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Application of Queueing Theory to Customers Purchasing Premium Motor Spirit (PMS) at a Filling Station

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ABSTRACT

The formation of waiting lines is a prevalence scenario that happens whenever the immediate demand for a service surpass the current capacity to provide that service. This discrepancy may be temporal, but a queue accumulates during the period. Formation of a line causes an increase of customers waiting time, over-utilization of the available servers and loss of customer goodwill. Application of Queueing theory determines the measures of performance of the service facility; this can be used to design the appropriate service facility. Data for this study was collected at Nigeria National Petroleum Corporation (NNPC) Mega Station Jos for seven consecutive days between the hours of 7am-6pm daily through observations, interviews, and records of customers purchasing PMS only. The multi-server model was adopted for the study of the existing structure has eight servers. The data was analyzed using descriptive analysis; Minitab-16 and TORA-2.0 software. The arrival rate λ=2.7483 customers/min is greater than the service rate μ=0.4137 customer/min showing that queue exists. There are Poisson arrivals and exponential service distributions as validated by a Chi-square goodness of fit test. The calculated mean of utilization factors for five scenarios is 67.808%.The utilization factor of 66.432 % obtained for M/M/10: FCFS/∞/∞ is the closest to this mean value and hence selected as the average utilization factor. This model that yielded an average queue time of 0.12353 minute and an average queue length of 0.33948 customers was formulated. M/M/10 gave optimal results and were proposed for adoption and to be used for solving similar problems. Management should open up two more servers. Incentives should be given to creating over time that will increase or sustain the acceptable utilization factor. Any utilization factor value below 66.432 % is not encouraged for this system as it will increase idle time.

INTRODUCTION

Background of Study

The queueing ugly scenario is a quest to strike a balance between the average waiting time for motorists, vehicles, etc. and the idle time of the attendants in the filling station. The problems of queues) are very popular in the day to day activities. Queues are often seen at the bus stop, ticket booths, petrol pump, bank counter, traffic lights and soon. Queueing theory deals with the mathematical description of the behavior of queues and can apply to a variety of operational situations where it is not possible to predict the arrival rate of customers and service rate of service facilities accurately. Queueing theory has all the required tools for the analysis of queue system. It can be used to determine the level of service (either the service rate or the number of service facilities).

The search for controlling the widespread problems of irregularities, congestions and delays experienced in the queueing method of Nigeria National Petroleum Cooperation (NNPC) Mega Filling Station in Dogon-karfe Jos, Plateau state, Nigeria has been among the users of petroleum products in the area. These problems result in a long queue that are found in this filling station always on a daily basis. The long queues are usually experienced in the purchase of petroleum products like DPK and PMS. Queueing is an eye saw and cannot be avoided in as much as the number of persons arriving at the service center is greater than the capacity of the service capability. However, a proper queue management technique will control the congestion by reducing waiting time.

Queueing analysis helps to provide better services and achieves higher efficiency. In marketing, distribution and retailing of petroleum products in Nigeria, queueing allows the system to queue their customers request until free servers become available. To design an effective congestion control structure, a good knowledge of the relationship between the congestion and delay is needed. Queueing is due to the randomness in service time, and the principal actors are the customer and the server. The causes of congestion in petrol station may include [1];

(a) Faulty fuel pump dispensary (meter).

(b) Inaccurate metering and high cost of fuel at other filling station.

(c) Location of the filling station,

(d) Inadequate service space and channels in the retail outlets.

(e) Scarcity of petroleum products from the supply source, which results in;

(i) Creating chaotic situations at the service facility.

(ii) Customers having to wait for too long without being served.

(iii) Impatient customers are leaving thereby affecting their goodwill.

PMS, popularly known as petrol in this part of the world is a complex mixture of hydrocarbon produced from crude oil.

The study is undertaken at the petrol pump at NNPC mega station Yakubu Gowon way in Dogon-Karfe, Jos, Plateau State Nigeria, which is the major petrol pump station in Greater Jos metropolis and situated at the heart of the city. Due to its location in the city, long queues can easily be seen in the service area. Customers purchase petroleum products like Premium Motor Spirit (PMS), Automotive Diesel Oil (AGO), Dual Purpose Kerosene (DPK), and lubricants from the station. There is also free air to customers that patronizes the station. In this study, considerations are made only on PMS.

The problem of the queue has been studied, and the key reasons that result in long line have been identified. Ohaneme reviewed the suitable queueing model for a line that can be developed by studying the arrival and service pattern of customers, evaluate the performance measures, and determine the optimal utilization factor using simulation approach in a petrol station [1]. Akinnuli reviewed the use of queuing theory in a petrol station system with emphasis on improving customer satisfaction by reducing waiting times through provision of sufficient number of servers and attendants [2]. A research work studying the queuing system in a shopping plaza using single-line multiple-channel model was carried out and investigated in a Study of Waiting and Service Costs of a MultiServer Queuing Model in A Specialist Hospital with emphasis on determining the optimal service level at the facility [3,4].

