COMPARISON OF VARIOUS PWM TECHNIQUES FOR FIELD ORIENTED CONTROL VSI FED PMSM DRIVE

P. Ramana¹, B. Santhosh Kumar², Dr. K. Alice Mary³, Dr. M. Surya Kalavathi⁴

Associate Professor, Dept. of EEE, GMRIT, Rajam, Srikakulam, AP-532127, India¹
PG student, Dept. of EEE, GMRIT, Rajam, Srikakulam, AP-532127, India²
Professor and Principal, VIIT, Visakhapatnam, AP-520040, India³
Professor, Dept. of EEE, JNTUCE, Hyderabad, AP-500072, India⁴

ABSTRACT: Permanent Magnet Synchronous Motor (PMSM) drives have been increasingly applied in a variety of industrial applications which require fast dynamic response and accurate control over wide speed ranges. However, there still exist challenges to design position-sensor less vector control of PMSM operating in a wide speed range, which covers both constant-torque and constant-power region. Field oriented control (FOC) of permanent magnet synchronous motor (PMSM) is one of the widely used methods for the speed control of the motor. The feasibility and effectiveness of various pulse width modulation techniques implemented for PMSM are addressed in this paper and verified by computer simulation. The whole drive system is simulated in MATLAB/SIMULINK based on the mathematical model of the system devices including PMSM and inverter. The aim of the drive system is to have speed control over wide speed range. Simulation results show that the speed controller has a good dynamic response.

Keywords: Permanent Magnet Synchronous Motor, Field Oriented Control, Sine Pulse-width Modulation, Space Vector Pulse-width Modulation, Third harmonic injected Pulse-width Modulation

I. INTRODUCTION

Permanent magnet synchronous motor drives (PMSM) offers many advantages over the induction motor, such as overall efficiency, effective use of reluctance torque, smaller losses and compact motor size. In recent years many studies have been developed to find out different solutions for the PMSM drive control having the features of quick and precise torque response, and the field oriented control has been recognized as viable and robust solution to achieve these requirements.[2]

The practical application of the system, using direct torque control, is handicapped by the difficulty of starting under full load due to the unknown initial rotor position. A lot of efforts have been made to detect the initial rotor position. Among them, the most versatile method is to make use of the structural and magnetic saturation saliencies which exist in the PMSM. The structural saliency could be employed to acquire the position of the rotor axis, while the saturation saliency, which is generated by the rotor permanent magnets, can be used to detect the magnetic polarity[1]. The main objective of the vector control is achieved by using a d-q rotating reference frame synchronously with the rotor flux space vector. In ideally, field-oriented control, the rotor flux linkage axis is forced to align with the d-axes. In field-oriented control, the torque equation becomes analogous to the DC machine [3]. The inverter plays an important role to provide better sinusoidal voltage or current, speed control of machines becomes finer. It is possible only if inverter gets better gate pulses.[4]

This paper presents a nonlinear model of PMSM, which incorporates both the structural and saturation saliencies to enable the numerical simulation of new rotor position detection. In this model, the self and mutual differential inductances of the phase windings are expressed as functions of the rotor position and stator current. Based on the model, the field oriented control (FOC) scheme is simulated within the MATLAB/ SIMULINK environment.

II. MATHEMATICAL MODELLING OF PMSM

The voltage equations for the permanent magnet motor in rotor reference frame are

\[ v_{qs} = r_{qs}i_{qs} + l_{qs}p_{qs} + l_{ad}p_{qr} + \omega l_{ds}i_{ds} + \omega l_{dr}i_{dr} + \omega v \]  
--- (1)

\[ v_{ds} = r_{ds}i_{ds} + l_{ds}p_{ds} + l_{ad}p_{dq} - \omega l_{qs}i_{qs} - \omega l_{dq}i_{dq} \]  
--- (2)

\[ v_{qr} = r_{qr}i_{qr} + l_{qr}p_{qr} + l_{dq}p_{qs} \]  
--- (3)

\[ v_{dr} = r_{dr}i_{dr} + l_{dr}p_{dr} + l_{dq}p_{ds} \]  
--- (4)
Where, $\psi$- air gap flux linkage

