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Batch arrival Retrial G-queue and an unreliable server with delayed repair

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Abstract: Single server batch arrival retrial G-queue and an unreliable server with delayed repair are analyzed. Positive customers arrive in batches according to Poisson processes. If the server is idle, one of the positive customers in the batch enters for service while the rest join the orbit. Otherwise all the customers enter the orbit. Arrival of negative customer removes the positive customer being in service from the system and causes the server breakdown. The repair of the failed server starts after a random amount of time known as delay time. Expected system size, orbit size, availability and failure frequency of the server are derived. Stochastic decomposition law is verified. Numerical examples are presented to illustrate the influence of the parameters on several performance characteristics.

Keywords: Retrial queue, G-queue, server breakdown, delayed repair and stochastic decomposition.

I. INTRODUCTION

Queueing systems with repeated attempts are characterized by the fact that a customer finding all the servers busy upon arrival must leave the service area and repeat his request for service after some random time. Between trials, the blocked customers join a pool of unsatisfied customers called 'orbit'. The retrial queueing system has been studied extensively due to its wide applicability in telephone switching system, telecommunication and computer networks. Recent works on retrial queue includes Aissani [1], Arivudainambi and Godhandaraman [2] and Artalejo and Li [3].

During the last decade, there has been an increasing interest in queueing system with negative customers. Queue with negative arrivals, called G-queue was first introduced by Gelenbe [5] with a view to modeling neural networks. Arrival of a negative customer induces positive customers to leave the system immediately. For a comprehensive analysis of queueing systems with negative arrivals, readers may refer to Gelenbe[6-8], Artalejo [4], Wu et.al [9] and Wu et.al [10]. In this paper, an unreliable batch arrival retrial G-queue with delayed repair is discussed.

II. MODEL DESCRIPTION

Consider a single server retrial queueing system with two types of independent arrivals, positive and negative. Positive customers arrive in batches according to Poisson process with rate λ^{+}. The batch size Y is a random variable with distribution function P(Y=k) $=c_k$, k=1,2,….., and probability generating function $C(z)=\sum_{k=1}^{\infty} c_k z^k$ having first two moments m_1 and m_2. If an arriving batch of positive customers finds the server free, one of the arrivals begins his service and others join the orbit. The retrial time of the customers in the retrial queue is generally distributed with distribution function A(x) with corresponding Laplace Stieltjes transform $A^{*}(s)$ and conditional completion rate $\eta(x) = a(x)/(1-A(x))$. The Service time follows a general distribution with distribution function B(x), Laplace Stieltjes transform $B^{*}(s)$, n^{th} factorial moments μ_n and conditional completion rate $\mu(x) = b(x)/(1-B(x))$.

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Vol. 3, Issue 5, May 2014

Negative customers arrive singly according to Poisson process with rate λ^-. The arrival of a negative customer removes the positive customer being in service from the system and makes the server breakdown. When the server fails, it stops providing service and waits for the repair to start. This waiting time of the server is known as delay time. The Delay time follows a general distribution with distribution function D(x), Laplace Stieltjes transform $D^*(s)$, n^{th} factorial moments θ_n and conditional completion rate $\theta(x) = w(x)/(1 - W(x))$.

The repair time also follows a general distribution with distribution function R(x), Laplace Stieltjes transform $R^*(s)$, n^{th} factorial moments β_n and conditional completion rate $\beta(x) = f(x)/(1 - F(x))$. As soon as the repair of the server is completed, the server enters the system and waits for a new customer.

III. ANALYSIS OF THE STEADY STATE DISTRIBUTION

Let N(t) denotes the number of customers in the orbit at time t and C(t) denotes the state of the server defined as

$$C(t) = \begin{cases} 0, \text{if the server is idle} \\ 1, \text{if the server is busy} \\ 2, \text{if the server is waiting for repair} \\ 3, \text{if the server is under repair} \end{cases}$$

For $t \geq 0$, define the supplementary variable as follows

(i) if C(t) = 0, $\xi(t)$ represents the elapsed retrial time at time t ;
(ii) if C(t) = 1, $\xi(t)$ represents the elapsed service time at time t ;
(iii) if C(t) = 2, $\xi(t)$ represents the elapsed delay time at time t ;
(iv) if C(t) = 3, $\xi(t)$ represents the elapsed repair time at time t .

Then the process $\{ X(t), t \geq 0 \} = \{C(t), N(t), \xi(t), t \geq 0 \}$ is a Markov Process.