This study will use simple descriptive statistics (mean) and least squared deviations from the average towards determining the optimal service level of the facility.

Statement of the Problem

Many previous studies have been conducted to investigate how waiting in line in a service system affect customer's goodwill. Queues occur when there is too much demand on service facilities or an inadequate number of service facilities. That is, when the request for a service facility exceeds the capacity of that facility, the servers spend much time in serving just some of customers. It is on this note that many customers are not able to receive adequate services on time so that we say that there is an excess of waiting time. Also, there is a queue problem when there is less demand on service facilities leading to too much idle facility time or too many facilities. This warrant the customers to wait before service or the service facilities stand idle and wait for customers. Queues or waiting lines are regular happenings that occur in our everyday life and different organizations or different settings like fuel stations and varieties of business situations.

Observing the fact that one major problem customers encounter in a fuel station are waiting time, it becomes relevant that a study of this process is conducted to find a solution to the long waiting time in the station.

Aims and Objectives of the Study

(1) To determine the optimal service level for the facility.

(2) Determine the arrival and service rate distributions of customers purchasing PMS from the petrol station.

(3) Use the existing model to obtain the values for the queueing parameters.

(4) Develop alternative models with their parameters values.

(5) Carry out a comparative analysis of the models and provide the best model based on the values of the parameters obtained.

(6) Perform general decision making using the appropriate model.

LITERATURE REVIEW

On etymology, the word queue comes via French, from the Latin word "cauda" meaning tail. The spelling "Queueing" over "Queuing" is typically encountered in academic research field [5].

A queue is defined as an aggregate of items awaiting service. Queueing theory is considered as a branch of operations research because the result is often used when making business decisions about the resources needed to provide services.

The origin of queueing theory is traced to the work of A.K Erlang in 1903. However, the work of D.G. Kendall in 1953 formed the basis for analytical calculations and the naming convention of queueing models being used today [6].

Multiple servers with a single line is a situation where the customers arrive from a calling population and joins a queue of clients to be served by more than one server. This a logical extension of a single-server waiting line is to have multiple servers. In this case, customers wait in a single line and move to the next available server as illustrated in **Figure 1** below.

Here, each one of the service stations can deliver the same type of service (i.e., the system has a single-phase) and is equipped with the same kind of facilities. The customer who selects one station makes this decision without any external pressure from anywhere. Due to this fact, the queue is single. The single queue usually breaks into smaller queues in front of each station.

Little's theorem (Little's queueing formula L=λW)

For any queueing system or any subset of a queueing system, the following quantities are defined as λ_{eff} = average number of arrivals when all arriving customers can join the system (i.e., effective arrival rate at the system)

λ= average number of arrivals entering the system per unit time

L_q= average number of customers waiting in line

L_s = average number of customers present in the queueing system

W_q = average time a customer spends in queue

W_s = average time a customer spends in the system

In these definitions, all averages are steady-state averages.

In general, for a queueing system at equilibrium with arrival rate λ (i.e., $\lambda$is equal to λ_{eff}). Mean queue length L, and mean wait time W Little's laws, states that the long-term average number of customers in a stable system is equal to the long-term average effective arrival rate multiplied by the average time a customer spend in the system. $L=\lambda W$

Often we are interested in the amount of time that a typical customer spends in a queueing system. Both W_s and W_q are computed under the assumption that the steady state has been reached. By using the powerful result known as Little's queueing formula, W_s and W_q may easily be computed from L_s and L_q. The relationship between L_s and W_s (also L_q and W_q) is valid under the conditions that the effective arrival rate at the system λ_{eff} is equal to the nominal arrival rate λ (when all arriving customers can join the system). Otherwise, if some customers cannot enter because the system is full, then $\lambda_{eff} < \lambda$ The Little's formula is express as; $L_q = \lambda W_q$, $L_s = \lambda W_s$

The profoundness of this formula is because it holds for virtually all queueing systems under very general conditions [7].