The eqn. (1) can be rewritten as

$$v_{qs} = (v_{qs} - \omega l_{ad} i_{qs}) = r_{a} i_{qs} + l_{ad} p i_{qr} + l_{aq} p i_{qs} + \omega l_{dr} i_{dr} + \omega l_{ds} i_{ds} + \omega l_{ds} i_{ds}$$  --- (5)

The electrical torque developed is

$$T_e = \frac{3}{2} P \left[ l_{ad} (l_{ad} - l_{aq}) i_{qs} i_{ds} + l_{ad} i_{qs} i_{dr} - l_{aq} i_{qr} i_{ds} + \psi i_{qs} \right]$$  --- (6)

The torque balance equation is

$$\frac{2}{P} i_p \omega_0 = T_e - \frac{2}{P} \beta \omega_0$$  --- (7)

Where all voltages ($v$) and currents ($i$) refer to the rotor reference frame. The subscripts $qs$, $ds$, $qr$ and $dr$ correspond to $q$ and $d$ axis quantities for the stator($s$) and rotor($r$) in all combinations, $r_a$ denotes the armature resistance, $l_{ad}$ denotes quadrature axis inductance, $l_{ds}$ denotes direct axis inductance etc. and $T_e$ is the developed torque. The rotor speed is given by $\omega_0$ and the load torque by $T_l$. $J$ is moment of inertia, $P$ is the number of poles and $\beta$ is the co-efficient of viscous friction. The derivative operator is represented by the symbol $p$.

Representing the voltage eqns. (1)-(5) into a state space representation as given below:

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} r_a & \omega l_{ad} & 0 & 0 \\ -\omega l_{aq} & r_a & 0 & 0 \\ 0 & 0 & r_{qr} & 0 \\ 0 & 0 & 0 & r_{dr} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} + \begin{bmatrix} l_{ad} & 0 & l_{aq} & 0 \\ 0 & l_{ds} & 0 & l_{ad} \\ 0 & l_{aq} & l_{qr} & 0 \\ 0 & l_{ad} & 0 & l_{dr} \end{bmatrix} \begin{bmatrix} p i_{qs} \\ p i_{ds} \\ p i_{qr} \\ p i_{dr} \end{bmatrix}$$  --- (8)

### III. FIELD ORIENTED CONTROL (FOC) OF PMSM

Field Oriented Control demonstrates that an induction motor or synchronous motor could be controlled like a separately excited dc motor by the orientation of the stator magneto motive forces.or current vector in relation to the rotor flux to achieve a desired objective. It usually refers to controllers which maintain a 90° electrical angle between rotor and stator field components. In DC motors, the flux and torque producing currents are orthogonal and can be controlled independently. The magneto motive forces, developed by these currents are also held orthogonal. The torque developed is given by the equation

$$T_e = K_a \phi (I_f) I_a$$  --- (9)

Hence the flux is only dependent on the field winding current. If the flux is held constant, then the torque can be controlled by the armature current. For this reason DC machines are said to have decoupled or have independent control of torque and flux. In AC machines, the stator and rotor fields are not orthogonal to each other. The only current that can be controlled is the stator current. Field Oriented Control is the technique used to achieve the decoupled control of torque and flux by transforming the stator current quantities (phase currents) from stationary reference frame to torque and flux producing currents components in rotating reference frame.

**Advantages of FOC:**
- Transformation of a complex and coupled AC model into a simple linear system
- Independent control of torque and flux, similar to a DC motor
- Fast dynamic response and good transient and steady state performance
- High torque and low current at start up
- High Efficiency
- Wide speed range through field weakening

### IV. PULSE WIDTH MODULATION

**A. Principle of Pulse Width Modulation (PWM)**

Fig.1 shows circuit model of a single-phase inverter with a centre-taped grounded DC bus, and Fig.2 illustrates principle of pulse width modulation.
Fig. 1 Circuit model of a single-phase inverter

