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Application of First Order Linear Homogeneous Difference Equations to the Real Life and Its Oscillatory Behavior

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ABSTRACT: In this paper we study the conversion of differential equation to difference equation, real time application of first order linear difference equation and the oscillatory properties

KEY WORDS: Differential equation; difference equation; homogeneous; linear; sequence; Oscillation and Non oscillation.

I. INTRODUCTION

Let us consider the n^{th} term of sequence as $a_n = f(n)$, for some unknown function f.
For example, if

$$a_n = \frac{n+1}{n^2+5} \quad ,$$

then it is easy to compute explicitly, say, $a_0 = \frac{1}{5}; a_{10} = \frac{11}{105}, a_{100} = \frac{101}{10005}$. In such cases we are able to compute any given term in the sequence without reference to any other terms in the sequence. However it is often in the case of application that we do not begin with an explicit formula for the terms of a sequence; rather, we may know only some relationship between the various terms.
An equation which expresses a value of a sequence as a function of the other terms in the sequence is called a difference equation. In particular, an equation which expresses the value a_n of a sequence $\{a_n\}$ as a function of the term a_{n-1} is called a first order difference equation.

II. RELATED WORK

In the recent years there has been a lot of interest in the study of oscillatory and non oscillatory properties of difference equations and functional difference equations. In the year 1966, S.N.Elaydi [4] was given some basic introduction about difference equations and briefly explained their oscillatory behaviors of solutions of difference equations. In the year 1999, R.P.Agarwal, and J.Popenda [2] were explained several new fundamental concepts in this fast developing area of research. These concepts are explained thorough examples and supported by simple results. In the year 2000, Dan Sloughter [3] was explained the applications of difference equations with some real time examples .

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III. RESULT AND DISCUSSION

Consider a homogeneous, first order, linear, differential equation of the form

$$2\frac{d\,y(t)}{dt}+7y(t)=0 \quad \ldots\ldots\ldots\ldots\ldots \qquad (1)$$

in equation (1) t is the independent variable and y is the dependent variable , a function of t.
We can do this by approximating derivatives by finite differences. Recall these definition of a derivative,

$$\frac{d\,y(t)}{dt}=\lim_{\Delta t\to 0}\frac{y(t+\Delta t)-y(t)}{\Delta t} \quad \ldots\ldots\ldots \qquad (2)$$

at any point at which y(t) is differentiable. A derivative in continuous time can be approximated by finite differences in discrete time by

$$\frac{y((n+1)\Delta t)-y(n\Delta t)}{\Delta t}$$

This is called a forward difference because it uses the present or current value of y of y(nΔt) and the next or future value of y of y((n+1)Δt). Similarly

$$\frac{y((n\Delta t)-y((n-1)\Delta t)}{\Delta t}$$ is a backward difference and $$\frac{y((n+1)\Delta t)-y((n-1)\Delta t)}{2\Delta t}$$

is a central difference. In the limit as Δt approaches zero these are all the same, but in discrete time, Δt is fixed and is not zero and these three approximations to a continuous time derivative are , in general ,different.

As an illustration we will convert the differential equation (1) , into a difference equation by difference approximation,

$$2\frac{y((n+1)\Delta t)-y(n\Delta t)}{\Delta t}+7y(n\Delta t)=0\ .\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

To simplify the notation, let y[n] = y (nΔt) where the square brackets, [.], distinguish a function of discrete time from a function of continuous time which is indicated y using parenthesis, (.). In this notation, time is not explicitly indicated but, since the time between consecutive discrete-time values of the function, y is always Δt, we do not need to explicitly indicate time. Using the simplified notation, (3) becomes

$$2\frac{y[n+1]-y[n]}{\Delta t}+7y[n]=0$$

2(y [n+1] – y[n]) + 7Δty[n] =0 ………… (4)
Which is a homogeneous difference equation.