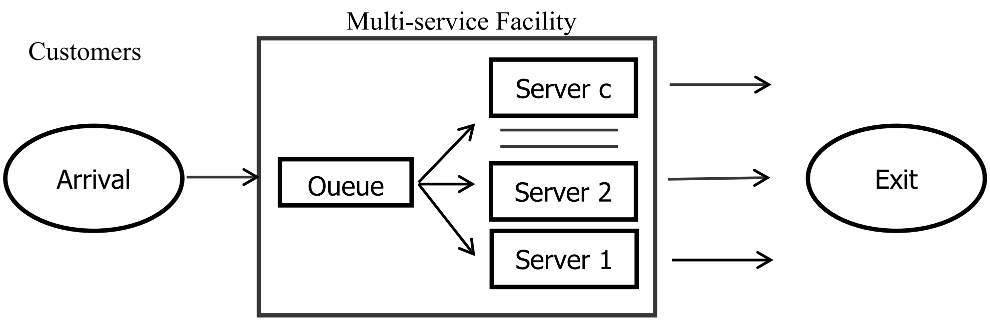

Figure 1. Customers multi-service facility.

METHODS

The data used for this research comprises of recording the number of arrivals and departures from direct observation of customers that come to purchase PMS for seven consecutive working days, between the hours of 7:00 am to 6:00 pm. The duration and time interval was structured to enable the researcher collect a representative data that considered traffic in weekdays and weekend (and also slacks/idle times and peak periods). Because of error, a stopped clock, pen and notebook were used to record the observations. During data collection, data were collected based on the arrival rate and departure rate of customers at the fuel station for the eight servers that were serving customers among the twelve reserved for the purchase of PMS by customers.

Also, formal and informal interviews at an individual level of discussions were held to obtain adequate clarification about other variables that can influence the development of the queueing model for customers' congestion at the PMS terminals. The interview conducted with the PMS Pump Operators and Manager by the researcher revealed that an average of 1600 customers arrives for PMS in a day. He also revealed that the waiting time of customers in the system increases during the peak hours and that the service discipline was on First Come First Serve (FCFS) basis.

Method of Data Analysis Technique

The main statistical methods used are:

1. The simple frequency table for finding the mean and variance of the arrival and service data.

2. The collected data for arrivals and departures will be analyzed for the goodness of fit to see how good it fits the Poisson distribution and exponential distribution respectively using the chi-square (χ^2) goodness of fit technique before computing the performance measures of the system.

Formulation of the Queueing Model for the Existing Structure

At the PMS refueling station, it is a situation where the customers arrive from a calling population and joins a queue of customers to be served by any of the eight servers that are regularly available on a first-come, first served (FCFS) basis. There is no limit on customers joining this particular queue, and the calling source is infinite. The primary data collection from direct observations shows that there was complete randomness in arrival pattern as well as service pattern. Therefore, an M/M/8: FCFS/∞/∞ queueing model has been proposed for the queue. M/M/8: FCFS/∞/∞ queueing system is a multi-server queueing system with Poisson input, exponential departure distribution, first come first served service discipline, unlimited number of calling population and waiting positions. This model was structured to accommodate the main characteristics of the queueing system the petrol station.

Assumptions of the Queue Model

In most models certain assumptions are used to simplify the modeling process. Since otherwise, in an attempt to model every detail of the real world scenario, the model would become too complicated to understand and useless for any practical purpose. Here, the following assumptions were incorporated in the model as:

1. The system is assumed to be in a steady state.

2. Arrivals at the system are completely random and occur in a single term (not in batches), neither balking nor reneging occurred and that the calling source (population) is infinite with no blocking.

3. The capacity of the queue is infinite (any number of customers can join the queue), and only one line comes up.

4. The queue discipline is First-Come, First-Served (FCFS) basis by any of the servers. There is no preference classification for any arrival.

5. The mean arrival rate is constant. This rate is independent of the number of customers already serviced, queue length or any other random property of the line.

6. The servers here represent only the PMS pumps but not pumps for other products.

7. The service providers are working to their full capacity.

8. The mean service rate μ is constant. This rate is independent in the sense that server won't speed up when the line is longer.

9. The average arrival rate is greater than average service rate.

10. Both the arrival and departure rates are state dependent, meaning that they depend on the number of customers in the service facility.

Derivations from Little's queueing theorem L=λW

Before deriving these significant results, we present an intuitive justification of $L = \lambda W$. First, we note that both sides of $L=\lambda W$ have the same unit (we assume that the unit of time is hours). This follows because L is expressed regarding many customers, is expressed regarding customers per hour, and W is expressed in hours. Thus, λW has the same unit (customers) as L. Hence, $L=\lambda W$ virtually independent of the number of servers, the arrival distribution, the service discipline, and the service time distribution. We may apply $L=\lambda W$ to any queueing system and thus the proposed model *M/M/C: FCFS/∞/∞..*

For an M/M/C: FCFS/∞/∞ system, $\lambda_{eff}=\lambda$ since there are no blocked customers. Therefore,

$L = \lambda_{eff}\,.\,W$ Explicitly, $L_q = \lambda_{eff}\,.\,W_q$ and $L_s = \lambda_{eff}\,.\,W_s$.