Define the following probabilities

$I_0(t) = P\{C(t)=0, N(t) = 0\}, t > 0$

$I_n(x,t)dx = P\{C(t)=0, N(t) = n, x < \xi(t) \leq x+dx\}, n \geq 1, t \geq 0, x \geq 0$

$P_n(x,t)dx = P\{C(t)=1, N(t) = n, x < \xi(t) \leq x+dx\}, n \geq 0, t \geq 0, x \geq 0$

$D_n(x,t)dx = P\{C(t)=2, N(t)= n, x < \xi(t) \leq x+dx\}, n \geq 0, t \geq 0, x \geq 0$

$R_n(x,t)dx = P\{C(t)=3, N(t)= n, x < \xi(t) \leq x+dx\}, n \geq 0, t \geq 0, x \geq 0$

Define the steady state probabilities

$I_0 = \lim_{t\to\infty} I_0(t)$ $\quad$ $I_n(x) = \lim_{t\to\infty} I_n(x,t), x \geq 0, n \geq 1,$

$P_n(x) = \lim_{t\to\infty} P_n(x,t), x \geq 0, n \geq 0,$ $\quad$ $D_n(x) = \lim_{t\to\infty} D_n(x,t), x \geq 0, n \geq 0,$

$R_n(x) = \lim_{t\to\infty} R_n(x,t), x \geq 0, n \geq 0.$

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(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

The system of equilibrium equations governing the model, using supplementary variable technique are given below:

$$\lambda^{+} I_0 = \int_0^\infty P_0(x)\mu(x)dx + \int_0^\infty R_0(x)\beta(x)dx \qquad (1)$$

$$\frac{d}{dx} I_n(x) = -(\lambda^{+} + \eta(x)) I_n(x), n \geq 1 \qquad (2)$$

$$\frac{d}{dx} P_n(x) = -(\lambda + \mu(x)) P_n(x) + \sum_{k=1}^{n} \lambda^{+} c_k P_{n-k}(x), n \geq 0 \qquad (3)$$

$$\frac{d}{dx} D_n(x) = -(\lambda^{+} + \theta(x)) D_n(x) + \sum_{k=1}^{n} \lambda^{+} c_k D_{n-k}(x), n \geq 0 \qquad (4)$$

$$\frac{d}{dx} R_n(x) = -(\lambda^{+} + \beta(x)) R_n(x) + \sum_{k=1}^{n} \lambda^{+} c_k R_{n-k}(x), n \geq 0 \qquad (5)$$

The boundary conditions are

$$I_n(0) = \int_0^\infty P_n(x)\mu(x)dx + \int_0^\infty R_n(x)\beta(x)dx, n \geq 1 \qquad (6)$$

$$P_0(0) = \lambda^{+} c_1 I_0 + \int_0^\infty I_1(x)\eta(x)dx \qquad (7)$$

$$P_n(0) = \lambda^{+} c_{n+1} I_0 + \int_0^\infty I_{n+1}(x)\eta(x)dx + \sum_{k=1}^{n} \lambda^{+} c_k \int_0^\infty I_{n-k+1}(x)dx, n \geq 1 \qquad (8)$$

$$D_n(0) = \lambda^{-} \int_0^\infty P_n(x)dx, n \geq 0 \qquad (9)$$

$$R_n(0) = \int_0^\infty D_n(x)\theta(x)dx, n \geq 0 \qquad (10)$$

Normalization Condition is

$$I_0 + \sum_{n=1}^{\infty} \int_0^\infty I_n(x)dx + \sum_{n=0}^{\infty} \int_0^\infty P_n(x)dx + \sum_{n=0}^{\infty} \int_0^\infty D_n(x)dx + \sum_{n=0}^{\infty} \int_0^\infty R_n(x)dx = 1 \qquad (11)$$

Define the following probability generating functions, for $|z| \leq 1$

$$I(x,z) = \sum_{n=1}^{\infty} I_n(x) z^n, \qquad P(x,z) = \sum_{n=0}^{\infty} P_n(x) z^n,$$

$$D(x,z) = \sum_{n=0}^{\infty} D_n(x) z^n \quad \text{and} \quad R(x,z) = \sum_{n=0}^{\infty} R_n(x) z^n.$$

