

# Analysis of micro-Doppler Radar Signatures of Rotating Targets Using Gabor-Wigner Transform

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**Abstract**— Micro-Doppler signatures provide unique information about properties of the target. These micro-Doppler features can be used for real time target recognition in military applications and surveillance operations. In this paper, we present Gabor-Wigner transform for extracting micro-Doppler features from the radar returns. The effectiveness of the Gabor-Wigner transform in extracting micro-Doppler features has been compared with short-time Fourier transform, Wigner Distribution and S-method. The efficiency of the Gabor-Wigner transform in micro-Doppler feature extraction is demonstrated by applying it to different experimental data sets.

**Keywords**— Time-Frequency analysis, micro-Doppler effect, Short-Time Fourier Transform, Wigner Distribution, S-method, Gabor-Wigner Transform

## I. INTRODUCTION

In many cases, a target or structure on a target may have rotations or vibrations. These micro-motions of the target may induce frequency modulation on the radar returned signal. This may lead to generation of side bands about the center of the Doppler shifted carrier frequency. The frequency modulation due to target's rotating or vibrating motion is called micro-Doppler (m-D) effect. [1]. Joint time-frequency methods can be used to extract m-D features [2]-[14]. The short-time Fourier transform (STFT) is the most widely used time-frequency representation. In STFT, the time and frequency resolution depends on the size of the window function used in calculating STFT. In order to improve time-frequency concentration, various quadratic time-frequency methods have been proposed. Wigner distribution (WD) is a well-known time-frequency method among the quadratic time-frequency methods, however it suffers from cross-term interferences. Stankovic et.al proposed S-method for improved distribution concentration in the time-frequency plane. S-

method (SM) has a property that, under certain assumptions, its value for multicomponent signals is nearly equal to the sum of the Wigner distributions of individual signal components [3]. The S-method is also numerically very simple and requires just a few more operations than the standard FT-based algorithm [4]. In 2007, S.C Pei et.al proposed Gabor-Wigner transform for the suppression of cross-terms in the output of WD function [5]. In this paper, we use Gabor-Wigner transform for the extraction of m-D features.

## II. GABOR-WIGNER TRANSFORM

STFT is the simplest time-frequency representation introduced for time localization of the frequency contents of a signal by using a suitable window. Basic principle behind STFT is segmenting the signal into narrow time intervals using a window function and taking Fourier transform of each segment.

$$STFT(t, \omega) = \int_{-\infty}^{\infty} x(\tau) \gamma(t - \tau) \exp(-j\omega\tau) d\tau \quad (1)$$

Where  $x(t)$  is the signal to be analyzed and  $\gamma(t-\tau)$ , windowing function centered at  $t = \tau$ . STFT has limited time frequency resolution which is determined by the size of the window used. The STFT with Gaussian window is called a Gabor transform [5]. Mathematically Gabor transform (GT) of  $x(t)$  is defined as:

$$GT(t, \omega) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(\tau-t)^2}{2}} e^{-j\omega(\tau-\frac{t}{2})} x(\tau) d\tau \quad (2)$$

The WD was originally developed in the area of quantum mechanics by Wigner [6] and then introduced for signal analysis by Ville [7]. It is defined as:

$$WVD(t, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) e^{-j\omega\tau} d\tau \quad (3)$$

Compared to STFT, WD has much better time and frequency resolution. But the main drawback of the WD is the cross-term interference. This interference phenomenon shows frequency components that do not exist in reality and considerably affect the interpretation of the time frequency plane. Cross-terms are oscillatory in nature and are located midway between the two components [8]. Presence of cross-terms severely limits the practical applications of WD. S. C. Pei and J. J. Ding proposed Gabor-Wigner (GW) transform that is defined as a multiplication of STFT with WD, can significantly remove the cross-terms without affecting the quality of auto-terms much[5]. Since cross-terms do not appear in GT, the time-frequency distribution of the GT can be used as a filter, to filter out the cross-terms in the output of the WD. There are many different combinations to define the GW transform. Here four different definitions are given.

$$GW(t, \omega) = GT(t, \omega)W D(t, \omega) \quad (4)$$

$$GW(t, \omega) = \min\{|GT(t, \omega)|^2, |W D(t, \omega)|\} \quad (5)$$

$$GW(t, \omega) = W D(t, \omega) \{ |GT(t, \omega)| > 0.25 \} \quad (6)$$

$$GW(t, \omega) = GT^{2.6}(t, \omega)W D^{0.6}(t, \omega) \quad (7)$$

GW transform combines the advantages of GT and WD by eliminating cross-terms while maintaining clarity as good as WD. However GW transform fails to provide useful results once the cross terms are superimposed on the auto components of WD in a multi component signal [9].