Thus, $L_q = \lambda\,.\,W_q$ and $L_s = \lambda\,.\,W_s$

Determination of Performance Measures M/M/C: FCFS/∞/∞ Queueing Model

In this model, there are multiple but identical servers in parallel to handle servicing customers. If there are n customers in the queueing system at any point in time, a single line will break up into shorter lines in front of each server and the number of customers in the queue at a particular time can take one of these two values:

1) All arrivals are being serviced because there is no queue, that is, n < c (the number of customers in the system at a time is less than the number of servers). However, (c – n) numbers of servers are not busy. The combined service rate will then be: $\mu n = n\mu$; $n < c$

(2) Service demanded by customers is greater than the capability of servers, and so a queue is formed, that is, n ≥ c (the number of customers in the system at a time is greater than or equal to the number of servers). Therefore, all servers will be busy, and the maximum number of customers in the queue will be (n – c). The combined service rate will be: $\mu n = c\mu$; $n \ge c$. Thus to derive the results for this model, we first defining accurately the following:

n = Number of customers in the system,

λ_n = Arrival rate given n customers in the system,

μ_n = Service rate given *n* customers in the system,

P_n = Steady-state probability of *n* customers in the system

We have for $\lambda_n = \lambda$; for all $n \ge 0$ and equilibrium $\rho < 1$ then:

The utilization factor ρ for the whole system is the ratio between the mean arrival rate λ and the maximum possible rate of service of all the channels; hence $\rho=\frac{\lambda}{c\mu}$ and

$$\text{Percent utilization} = \frac{\lambda}{c\mu}\times 100$$

Solving Difference Equations of the System by applying the iterative method, the probability of *n* customers Pn in the system have been developed and is given by $P_n = \frac{\lambda^n}{\mu(2\mu)(3\mu)\ldots(n\mu)}P_0 = \frac{\lambda^n}{n!\mu^n}P_0$

Thus, $P_n = \frac{1}{n!}\left(\lambda/\mu\right)^n P_0;\ n=0,1,\ldots,c-1 \text{ i.e. } n<c$ ....(i) and

$P_n = \frac{\lambda^n}{(\prod_{i=1}^{c} i\mu)(c\mu)^{n-c}}P_0 = \frac{1}{c!c^{n-c}}\left(\lambda/\mu\right)^n P_0$ for $n \ge c$ ...(ii) [8]

1. The probability of having no customers (empty or idle facility) P_o in a multi-channel system is determined using the relation $\sum_{n=0}^{\infty} P_n = 1$ (sum of probabilities is unity) and the equations (i) and (ii) above, it implies that $\sum_{n=0}^{c-1} P_n + \sum_{n=c}^{\infty} P_n = 1$

$$\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n P_0 + \sum_{n=c}^{\infty}\frac{1}{c!c^{n-c}}\left(\lambda/\mu\right)^n P_0 = 1$$

$$\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n + \sum_{n=c}^{\infty}\frac{c^c}{c!c^{n-c}}\left(\lambda/\mu\right)^n\right] = 1$$

$$P_0\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n + \frac{c^c}{c!}\sum_{n=c}^{\infty}\left(\lambda/\mu\right)^n\right] = 1$$

$$P_0\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n + \frac{c^c}{c!}\left\{\left(\lambda/c\mu\right)^c + \left(\lambda/c\mu\right)^{c+1} + \left(\lambda/c\mu\right)^{c+2} + \ldots\right\}\right] = 1$$

$$P_0\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n + \frac{c^c}{c!}\left(\lambda/c\mu\right)^c\left\{1 + \left(\lambda/c\mu\right) + \left(\lambda/c\mu\right)^2 + \ldots\right\}\right] = 1$$

$$P_0\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n + \frac{1}{c!}\left(\lambda/c\mu\right)^c\left\{\frac{1}{1-\left(\lambda/c\mu\right)}\right\}\right] = 1$$ (Sum of infinite geometric series) or

$$P_0\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\lambda/\mu\right)^n+\frac{1}{c!}\left(\lambda/c\mu\right)^c\left\{\frac{cu}{cu-\lambda}\right\}\right]=1$$

Thus, the probability that the system shall be idle is

$$P_0\left[\sum_{n=0}^{c-1}\frac{\left(\lambda/\mu\right)^n}{n!}+\frac{\left(\lambda/\mu\right)^c}{c!}\frac{cu}{cu-\lambda}\right]^{-1} \qquad \text{(iii)}$$