Multiplying equations (2) – (10) by z^n and summing over all possible values of n, we obtain the following results:

$$\left[\frac{\partial}{\partial x} + (\lambda^{+} + \eta(x))\right] I(x,z) = 0 \qquad (12)$$

$$\left[\frac{\partial}{\partial x} + (\lambda - \lambda^{+} C(z) + \mu(x))\right] P(x,z) = 0 \qquad (13)$$

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(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

$$\left[\frac{\partial}{\partial x} + (\lambda^{+} - \lambda^{+}C(z) + \theta(x))\right]D(x,z) = 0 \qquad (14)$$

$$\left[\frac{\partial}{\partial x} + (\lambda^{+} - \lambda^{+}C(z) + \beta(x))\right]R(x,z) = 0 \qquad (15)$$

$$I(0,z) = \int_0^\infty P(x,z)\mu(x)dx + \int_0^\infty R(x,z)\beta(x)dx - \lambda^{+}P_0 \qquad (16)$$

$$P(0,z) = \frac{1}{z}\int_0^\infty I(x,z)\eta(x)dx + \frac{\lambda^{+}C(z)}{z}\left[\int_0^\infty I(x,z)dx + I_0\right] \qquad (17)$$

$$D(0,z) = \lambda^{-}\int_0^\infty P(x,z)dx \qquad (18)$$

$$R(0,z) = \int_0^\infty D(x,z)\theta(x)dx \qquad (19)$$

The solutions of equations (12) – (15) are obtained as

$$I(x,z) = I(0,z)\exp\{-\lambda^{+}x\}[1-A(x)] \qquad (20)$$
$$P(x,z) = P(0,z)\exp\{-(\lambda-\lambda^{+}C(z))x\}[1-B(x)] \qquad (21)$$
$$D(x,z) = D(0,z)\exp\{-(\lambda^{+}-\lambda^{+}C(z))x\}[1-W(x)] \qquad (22)$$
$$R(x,z) = R(0,z)\exp\{-(\lambda^{+}-\lambda^{+}C(z))x\}[1-F(x)] \qquad (23)$$

Using equations (21) and (23), equation (16) yields

$$I(0,z) = P(0,z)B^{*}(g(z)) + R(0,z)F^{*}(h(z)) - \lambda^{+}I_0 \qquad (24)$$

where $g(z) = \lambda - \lambda^{+}C(z), h(z) = \lambda^{+} - \lambda^{+}C(z)$

Using equations (20) and (24), equation (17) yields

$$P(0,z) = \frac{\lambda^{+}I_0C(z) + I(0,z)\left[C(z) + (1-C(z))A^{*}(\lambda^{+})\right]}{z} \qquad (25)$$

Substituting the expression of P(0,z) in equation (21) and using the resultant expression, equation (18) gives

$$D(0,z) = \lambda^{-}\frac{\lambda^{+}I_0C(z) + I(0,z)[C(z) + (1-C(z))A^{*}(\lambda^{+})](1-B^{*}(g(z)))}{z(g(z))} \qquad (26)$$

Substituting the expression of D(0,z) in equation (22) and using the resultant expression, equation (19) gives

$$R(0,z) = \lambda^{-}\frac{\lambda^{+}I_0C(z) + I(0,z)[C(z) + (1-C(z))A^{*}(\lambda^{+})](1-B^{*}(g(z)))W^{*}(h(z))}{z(g(z))} \qquad (27)$$

Using equation (25) and (27), equation (24) yields

$$I(0,z) = \frac{\lambda^{+}I_0[C(z)(g(z)B^{*}(g(z)) + \lambda^{-}(1-B^{*}(g(z)))W^{*}(h(z))F^{*}(h(z))) - zg(z)]}{zg(z) - [C(z) + (1-C(z))A^{*}(\lambda^{+})][g(z)B^{*}(g(z)) + \lambda^{-}(1-B^{*}(g(z)))W^{*}(h(z))F^{*}(h(z))]} \qquad (28)$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

The partial probability generating function of orbit size when the server is idle is given by

$$I(z) = \int_0^\infty I(x,z)dx = \frac{I_0[1-A^*(\lambda^+)][C(z)(g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))) - zg(z)]}{zg(z) - [C(z) + (1-C(z))A^*(\lambda^+)][g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))]} \quad (29)$$

The partial probability generating function of the orbit size when the server is busy is given by

$$P(z) = \int_0^\infty P(x,z)dx = \frac{I_0\lambda^+(C(z)-1)A^*(\lambda^+)(1-B^*(g(z))}{zg(z) - [C(z) + (1-C(z))A^*(\lambda^+)][g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))]} \quad (30)$$

