXXXXXXXXXXXXXXX
  3   0.998                         XXXXXXXXXXXXXXXXXXXXXX
  4   0.776                         XXXXXXXXXXXXXXXX
  5   0.629                         XXXXXXXXXXXXXX
  6   0.191                         XXXX
  7  -0.137                       XXX
  8  -0.220                    XXXXXX
  9  -0.164                      XXXX
 10   0.074                          XXX
 11   0.036                          XX
 12  -0.013                          X
 13  -0.044                         XX
 14  -0.074                        XXX
 15   0.023                          XX
 16  -0.004                          X
 17  -0.060                        XXX
```

Figure 2. Cross Correlation function (CCF) for α_t, β_{t+k}

Table 1. Cross Correlation and Transfer Function Weight.

Lag	Cross Correlation	Weight $\hat{V}_j$
0	0	0
1	0	0
2	0.837	0.5750
3	0.998	0.6856
4	0.776	0.5331
5	0.629	0.4321
6	0	0

$$\left(1-\delta_1 B-\delta_2 B^2\right)\sum_{j=0}^{\infty} v_j B^j=\left(\omega_0-\omega_1\right)B^2$$

$$\Rightarrow \sum_{j=0}^{\infty} v_j B^j-\delta_1\sum_{j=0}^{\infty} v_j B^{j+1}-\delta_2\sum_{j=0}^{\infty} v_j B^{J+2}=\left(\omega_0 B^2-\omega_1 B^3\right)$$

Solving the above equation, we have

δ_1=0.7157, δ_2=0.5750, ω_0=0.5750; ω_0= - 0.2741,

$$\Rightarrow v(B)=\frac{0.5750B^2+0.274B^3}{1-0.7157B-0.0737B^2}$$

Estimation of the Noise n_t

The noise $\hat{n}_t$ is estimated as

$$\hat{n}_t = y_t - \hat{y}_t$$

$$= y_t - (\hat{\delta}_1\hat{y}_{t-1}+\hat{\delta}_2\hat{y}_{t-2}\ldots+\hat{\delta}_r\hat{y}_{t-r}+\hat{\omega}_0 x_{t-b}-\hat{\omega}_1 x_{t-b-1}\ldots-\hat{\omega}_s x_{t-b-s})$$

$$= y_t - \left(0.715\hat{y}_{t-1}+0.0737\hat{y}_{t-2}+0.575x_{t-2}-0.274x_{t-3}\right)$$

Model Identification for the Noise Series n_t

Autocorrelation and partial autocorrelation function for n_t: The autocorrelation function decays exponentially to 0, while the partial autocorrelation function cut off after lag 1 **(Figures 3 and 4).** Thus the identified model is ARIMA (1,0,0) which is equivalent to AR(1) **(Table 2).**

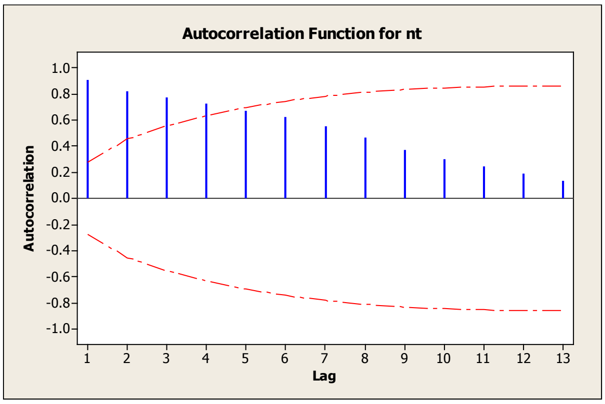

Figure 3. Autocorrelation function for $n_{t.}$

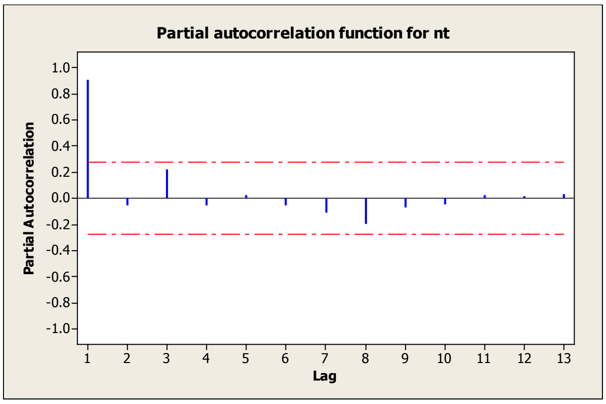

Figure 4. Partial autocorrelation function for $n_{t.}$

Table 2. The parameter estimates for the model.