The equation (4) is written as 7 y (n) + 2(y (n+1) – y (n)) = 0, with y (0) =1
The solution of this equation is

$$y(n)=\left(\frac{-5}{2}\right)^{n}$$ And the graph of the equation is

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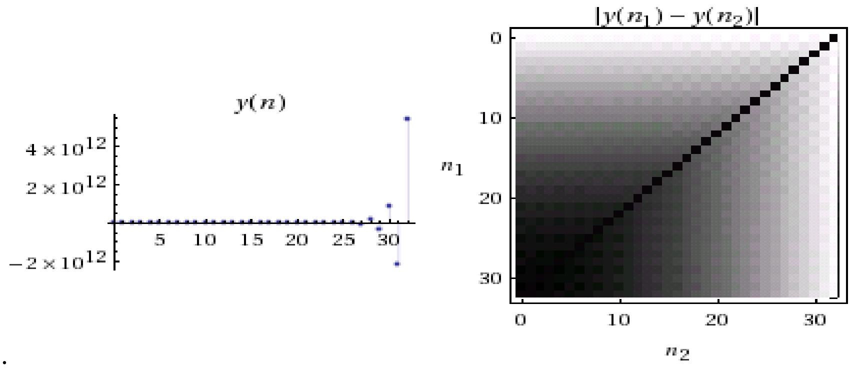

Figure 1

n	0	1	2	3	4
$y(n)$	1	−2.5	6.25	−15.625	39.0625

IV. METHODOLOGY

Many of the results obtained in this paper are by employing one or more techniques such as difference equations, Oscillatory properties and the graphs are drawn from wolfram alpha (computational knowledge engine).

V. EXPERIMENTAL RESEARCH

Definition:

A sequence $y = \{y(m,n)\}_{m=1}^{\mu}{}_{n=1}^{\infty}$ is said to be nonoscillatory around 0 if there exist a positive integer ν such that

y (m, n) > 0 (positive sequence) for all $m \le \mu$, $n \ge \nu$

or

y (m, n) < 0 (negative sequence) for all $m \le \mu$, $n \ge \nu$.

Otherwise the sequence y is called oscillatory.

We shall consider sequence y which are defined N^2. It is clear that the sequence y is nonoscillatory.

For example,

Suppose a cup of tea, initially at a temperature of 180^0 F, is placed in a room which is held at a constant temperature of 80^0 F. Moreover, suppose that after one minute the tea has cooled to 175^0 F. What will the temperature be after 20 minutes?

Solution:

If we let t_n be the temperature of the tea after n minutes and we let s be the temperature of the room , then we have $t_0 = 180$, $t_1 = 175$ and s =80.

Newton's law of cooling states that $t_{n+1} - t_n = k(t_n - 80)$, n= 0,1,2,3,… ……….. (5)

where k is the a constant which we will have to determine . To do so, we make use of the information given about the change in the temperature of the tea during the first minute.

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Applying (5) with n=0, we must have

$$t_1 - t_0 = k(t_0 - 80). \quad .... \quad (6)$$

That is,

175-180 =k (180-80). Hence k = -0.05

thus (6) becomes

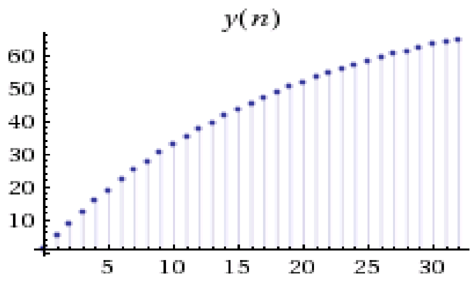
$$t_{n+1} - t_n = -0.05(t_n - 80) = -0.05\, t_n + 4.$$

Hence

$$t_{n+1} = t_n - 0.05 t_n + 4 = 0.95 t_n + 4. \quad .... (7)$$

The equation (7) is written as y(n+1)-0.95y(n)-4 = 0 with initial condition y(0) =1

The solution of the difference equation is $y(n) = 80 - 70\left(\frac{19}{20}\right)^n$ and the graph of the solution is