The partial probability generating function of the orbit size when the server is waiting for repair is given by

$$D(z) = \int_0^\infty D(x,z)dx = \frac{-I_0\lambda^-A^*(\lambda^+)(1-B^*(g(z))(1-W^*(h(z))}{zg(z) - [C(z) + (1-C(z))A^*(\lambda^+)][g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))]} \quad (31)$$

The partial probability generating function of the orbit size when the server is under repair is given by

$$R(z) = \int_0^\infty R(x,z)dx = \frac{-I_0\lambda^-A^*(\lambda^+)(1-B^*(g(z))(1-F^*(h(z))W^*(h(z))}{zg(z) - [C(z) + (1-C(z))A^*(\lambda^+)][g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))]} \quad (32)$$

IV. PERFORMANCE MEASURES

Probability that the server is idle is given by

$$I = \lim_{z\to 1} I(z) = \frac{(1-A^*(\lambda^+))[\lambda^-(m_1-1) + \lambda^+ m_1(1+\lambda^-\beta_1 + \lambda^-\theta_1)(1-B^*(\lambda^-))]}{\lambda^- A^*(\lambda^+)} \quad (33)$$

Probability that the server is busy is given by

$$P = \lim_{z\to 1} P(z) = \frac{\lambda^+ m_1(1-B^*(\lambda^-))}{\lambda^-} \quad (34)$$

Probability that the server is waiting for repair is given by

$$D = \lim_{z\to 1} D(z) = \lambda^+ m_1\theta_1(1-B^*(\lambda^-)) \quad (35)$$

Probability that the server is under repair is given by

$$R = \lim_{z \to 1} R(z) = \lambda^{+} m_1 \beta_1 (1 - B^{*}(\lambda^{-})) \tag{36}$$

The unknown constant I_0 can be determined by using the normalization condition (11) as

$$I_0 = \frac{\lambda^{-}(1 - m_1 + m_1 A^{*}(\lambda^{+})) - \lambda^{+} m_1 (1 + \lambda^{-}\beta_1 + \lambda^{-}\theta_1)(1 - B^{*}(\lambda^{-}))}{\lambda^{-} A^{*}(\lambda^{+})} \tag{37}$$

Probability generating function of the number of customers in the orbit is given by

$P_q(z) = I_0 + I(z) + P(z) + D(z) + R(z)$

$$= \frac{I_0 A^{*}(\lambda^{+})[[z - B^{*}(g(z))]g(z) + [\lambda^{+}(c(z) - 1) - \lambda^{-}](1 - B^{*}(g(z)))]]}{zg(z) - [C(z) + (1 - C(z))A^{*}(\lambda^{+})][g(z)B^{*}(g(z)) + \lambda^{-}(1 - B^{*}(g(z)))W^{*}(h(z))F^{*}(h(z))]} \tag{38}$$

Mean number of customers in the orbit L_q under steady state condition is given by

$$L_q = \lim_{z \to 1} \frac{d}{dz} P_q(z) = \frac{Dr_1' Nr_1'' - Nr_1' Dr_1''}{2(Dr_1')^2}$$

Where, $Nr_1(z)$ and $Dr_1(z)$ represent the numerator and denominator of $P_q(z)$

$$Nr_1' = I_0 A^{*}(\lambda^{+})\lambda^{-}$$

$$Nr_1'' = -2 I_0 A^{*}(\lambda^{+})\lambda^{+} m_1$$

$$Dr_1' = \lambda^{-}(1 - m_1 + m_1 A^{*}(\lambda^{+})) - \lambda^{+} m_1 (1 + \lambda^{-}\beta_1 + \lambda^{-}\theta_1)(1 - B^{*}(\lambda^{-}))$$

$$\begin{aligned} Dr_1'' = & -\lambda^{+} m_2 (1 - B^{*}(\lambda^{-})) - 2\lambda^{+} m_1 - \lambda^{-} m_2 (1 - A^{*}(\lambda^{+})) - 2 m_1 (1 - A^{*}(\lambda^{+}))[-\lambda^{+} m_1 B^{*}(\lambda^{-}) + \lambda^{+}\lambda^{-} m_1 (1 - B^{*}(\lambda^{-}))(\beta_1 + \theta_1)] \\ & + 2\lambda^{+2} m_1^2 \mu_1 - 2\lambda^{+2}\lambda^{-} m_1^2 \beta_1 \theta_1 (1 - B^{*}(\lambda^{-})) + 2\lambda^{+2}\lambda^{-} m_1^2 \mu_1 (\beta_1 + \theta_1) - \lambda^{+}\lambda^{-} m_2 (\beta_1 + \theta_1)(1 - B^{*}(\lambda^{-})) \\ & - \lambda^{+2}\lambda^{-} m_1^2 (\beta_2 + \theta_2)(1 - B^{*}(\lambda^{-})) \end{aligned} \tag{39}$$