The mean length of the waiting line (L_q) or average queue length, excluding the customers under service, is obtained as, $L_q=\sum_{n=c+1}^{\infty}(n-c)P_n=\sum_{n=c+1}^{\infty}(n-c)\frac{1}{c!c^{n-c}}\left(\lambda/\mu\right)^n P_0$ then, by change of variable technique, let

$$y=n-c\Rightarrow n=y+c\Rightarrow$$

$$L_q=\sum_{y=1}^{\infty}y\frac{1}{c!c^y}\left(\lambda/\mu\right)^{y+c}P_0=\frac{1}{c!}\left(\lambda/\mu\right)^c P_0\sum_{y=1}^{\infty}y\frac{1}{c^y}\left(\lambda/\mu\right)^y$$

$$=\frac{1}{c!}\left(\lambda/\mu\right)^c P_0\sum_{y=1}^{\infty}y\left(\lambda/c\mu\right)^y\Rightarrow L_q\ \frac{1}{c!}\left(\lambda/\mu\right)^{c+1}P_0\sum_{y=1}^{\infty}y\left(\lambda/c\mu\right)^{y-1}$$

Now, from sum of geometric series and its derivative,

$$L_q=\frac{1}{c!c}\left(\lambda/\mu\right)^{c+1}P_0\left\{\left[1-\left(\lambda/c\right)\right]^2\right\}=\frac{1}{(c-1)!c^2}\left(\lambda/\mu\right)^c\left(\lambda/\mu\right)P_0\left\{\frac{(c\mu)^2}{(c\mu-\lambda)^2}\right\},$$

which yields: $L_q=\frac{\lambda\mu\left(\lambda/\mu\right)^c}{(c-1)!(c\mu-\lambda)^2}P_0$

The average number of customers in the multi-channel system is: $L_s=\frac{\lambda\mu\left(\lambda/\mu\right)^c}{(c-1)!(c\mu-\lambda)^2}P_0+\frac{\lambda}{\mu}$

i.e., $L_s=L_q+\frac{\lambda}{\mu}$ (by Little's theorem)

2. The average time a customer spends in the system is

$$W_s=\frac{L_s}{\lambda}=\frac{\mu\left(\lambda/\mu\right)^c}{(c-1)!(c\mu-\lambda)^2}P_0+\frac{1}{\mu} \quad \text{(by Little's theorem)}$$

3. The average time a customer spends in the queue is

$$W_q=\frac{L_q}{\lambda}=\frac{\mu\left(\lambda/\mu\right)^c}{(c-1)!(c\mu-\lambda)^2}P_0 \quad \text{(by Little's theorem)}$$

4. Average waiting time for an arrival not immediately served

$$W_a=\frac{1}{c\mu-\lambda}$$

In the multi-channel system, the probability that a customer approaching the servers must wait to be serviced (P_w) coincides with the probability that there is c or more customers in the system and is given as $P(n\geq c)=\sum_{n=c}^{\infty}P_n=\sum_{n=c}^{\infty}\frac{1}{c!c^{n-c}}\left(\lambda/\mu\right)^n P_0$

By change of variable technique, let $y=n-c\Rightarrow n=y+c$,

$$\Rightarrow P(n\geq c)=\sum_{y=0}^{\infty}\frac{1}{c!c^y}\left(\lambda/\mu\right)^{y+c}P_0=\frac{1}{c!}\left(\lambda/\mu\right)^c P_0\sum_{y=0}^{\infty}\left(\lambda/c\mu\right)^y$$

$$=\frac{1}{c!}\left(\lambda/\mu\right)^c P_0\left\{\frac{1}{1-\left(\lambda/c\mu\right)}\right\} \quad \text{(infinite sum of a geometric series)}$$

$$=\frac{1}{c!}\left(\lambda/\mu\right)^c P_0\left\{\frac{cu}{cu-\lambda}\right\}=\frac{1}{c(c-1)!}\left(\lambda/\mu\right)^c P_0\left\{\frac{cu}{cu-\lambda}\right\}$$

$$P(n\geq c)=\frac{\mu\left(\lambda/\mu\right)^c}{(c-1)!(c\mu-\lambda)}P_0. \quad \text{Thus, } P(n\geq c)=P_w=\frac{W_q}{W_a}$$

Table 1. TORA Optimization System, Windows®-version 2.00.