As depicted in Fig. 2, the inverter output voltage is determined in the following:
- When $V_{\text{control}} > V_{\text{in}}$, $V_{A0} = V_{\text{dc}}/2$
- When $V_{\text{control}} < V_{\text{in}}$, $V_{A0} = -V_{\text{dc}}/2$

Also, the inverter output voltage has the following features:
- PWM frequency is the same as the frequency of $V_{\text{in}}$
- Amplitude is controlled by the peak value of $V_{\text{control}}$
- Fundamental frequency is controlled by the frequency of $V_{\text{control}}$

B. Principle of Sinusoidal PWM

Fig. 3 shows circuit model of three-phase PWM inverter and Fig. 4 shows waveforms of carrier wave signal ($V_{\text{in}}$) and control signal ($V_{\text{control}}$). Inverter output line to neutral voltages are $V_{A0}$, $V_{B0}$, $V_{C0}$, inverter output line to line voltages are $V_{AB}$, $V_{BC}$, $V_{CA}$ respectively.

Fig. 3 Three-phase PWM Inverter

As described in Fig.4, the frequency of $V_{\text{in}}$ and $V_{\text{control}}$ is:
- Frequency of $V_{\text{in}} = f_s$
- Frequency of $V_{\text{control}} = f_1$

Where, $f_s$ = PWM frequency and $f_1$ = Fundamental frequency

The inverter output voltages are determined as follows:
- When $V_{\text{control}} > V_{\text{in}}$, $V_{A0} = V_{\text{dc}}/2$
- When $V_{\text{control}} < V_{\text{in}}$, $V_{A0} = -V_{\text{dc}}/2$

Where, $V_{AB} = V_{A0} - V_{B0}$, $V_{BC} = V_{B0} - V_{C0}$, $V_{CA} = V_{C0} - V_{A0}$

C. Principle of Space Vector PWM

The circuit model of a typical three-phase voltage source PWM inverter is shown in Fig. 5. $S_1$ to $S_6$ are the six power switches that shape the output, which are controlled by the switching variables $a$, $a'$, $b$, $b'$ and $c$, $c'$. When an
upper transistor is switched on, i.e., when a, b or c is 1, the corresponding lower transistor is switched off, i.e., the corresponding a', b' or c' is 0. Therefore, the on and off states of the upper transistors S1, S3 and S5 can be used to determine the output voltage.

Fig. 5 Three-phase voltage source PWM Inverter.

The relationship between the switching variable vector \([a, b, c]^T\) and the line-to-line voltage vector \([V_{ab}, V_{bc}, V_{ca}]^T\) is given as follows

\[
\begin{bmatrix}
V_{ab} \\
V_{bc} \\
V_{ca}
\end{bmatrix} =
\begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix} = V_{dc} \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix} \quad \text{--- (10)}
\]

Also, the relationship between the switching variable vector \([a, b, c]^T\) and the phase voltage vector \([V_a, V_b, V_c]^T\) can be expressed below.

\[
\begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix} =
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix} = \frac{V_{dc}}{3} \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix} \quad \text{--- (11)}
\]

As illustrated in the Fig 6, there are eight possible combinations of on and off patterns for the three upper power switches. The on and off states of the lower power devices are opposite to the upper one and so are easily determined once the states of the upper power transistors are determined. According to equations stated above the eight switching vectors, output line to neutral voltage (phase voltage), and output line-to-line voltages in terms of DC-link \(V_{dc}\), are given in Table 1 and Fig.8 shows the eight inverter voltage vectors \((V_0 \text{ to } V_7)\).

<table>
<thead>
<tr>
<th>TABLE 1 Switching Vectors, Phase Voltages and Output Line to Line Voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Vectors</td>
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<tr>
<td>-----------------</td>
</tr>
<tr>
<td></td>
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<tr>
<td>V_0</td>
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<tr>
<td>V_1</td>
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<tr>
<td>V_2</td>
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<tr>
<td>V_3</td>
</tr>
<tr>
<td>V_4</td>
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<tr>
<td>V_5</td>
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<tr>
<td>V_6</td>
</tr>
<tr>
<td>V_7</td>
</tr>
</tbody>
</table>

**Note:** The respective voltage should be multiplied by \(V_{dc}\).
The fundamental component is higher than the DC supply voltage. Consequently, it provides a better utilization of the DC supply voltage.