Probability generating function of the number of customers in the system is given by

$P_s(z) = I_0 + I(z) + zP(z) + D(z) + R(z)$

$$= \frac{I_0 A^{*}(\lambda^{+})[[z - B^{*}(g(z))]g(z) + [\lambda^{+} z(c(z) - 1) - \lambda^{-}](1 - B^{*}(g(z)))]]}{zg(z) - [C(z) + (1 - C(z))A^{*}(\lambda^{+})][g(z)B^{*}(g(z)) + \lambda^{-}(1 - B^{*}(g(z)))W^{*}(h(z))F^{*}(h(z))]} \tag{40}$$

Mean number of customers in the system L_s under steady state condition is given by

$$L_s = \lim_{z \to 1} \frac{d}{dz} P_s(z) = L_q + P \tag{41}$$

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Vol. 3, Issue 5, May 2014

V. STOCHASTIC DECOMPOSITION

Theorem : The number of customers in the system under steady state (L_s) can be expressed as the sum of two independent random variables, one of which is the total number of customers (L) in the batch arrival classical G-queue and unreliable server with delayed repair and the other is the number of customers in the orbit given that the server is idle (L_I).

Proof:

The probability generating function $\pi(z)$ of the system size in the classical batch arrival G-queue and unreliable server with delayed repair is given by

$$\pi(z) = \frac{[zg(z) - g(z)B^*(g(z))) + \lambda^+ z(c(z)-1)(1-B^*(g(z)) - \lambda^-(1-B^*(g(z))][\lambda^- - \lambda^+ m_1(1-B^*(\lambda^-))(1+\lambda^-\beta_1 + \lambda^-\theta_1)]}{\lambda^-[zg(z) - [g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))]]} \tag{42}$$

The probability generating function $\chi(z)$ of the number of customers in the orbit when the system is idle is given by

$$\chi(z) = \frac{I_0 + I(z)}{I_0 + I(1)} = \frac{\begin{array}{c}[[zg(z) - (g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))]]* \\ [\lambda^-(1-m_1+m_1A^*(\lambda^+)) - \lambda^+ m_1(1+\lambda^-\beta_1+\lambda^-\theta_1)(1-B^*(\lambda^-))]\end{array}}{\begin{array}{c}\{zg(z) - [g(z)B^*(g(z)) + \lambda^-(1-B^*(g(z)))W^*(h(z))F^*(h(z))](C(z)+(1-C(z))A^*(\lambda^+))\} \\ \{\lambda^- - \lambda^+ m_1(1+\lambda^-\beta_1+\lambda^-\theta_1)(1-B^*(\lambda^-))\}\end{array}} \tag{43}$$

From equation (42) and (43), we see that $P_s(z) = \pi(z)\,\chi(z)$.

Hence, $L_s = L + L_I$.

VI. RELIABILITY INDICES

Let A(t) be pointwise availability of the server at time 't', that is the probability that the server is either serving a customer or idle. We define the steady state availability of the server as $A = \lim_{t\to\infty} A(t)$.

Availability of the server is given by

$$A = 1 - D(1) - R(1) = 1 - \lambda^+ m_1 (1 - B^*(\lambda^-))(\beta_1 + \theta_1)$$

Failure frequency of the server is given by

$$F = \lambda^- P(1) = \lambda^+ m_1 (1 - B^*(\lambda^-))$$

VII. NUMERICAL RESULTS

In this section, we give some numerical results to illustrate the effect of parameters on the performance characteristics. For numerical calculation, we assume that the distributions of service time, retrial time, delay time and repair time follow exponential with rate μ,η,θ and β.

The effect of varying arrival rate λ^+ and λ^- with the fixed values $(\mu, \eta, \theta, \beta) = (20, 0.5, 6, 3)$ on I_0-probability that the server is idle in the empty system, L_s-Mean number of customers in the system, L_q -Mean number of customers in the orbit, A -Availability of the server and F - Failure frequency of the server is given in Table (1). From the table, it is clear that increase in λ^+ and λ^- , increases L_s ,L_q and decreases the other measures.