Scenario	C	Lambda	Mu	L'da eff	P_0	Ls	Lq	Ws	Wq
1	8	2.7483	0.4137	2.7483	0.00095	9.21787	2.57465	3.35403	0.93682
2	9	2.7483	0.4137	2.7483	0.00116	7.51294	0.86972	2.73367	0.31646
3	10	2.7483	0.4137	2.7483	0.00125	6.9827	0.33948	2.54074	0.12353
4	11	2.7483	0.4137	2.7483	0.00128	6.78069	0.13747	2.46723	0.05002
5	12	2.7483	0.4137	2.7483	0.00129	6.6987	0.05548	2.4374	0.02019

Investigation to Proposing a Better Model

To formulate a suitable model with characteristics that will enable a solution to this problem of the long wait to be achieved, a modification of the original model in the area of the number of servers (level of service) will be made. By applying the principle of (M/M/C) the multi-channel system. Here, the researcher will embark on an investigation for a much more workable model using some trial models with a level of service C= 9,10,11,12 for the best/optimum utilization of the facility. On analysis, the proposed model should prove workable as to producing the desired result of eliminating or reducing queueing time, service time, system length, etc. to the barest minimum. Therefore, the proposed model should provide an ideal service facility for comparison with the existing model. This comparative analysis of the entire five scenarios queueing model will provide a quantitative basis for analyzing the queueing phenomena at the petrol station.

Determination of the Acceptable Utilization Factor

The mean for the servers' utilization factor is to be determined. The model with the least squared deviation from this calculated mean is the minimum acceptable utilization factor for the policy making in this system, and it is also known to be the optimal utilization factor and the corresponding number of servers is required to provide the optimal service level. Any value below this value is at this moment rejected because utilization factor below the acceptable factor causes an increment of idle time, less production time, and decrease in service rate.

Some Techniques for Model Solution

One of the advantages of the application of queueing model for solving waiting line problem is the availability of software packages to handle and manipulate large data to get the solution for effective and efficient decision-making. Where queueing results involve computationally complex formulas or multiple channels, it is recommended we use software packages to carry out these calculations. The software packages also provide the user the opportunity for flexibility of the model in that various additional constraint can be added to take into account other constraints that may be peculiar to different situations in real life problem. Minitab-16 will be employed for the computation of descriptive statistics and chi-square goodness of fit test of arrival and service distributions in the existing system while TORA (Techniques of Operations Research Applications) Windows®-version 2.00 menu driven optimization software package will be used in obtaining the performance measures of both existing and trial models **(Table 1)**.

ANALYSIS OF DATA

(1) Average arrival rate $\lambda = \frac{\text{Total Number of arrivals}}{\text{Total arrival time}} = \frac{12697}{77} = 164.8961$

λ = 164.8961 customers per hour.

Thus, $\lambda = \frac{164.8961}{60} = 2.7483$ customers/minute

Employing minitab-16 Software using the Poisson probability mass function

$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0,1,2....$ With λ = 164.8961,

the chi-square goodness of fit test yields a P-Value of 0.995.

The hypothesis is:

H_o: The arrival rate follows a Poisson distribution with mean rate λ.

H_1: The arrival rate does not follow the Poisson distribution with mean rate λ.

The level of significance =5% If p-value < α, reject H_o otherwise do not reject H_o at α-level of significance.

(2) Mean $(\bar{x}) = \frac{\sum fx}{\sum f} = \frac{11167}{77} = 145.026$

The average service rate μ of individual channel of customers completing a service per hour

$\grave{\imath} = 1/\bar{x} = \frac{1}{145.026} = 0.00689515$ customers/hr

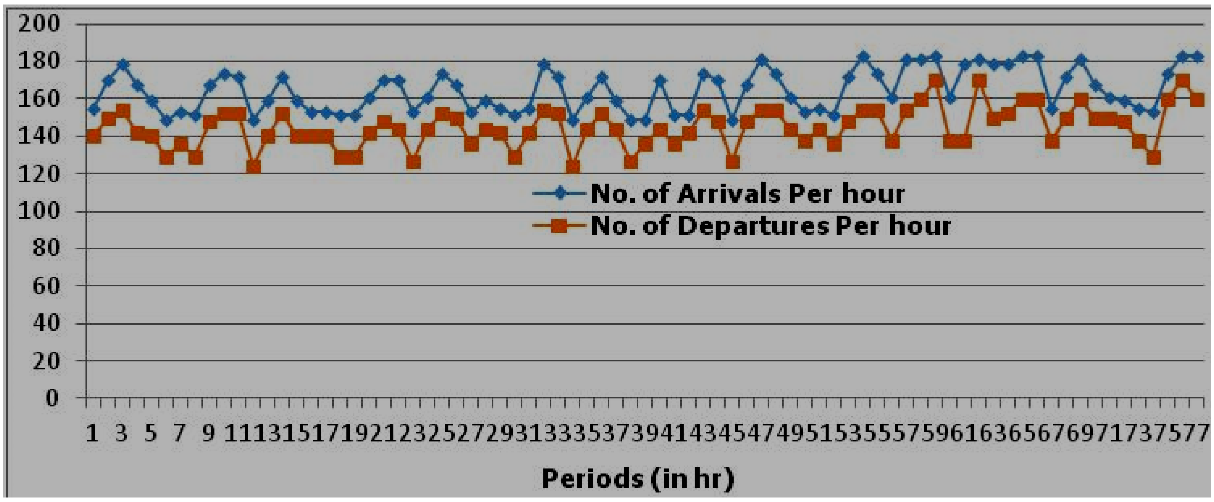

Figure 2. Number of arrivals and departures per hour, for the petrol station.