Harmonic Injection PWM

The sinusoidal PWM is the simplest modulation scheme to understand but it is unable to fully utilize the available DC bus supply voltage. Due to this problem, the third-harmonic injection pulse-width modulation (THIPWM) technique was developed to improve the inverter performance. The THIPWM is implemented in the same manner as the SPWM, that is, the reference waveforms are compared with a triangular waveform. As a result, the amplitude of the reference waveforms does not exceed the DC supply voltage \( V_{dc} \), but the fundamental component is higher than the supply voltage \( V_{dc} \). As mentioned above, this is approximately 15% to 5% higher in amplitude than the normal sinusoidal PWM. Consequently, it provides a better utilization of the DC supply voltage.

\[
S_1 = T_1 + T_2 + T_0/2
\]

\[
S_2 = T_0/2
\]

\[
S_3 = T_0/2
\]

\[
S_4 = T_0/2
\]

\[
S_5 = T_0/2
\]

\[
S_6 = T_0/2
\]

\[
\therefore T_0 = T_2 - (T_1 + T_2), \quad 0 \leq \alpha \leq 60^\circ
\]

\[
\therefore T_1 = \sqrt{3} T_2 V_{ref} V_{dc}
\]

\[
\therefore T_2 = \frac{\sqrt{3} T_2 V_{ref} V_{dc}}{\sin (\alpha - n - \frac{1}{3}) \pi)
\]

**TABLE II Switching Time Calculation at Each Sector**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Upper Switches(S1, S2, S3)</th>
<th>Lower Switches(S4, S5, S6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( S_1 = T_1 + T_2 + T_0/2 )</td>
<td>( S_2 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_2 = T_0/2 )</td>
<td>( S_3 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_3 = T_0/2 )</td>
<td>( S_4 = T_0/2 )</td>
</tr>
<tr>
<td>2</td>
<td>( S_1 = T_1 + T_2 + T_0/2 )</td>
<td>( S_2 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_2 = T_0/2 )</td>
<td>( S_3 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_3 = T_0/2 )</td>
<td>( S_4 = T_0/2 )</td>
</tr>
<tr>
<td>3</td>
<td>( S_1 = T_0/2 )</td>
<td>( S_2 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_2 = T_0/2 )</td>
<td>( S_3 = T_0/2 )</td>
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<tr>
<td></td>
<td>( S_3 = T_0/2 )</td>
<td>( S_4 = T_0/2 )</td>
</tr>
<tr>
<td>4</td>
<td>( S_1 = T_0/2 )</td>
<td>( S_2 = T_0/2 )</td>
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<tr>
<td></td>
<td>( S_2 = T_0/2 )</td>
<td>( S_3 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_3 = T_0/2 )</td>
<td>( S_4 = T_0/2 )</td>
</tr>
<tr>
<td>5</td>
<td>( S_1 = T_2 + T_0/2 )</td>
<td>( S_2 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_2 = T_0/2 )</td>
<td>( S_3 = T_0/2 )</td>
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<tr>
<td></td>
<td>( S_3 = T_0/2 )</td>
<td>( S_4 = T_0/2 )</td>
</tr>
<tr>
<td>6</td>
<td>( S_1 = T_2 + T_0/2 )</td>
<td>( S_2 = T_0/2 )</td>
</tr>
<tr>
<td></td>
<td>( S_2 = T_0/2 )</td>
<td>( S_3 = T_0/2 )</td>
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<tr>
<td></td>
<td>( S_3 = T_0/2 )</td>
<td>( S_4 = T_0/2 )</td>
</tr>
</tbody>
</table>
Fig. 7: Third Harmonic Injection

Fig. 8: Reference Voltages (a,b,c), Triangular Waveforms ($V_T$), and Output Voltage ($V_{a0}, V_{b0}, V_{c0}$).