### III. RESULTS

In this section, we demonstrate the application and effectiveness of GW transform as m-D feature extraction technique with two different types of experimental radar data obtained in different scenarios.

#### A. Rotating antenna in SAR

We extract the m-D features relating to a rotating antenna in a SAR target scenario. The m-D for such rotating target may be seen as a sinusoidal phase

modulation of the SAR azimuth phase history. The phase modulation may equivalently be seen as a time-varying Doppler frequency[10]. The Doppler smearing due to the rotating parts is often well localized in a finite number of range cells. It is reasonable to process the Doppler signal for each range cell independently. Since the ground truth of the target is already known, the data at the range cell 123 was analyzed using the GW transform. Figure 1a illustrates the time-frequency signature using STFT and Figure 1b illustrates the time-frequency signature using WD at range cell 123. Figure 1c illustrates the time-frequency signature using the S-method for L=4. Even though S method gives better resolution compared to STFT but still suffered from cross terms between close scatterers. Figure 1d shows that result from the proposed GW transform. Clearly it produces concentrated time-frequency signatures compare to STFT and WD. Using the time-frequency plot, the rotation rate of the antenna is estimated by measuring the time interval between peaks. The period is the time interval between peaks. As an example, in Figure 1d, there are 3 peaks. The time interval between peak 1 and peak 2, between peak 2 and peak 3, and between peak 1 and peak 3 are measured. The average value is then used to estimate the rotation rate. The estimated rotation rate is 4.8 seconds, which is very close to the actual value of 4.7 seconds.

#### B. Corner reflectors

High range resolution (HRR) profiles were collected using a stepped frequency waveform (SFWF) radar mode at X- band between 9.0 to 9.4 GHz, i.e., a synthetic bandwidth of 400 MHz; the frequency step size was 1 MHz. The test target was made up of four corner reflectors, three of which are stationary to provide a geometric reference and a contrast to the shape of the oscillating reflector in the HRR profiles. Out of three stationary reflectors, two were placed closer to each other than the third stationary reflector. The test target was located at 2km from the radar system. HRR scans were performed at 2kHz PRF. This data set is used to extract the m-D features using different time-frequency analysis. Figure 2a illustrates the time-frequency signature of the signal which is obtained using STFT. As expected time-frequency representation of STFT is cross term free but its time and frequency resolution is not good. Figures 2b and 2c illustrate the time frequency signatures of the same signal using WD and S-method time frequency representations respectively. Even though WD gives best resolution, it is corrupted with cross terms. On the other hand S method with L value 2 gives better time frequency resolution than STFT. But still there are some cross terms between the two stationary reflectors which

are close to each other. Figure 2d shows GW transform time-frequency representation of the signal. It can be seen that cross terms are removed between two close stationary reflectors. Results demonstrate that the GW transform is an effective tool to extract m-D features

where  $\alpha$  is the order of entropy. In the case of TFRs, low entropy stands for high concentration of auto-terms, where as high entropy means lower energy concentration of auto-terms. Ratio of norms (RN) is calculated by dividing fourth power norm of a TFR,  $\phi(t, \omega)$  by its second power norm [17]. Mathematically

**IV. PERFORMANCE ANALYSIS**

The performance of the time-frequency representations (TFRs) can be compared based on their readability, resolution, cross-terms suppression and energy concentration. Cross-terms suppression and energy concentration of a time-frequency representation is evaluated by visual inspection or on the basis of quantitative measures like entropy measures and ratio of norms [15].

Renyi entropy [16] of time-frequency representation,  $\phi(t, \omega)$  is defined as:

$$Entropy_{Renyi} = \frac{1}{1-\alpha} \log_2 \left( \sum_t \sum_\omega \phi^\alpha(t, \omega) \right) \tag{8}$$

$$RN = \frac{\sum_t \sum_\omega |\phi^\alpha(t, \omega)|^4}{\left( \sum_t \sum_\omega |\phi^\alpha(t, \omega)|^2 \right)^2} \tag{9}$$

Higher value of RN implies that signal auto-components are highly concentrated [17]. From table I and II, it is evident that GW transform has minimum value of entropy and maximum value for RN. Therefore the proposed GW transform shows good energy concentration property compared to other TFRs.