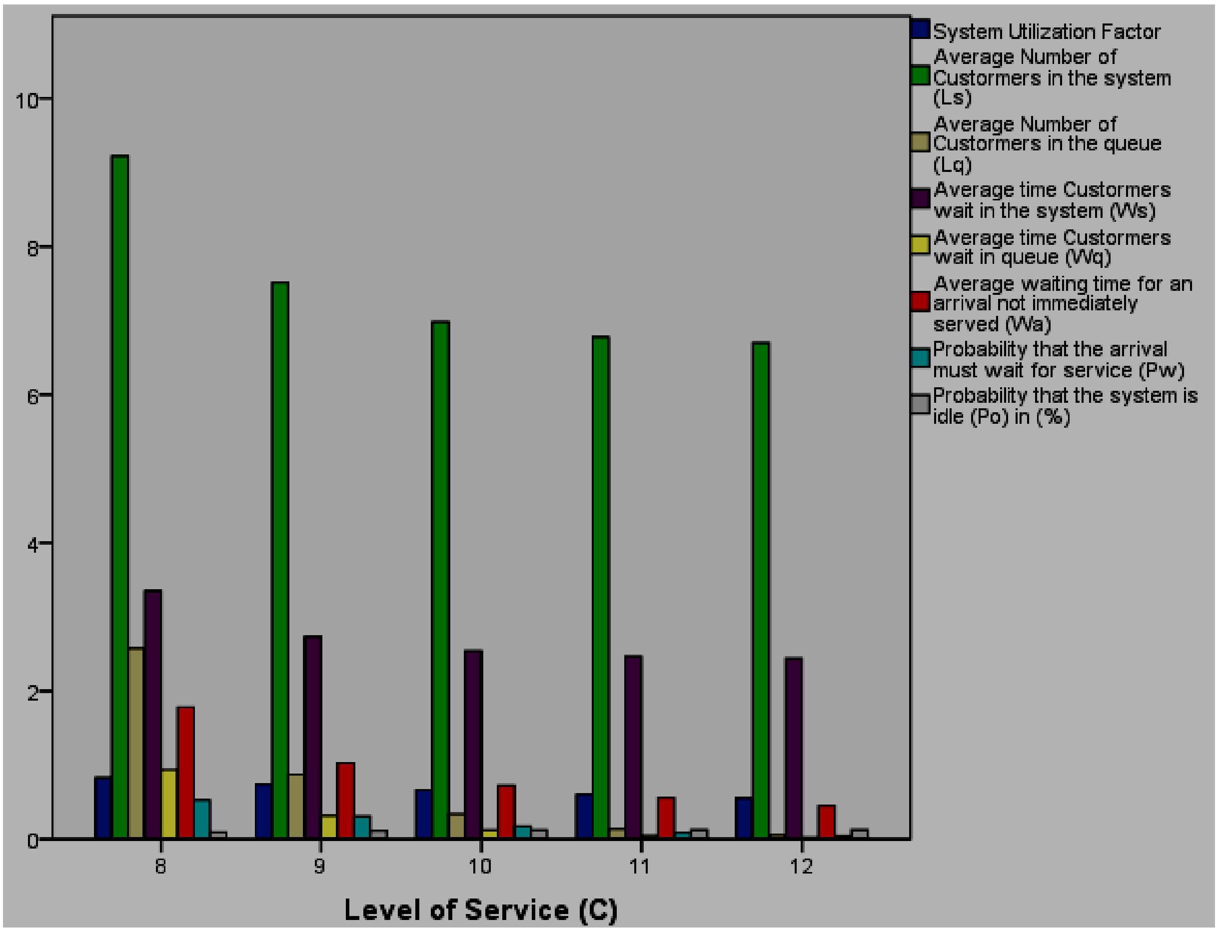

Figure 3. General analysis of the queueing systems.

Therefore, $\mu = 0.00689515 \times 60 = 0.4137$ customers/minute

From the Exponential probability density function $f(x) = \mu e^{-\mu x}$; $x > 0$, $\mu > 0$ with, $\mu = 0.006895$, the cumulative probabilities, $P(X £ x) = 1-e^{-\mu x}$ a chi-square goodness of fit test using Minitab-16 Software gives a P-Value of 0.848.