V. SIMULATION RESULTS

Fig. 9: Simulink Model for SVPWM based FOC of PMSM

Fig. 10: Torque characteristics for SVPWM based FOC of PMSM

Fig. 11: Speed characteristics for SVPWM based FOC of PMSM
Fig. 12: Simulink Model for SPWM based FOC of PMSM

Fig. 13: Torque characteristics for SPWM based FOC of PMSM

Fig. 14: Speed characteristics for SPWM based FOC of PMSM

Fig. 15: Simulink Model for Third Harmonic Injection PWM based FOC of PMSM
Comparison of SVPWM and SPWM based Field Oriented Control of PMSM

Table III Comparison of THD of various PWM techniques

<table>
<thead>
<tr>
<th>PWM technique</th>
<th>Sine PWM</th>
<th>Space Vector PWM</th>
<th>Third Harmonic Injection PWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>THD</td>
<td>28.79</td>
<td>12.47</td>
<td>4.59</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

This paper proposes a method for PMSM drive based on FOC using SVPWM, SPWM and Third Harmonic Injection PWM. The proposed predictive method estimates the stator current at the next sample using the motor equations. Also, on the basis of the field-oriented control, the reference currents of PMSM are calculated in terms of minimum torque ripples and fixed speed operation. Thereupon, the difference between estimated and calculated reference currents is applied to choose a proper switching vector based on SVPWM, and so as with the SPWM. But while considering the Third Harmonic Injection PWM it is highly non-linear, so it increases the non-linearity of the system. So it cannot be used in controlling the PMSM. Several numerical simulations using MATLAB-Simulink have been carried out in steady-state and transient-state. According to the results, the proposed technique is able to reduce torque ripple, speed error, and time to reach transient-state at abrupt mechanical load changes. In addition, we could have some other advantages like, constant switching frequency, fast transient response, and tunable output torque and speed with lower error.
REFERENCES


BIOGRAPHY

Sri P.Ramana received B.Tech (EEE) degree, First class with distinction from JNTU, Hyderabad in May 2001. He received M.Tech degree First class with distinction from JNTU, Hyderabad in 2006. He is in teaching profession for last 13 years. He is pursuing his Ph.D at JNTU, Hyderabad, A.P, India. He has published 16 Research Papers. His research interests include control systems and electrical machine drives. At present he is working as Associate Professor in GMR Institute of Technology, Rajam, AP, India.

B.Santhosh Kumar received B.Tech (EEE) degree, First class from JNTU, Kakinada in May 2011. At present he is pursuing his M.Tech (Power & Industrial Drives) at GMR Institute of Technology, Rajam, Affiliated to JNTU, Kakinada, A.P, India.

Dr K. Alice Mary received B.E (Electrical Power) degree in December 1981 from Mysore University. She received master’s degree M.E (power apparatus & electric drives) in1989 from IIT, Roorkee, UP. She received Ph.D from IIT Kharagpur, WB. She is in teaching profession for last 30 years. She has published 30 Research Papers and presently guiding 4 Ph.D. Scholars. She received best paper award at national system conference Tiruvananthapuram in 1996 for her research work. She is a recipient of “Mahila Jyothi” Award(National award) for her overall educational excellence by Integrated Council for Socio-Economic progress, New Delhi, 2002 and “Mother Teresa Excellence Award”(National award) in 2002 by Front for Nations Progress, Bangalore and Shatstra award and Vijeta award for academic excellence and authoring a Technical book. Her research interests includes control systems and power electronics control of electrical machine drives. At present she is working as Professor and Principal at VIIT, Duvvada, Visakhapatnam, AP, India.

Dr. M Surya Kalavathi received B.E (EE) degree in the 1988 from SV University, Tirupathi. She received master’s degree M.E (Power Systems) in1993 from SV University, Tirupathi, AP. She received Ph.D from JNTU, Hyderabad in 2006. She received Post Doctoral at Carnegie Mellon University, USA in 2008 She is in teaching profession for last 20 years. She has published 25 Research Papers and presently guiding 5 Ph.D. Scholars. She has specialised in Power Systems, High Voltage Engineering and Control Systems. Her research interests include Simulation studies on Transients of different power system equipment. At present she is working as Professor at JNTUCE, Hyderabad, AP, India.