IJIRSET

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Vol. 3, Issue 5, May 2014

Table (2) provides I_0,L_s,L_q, A and F by varying the values of μ and η for arbitrary values (λ^+, λ^-, θ, β)=(0.5, 0.5, 6, 3). Here, the table reveals that as μ and η increase, I_0 increases, L_s and L_q decreases. As μ increase, A increases and F decreases.

Table (3) shows that the effect of varying θ and β with the fixed values (λ^+, λ^- , μ, η)=(0.5, 0.5, 20, 0.5) on the performance measures. From this table we observe that increase in β and θ, increases I_0 and A, decreases L_s and L_q and has no effect on F.

Table 1. Performance Measures by varying λ^+ and λ^-

λ^+	λ^-	I_0	L_s	L_q	A	F
0.5	0.1	0.5269	0.9278	0.8930	0.9983	0.0035
	0.2	0.5238	0.9403	0.9057	0.9965	0.0069
	0.3	0.5207	0.9529	0.9184	0.9948	0.0103
	0.4	0.5176	0.9655	0.9312	0.9931	0.0137
	0.5	0.5146	0.9781	0.9440	0.9915	0.0171
0.4	0.1	0.6273	0.6127	0.5848	0.9986	0.0028
	0.2	0.6251	0.6186	0.5909	0.9972	0.0055
	0.3	0.6229	0.6246	0.5970	0.9959	0.0083
	0.4	0.6207	0.6305	0.6031	0.9945	0.0110
	0.5	0.6185	0.6364	0.6091	0.9932	0.0137
0.3	0.1	0.7249	0.3906	0.3697	0.9990	0.0021
	0.2	0.7234	0.3933	0.3725	0.9979	0.0042
	0.3	0.7219	0.3959	0.3752	0.9969	0.0062
	0.4	0.7215	0.3985	0.3779	0.9959	0.0082
	0.5	0.7190	0.4011	0.3807	0.9949	0.0102

Table 2. Performance Measures by varying μ and η

μ	η	I_0	L_s	L_q	A	F
10	0.3	0.1111	8.3570	8.2930	0.9833	0.0333
	0.4	0.3125	2.2954	2.2288	0.9833	0.0333
	0.5	0.4333	1.6329	1.2962	0.9833	0.0333
	0.6	0.5139	0.9848	0.9182	0.9833	0.0333
	0.7	0.5714	0.7800	0.7134	0.9833	0.0333
20	0.3	0.2195	3.6831	3.6490	0.9915	0.0171
	0.4	0.4040	1.5293	1.4952	0.9915	0.0171
	0.5	0.5146	0.9781	0.9440	0.9915	0.0171
	0.6	0.5884	0.7259	0.6917	0.9915	0.0171
	0.7	0.6411	0.5812	0.5417	0.9915	0.0171
25	0.3	0.2418	3.2334	3.2060	0.9931	0.0137
	0.4	0.4228	1.4091	1.3816	0.9931	0.0137
	0.5	0.5314	0.9109	0.8835	0.9931	0.0137
	0.6	0.6038	0.6784	0.6509	0.9931	0.0137
	0.7	0.6555	0.5437	0.5163	0.9931	0.0137

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Vol. 3, Issue 5, May 2014

Table 3. Performance Measures by varying θ and β

β	θ	I_0	L_s	L_q	*A*	*F*
3	1	0.4862	1.1505	1.1163	0.9772	0.0171
	2	0.5033	1.0368	1.0027	0.9858	0.0171
	3	0.5089	1.0058	0.9717	0.9886	0.0171
	4	0.5118	0.9916	0.9574	0.9900	0.0171
	5	0.5135	0.9834	0.9493	0.9909	0.0171
4	1	0.4890	1.1320	1.0979	0.9787	0.0171
	2	0.5061	1.0216	0.9874	0.9872	0.0171
	3	0.5118	0.9916	0.9574	0.9900	0.0171
	4	0.5146	0.9778	0.9437	0.9915	0.0171
	5	0.5163	0.9699	0.9358	0.9923	0.0171
5	1	0.4907	1.1213	1.0872	0.9795	0.0171
	2	0.5078	1.0128	0.9787	0.9880	0.0171
	3	0.5135	0.9834	0.9493	0.9909	0.0171
	4	0.5163	0.9699	0.9358	0.9923	0.0171
	5	0.5180	0.9622	0.9281	0.9932	0.0171

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