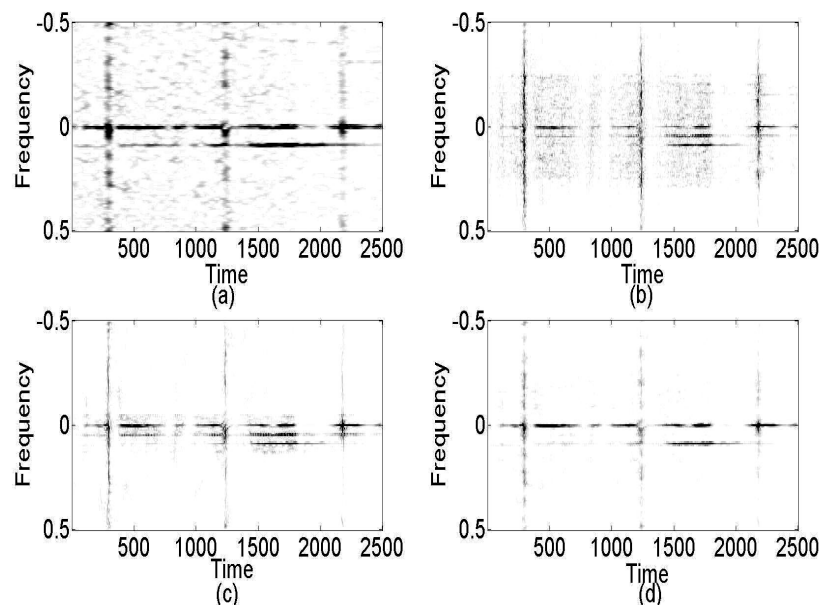


Fig 1. Time frequency signature of Original time series at range cell123  
(a) STFT (b)WD (c) S-method (d) GW Transform

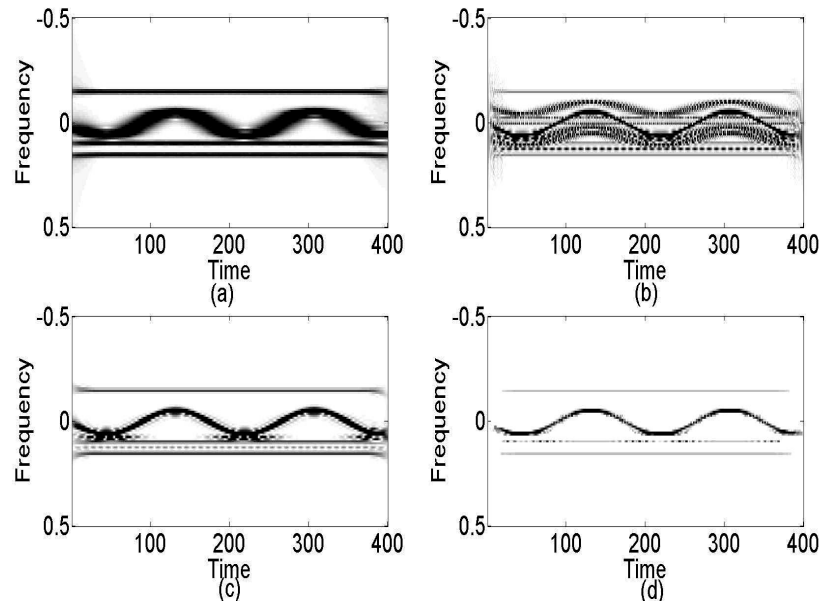


Fig 2. Time frequency signature of Original time series (a) STFT and (b)WD. (c) S-method (d) GW Transform

TABLE I. COMPARISON OF GW TRANSFORM WITH OTHER TFRs BASED ON ENTROPY

Performance measure based on Entropy				
Signal	STFT	WV	SM	GW
Rotating Antenna	28.165	26.199	26.962	10.046
Corner Reflectors	30.059	27.608	28.293	13.367

TABLE II. COMPARISON OF GW TRANSFORM WITH OTHER TFRs BASED ON RATIO OF NORMS

Performance measure based on Ratio of Norms( $\times 10^{-3}$ )				
Signal	STFT	WV	SM	GW
Rotating Antenna	0.562	0.236	0.572	5.172
Corner Reflectors	0.627	0.339	0.734	2.316

### V. CONCLUSION

This paper presents Gabor-Wigner transform approach for the extraction of micro-Doppler features of radar

returned signals from targets. The method combines the advantages of short-time Fourier transform and Wigner Distribution in order to extract the m-D features of radar target returns. By applying the proposed Gabor-Wigner transform to experimental data, the effectiveness of this analysis technique is confirmed. From the extracted m-D signatures, information about the target's micro-motion dynamics, such as rotation rate and period of oscillation, can be obtained.

### VI. ACKNOWLEDGEMENTS

This work is supported by the Defense Research & Development organization, Govt. of India, Ministry of Defense through the project ERIP/ER/ 1104582/M01/1369.

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