Parameters	Estimate	p-Value
ω_0	0.52	0.000
ω_1	-0.24	0.009
δ_1	0.69	0.000
δ_2	0.04	0.003
φ_1	0.55	0.000

Writing the model explicitly, we have

$$\phi_p(B)n_t = \varepsilon_t$$

So that $(1-\phi_1 B)n_t = \varepsilon_t$

and $n_t = \frac{1}{1-\phi_1 B}\varepsilon_t$

The Transfer Function-AR Noise Model

Combining the two models, we have

$$y_t = v(B)x_t + n_t$$

$$y_t = \frac{\omega(B)B^b}{\delta(B)}x_t + n_t$$

$$y_t = \frac{\omega_0 - \omega_1 B}{1-\delta_1 B - \delta_2 B^2}x_{t-b} + \frac{1}{1-\phi_1 B}\varepsilon_t$$

The final transfer function-AR noise model is obtained as;

$$y_t = \frac{\omega_0 - \omega_1 B}{1-\delta_1 B - \delta_2 B^2}x_{t-2} + \frac{1}{1-\phi_1 B}\varepsilon_t$$

Where $x_t = X_t - X_{t-1}$ and $y_t = Y_t - Y_{t-1}$

DIAGNOSIS

To check for model adequacy, the following diagnostic check was applied.

Autocorrelation Function for the Residual ε_t

The autocorrelation function of the residual term ε_t **(Figure 5)** below demonstrates no particular pattern or spikes. No autocorrelation is significant. This indicates that the residual follows a white noise process. Hence the fitted model is adequate.

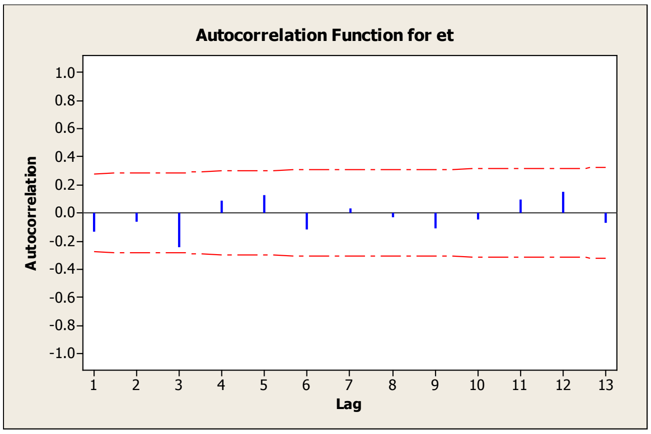

Figure 5. Partial autocorrelation function for $\varepsilon_{t.}$

Forecasting

Having tested the adequacy of the transfer function AR noise model and was found to be appropriate; next is to generate forecasts. The forecasts of the next ten years are presented in **Table 3.**

Table 3. Forecasts from period 54 at 95 percent limits.

Period	Forecast	Lower	Upper
54	148.103	132.110	163.096
55	148.152	127.194	168.110
56	148.270	124.189	170.351
57	149.101	121.117	176.085
58	149.206	121.260	176.152
59	149.281	121.401	176.161
60	149.474	121.672	176.276
61	150,690	119.310	181.070
62	152.215	119.517	183.913
63	152.410	119.708	184.112

SUMMARY / CONCLUSION

This work provides a solution to a dynamic relationship where the residual fails to follow a white noise process or violate its usual assumptions. The work modeled the residual obtained from fitting the transfer function (TF) model separately. The ARIMA model obtained from the TF noise is added to the major component of the model resulting in transfer function autoregressive-noise model. Data from naira exchange rate of US Dollars and Swiss Franc (1970-2013) was used to demonstrate the workability of the model. The model was found to be appropriate and forecasts for the Naira exchange of Swiss Franc were generated using the model.

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