The hypothesis is:

H_o: The service rate has an exponential distribution with mean rate μ.

H_1: The service rate does not have the exponential distribution with mean rate μ.

The level of significance =5% If p-value < α, reject H_o otherwise do not reject H_o at α-level of significance.

(3) The system capacity = $c\mu = 8 \times 0.4137 = 3.3096$ (service rate for eight servers). The queue will not explode since $c\mu > \lambda$.

Table 2. Summary analysis of the queueing models.

Performance Measure	8 Servers	9 servers	10Servers	11 Servers	12 Servers
Arrival rate	2.7483	2.7483	2.7483	2.7483	2.7483
Service rate	0.4137	0.4137	0.4137	0.4137	0.4137
System Utilization	83.04%	73.81%	66.43%	60.39%	55.36%
L_s	9.21787	7.51294	6.9827	6.78069	6.6987
L_q	2.57465	0.86972	0.33948	0.13747	0.05548
W_s (in mins)	3.35403	2.73367	2.54074	2.46723	2.4374
W_q (in mins)	0.93682	0.31646	0.12353	0.05002	0.02019
W_a	1.78158	1.02564	0.7201	0.55482	0.45124
P_w	0.52584	0.30855	0.17155	0.09016	0.04474
P_o	0.10%	0.12%	0.13%	0.13%	0.13%

Discussion of Findings/Results

i. Through the study it has been found that the most congested period is from 8:00 am to 11:00 am in morning and 4:00 pm to 6:00 p.m.. It's the time when most of the customers go to or come from their office, higher institutions and marketplace; or travelers through or within the state are in transit, thus increasing the inflow of customers at the petrol pump. There was congestion also on the Friday and the weekend.

ii. The arrival rate follows a Poisson distribution and the service rate follows an exponential distribution since P-Value = 0.995 > 0.05 and P-Value = 0.848 > 0.05 respectively.

iii. The existing queueing model is that of M/M/8: FCFS/∞/∞ and the number of arrival per hour is greater than the number of departures per hour as shown in **Figure 2**, leading to the arrival rate λ= 2.7483 customers/min that is greater than the service rate μ = 0.4137 customer/min showing that queue exists.

iv. The ρ = 66.432% obtained from the 10-servers model is closest to the mean utilization factor 67.808% as it has the least squared deviation from the average value for all models and hence selected as the minimum acceptable utilization factor for the system. It is apparent from the **Figure 3** and **Table 2** that this model provides a better result to this queueing problem in the existing 8-server model as the queue disappears and the waiting time reduces to a minimum. With this model, a customer spends an average duration of 2.54 minutes in the station and just 0.12 minute in the queue. Results further show that there is an average of 0.34 customers in the queue and 6.98 in the system with 33.568% of the time all the 10-servers are not busy. It is clear from the above results that customers spend little or no time in the queue to purchase PMS. In fact, a queue length of 0.34 shows that the queue has disappeared, and waiting time reduced to the barest minimum. The application of this model, however, implies the addition of two more servers which invariably involves, at least, two more attendants leading to additional cost, but the benefit derivable by this particularly outweighs this cost.

CONCLUSION

This study analyzed the existing queueing model quantitatively for customers purchasing PMS at the NNPC Mega Station; Jos then went ahead to formulate and propose a model that on application with system characteristics, gave optimal quantitative results on comparison with other models. The results of the analysis of the present and trial models of the service facility showed that average queue length, waiting time to customers as well as over-utilization of servers could be reduced when the number of servers is increased from eight to ten as it can also be seen in the results. Therefore, we now concluded that the aim of this project has been achieved through the objectives. The proposed model provides a tool to assist management of the service station and same service stations to take decisions more precisely as compared to decisions based on intuition and judgment about the number of facilities to be provided for efficient service delivery.

RECOMMENDATIONS

Following the analysis, observations and interviews carried out in this research work, the following recommendations are made:

(i) The Management should open two more servers for optimum usage and run of the facility.

(ii) Incentives can be given to creating over time; which will increase the utilization factor of the ten servers.

(iii)The management should train their attendants in multi-skills to enable them to offer free customer-friendly services of necessary vehicle maintenance such as checking type pressures, open air and topping up fluid levels as well as handling the petrol pump as quickly as possible. Because a fair proportion of customers especially the female motorists by observations and interviews have little or no inclination on how to carry out this routine maintenance. To improve customer satisfaction, loyalty, and goodwill leading to an improved state of the facility since there is still 33.568% of the time that all the 10-servers are not busy.

(iv)Any utilization value below the 66.432% minimum acceptable utilization rate of the proposed 10-server model is not encouraged as there will be an increment of idle time leading to less productive time.

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