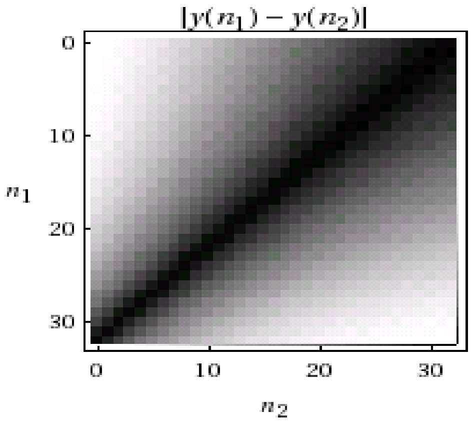


Figure 2.

n	0	1	2	3	4
$y(n)$	1	4.95	8.7025	12.2674	15.654

for n= 0,1,2,3,… now (7) is in the standard form of a first order linear equation, so

let us take the first order linear difference equation is of the form

$$t_n = \alpha^n t_0 + \beta\left(\frac{1-\alpha^n}{1-\alpha}\right) \quad .... \quad (8)$$

we know that the solution is

$$t_n = (0.95)^n(180) + 4\left(\frac{1-(0.95)^n}{1-0.95}\right)$$ where α =0.95, β =4 and t_0 =180

$$t_n = 80 + 100\,(0.95)^n \quad .... \quad (9)$$

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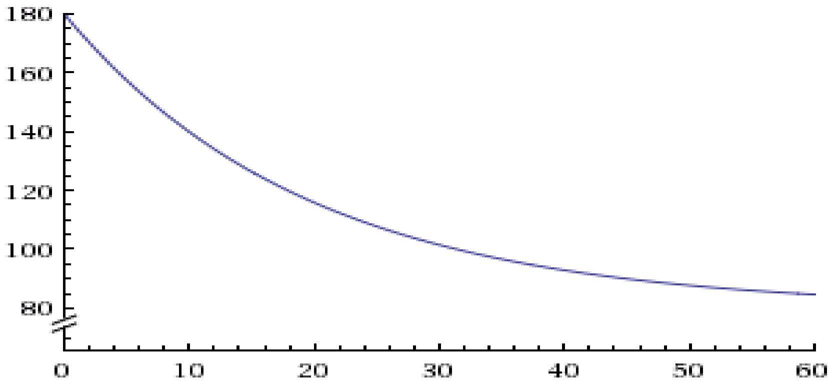

Figure 3 .Tea temperature decreases asymptotically towards room temperature

for n =0,1,2,3,… In particular,

$t_{20} = 80 + 100(0.95)^{20} = 115.85$,

where we have rounded the answer to two decimal places. Hence after 20 minutes the tea has cooled to just 116^0F. Also, since

$$\lim_{n\to\infty} (0.95)^n = 0,$$

we see that

$$\lim_{n\to\infty} t_n = \lim_{n\to\infty} (80+100(0.95)^n) = 80 \ldots\ldots \qquad (10)$$

that is , as we would expect, the temperature of the tea will approach an equilibrium temperature of 80^0 F, the room temperature . In figure 1 we have plotted temperature t_n verses time n for

n= 0,1,2,3,…,60, along with the horizontal line t = 80. As indicated by (10), we can see that t_n decreases asymptotically towards 80^0 F as n increases.

VI. CONCLUSION

By the definition of nonoscillatory and based on some theorems, the solution of the given difference equation

$$t_n = \alpha^n t_0 + \beta\left(\frac{1-\alpha^n}{1-\alpha}\right)$$ is nonoscillatory .

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Dr.P.Mohankumar is a master of science from the prestigious Indian Institute of Technology, Chennai. After that he has joined the Tamilnadu collegiate service as Assistant Professor of Mathematics. During the service he joined the college of Engineering, Guindy for M.Phil, and got M.Phil degree in first class. After 35 years of glorious service in Arts Colleges. He retired from service. After that he got his Ph.D in Mathematics Difference Equations from Salem Periyar University, under the guidance of Prof. E. Thandapani. The thesis is highly commended one. He is in VMU from 01/10/2008.