ABSTRACT: In this paper, a dynamic modelling of five-phase induction motor is described in a step by step approach. A dq model based on transformation theory for five-phase induction machine is presented. A detailed implementation of an indirect-type five-phase field oriented control including the hysteresis-type pulse width modulation (PWM) current regulator is described. Simulations have been carried out for different load conditions.

KEYWORDS: Dynamic Model, Five-Phase Induction Motor, Indirect Vector Control.

I. INTRODUCTION

The advantages of five-phase machines are found over three-phase machines such as an increase in current per phase without the need to increase the phase voltage, reduction in amplitude and increase in frequency of pulsating torques, fault tolerance, stability and lower current ripple [1]. It has also been shown that increasing the number of phases can result in an increase in the torque/ampere relation for the same volume machines [2]-[4]. Hence the analysis, design and application of machines with high phase numbers require adequate mathematical models to be established, through which their performance and advantages can be assessed.

S. Boora et al. [5] presented paper on dynamic d-q axis model of three-phase asynchronous motor in synchronously rotating frame. In this paper, authors have transferred three-phase voltage in to two-phase voltage axis using dq transformation. S. Gupta and S. Wadhwani [6] discussed modeling of induction motor using synchronously rotating reference frame. Mathematical modeling of three-phase induction motor was discussed in this paper. The steady-state and transient response of the motor have shown with varying load on motor. Five-phase induction motor performance under symmetrical and asymmetrical connection have discussed in [7]-[8]. In these papers, authors have mentioned that if one or two legs of the inverter are open circuited then also motor work satisfactory with the increment of fundamental and third harmonic currents. Simulink induction machine models are available in the literature [9]-[10], but they appear to be black-boxes with no internal details.

In this paper each block solves one of the model equations. The necessary differential equations describing the performance of five-phase induction machines in d-q frame of reference based on transformation theory are presented in this paper. A detailed implementation of an indirect-type five-phase field-orientation control including the hysteresis type of PWM current regulator is illustrated [11]. Simulations have been carried out for different load conditions.

II. DYNAMIC MODELLING OF FIVE-PHASE INDUCTION MOTOR

The five-phase stationary reference frame variables are transformed into two-phase stationary reference frame variables and then transform these variables into synchronously rotating reference frame and vice-versa. The voltage as-bs-cs-ds-es can be resolved into \( V_{ds} \) and \( V_{qs} \) components and can be represented in the matrix form using (1).

\[
\begin{bmatrix}
V_{ds} \\
V_{qs}
\end{bmatrix} = \frac{2}{5} \begin{bmatrix}
\cos\theta & \cos(\theta - 2\pi/5) & \cos(\theta - 4\pi/5) & \cos(\theta + 2\pi/5) \\
\sin\theta & \sin(\theta - 2\pi/5) & \sin(\theta - 4\pi/5) & \sin(\theta + 2\pi/5)
\end{bmatrix} \begin{bmatrix}
V_{ds} \\
V_{bs}
\end{bmatrix}
\]

(1)
\[
\begin{bmatrix}
I_{ds} \\
I_{qs} \\
I_{es}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
\cos \theta & \cos(\theta - 2\pi/5) & \cos(\theta - 4\pi/5) \\
\sin \theta & \sin(\theta - 2\pi/5) & \sin(\theta - 4\pi/5) \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix} \begin{bmatrix}
I_{as} \\
I_{bs} \\
I_{cs}
\end{bmatrix}
\]

(2)

where, \(V_{as}, V_{bs}, V_{cs}, V_{ds}\) and \(V_{qs}\) are stator phase voltages. \(V_{ds}\) and \(V_{qs}\) are the \(d\) and \(q\) axis stator fundamental voltages respectively. \(V_{as}\) is the zero sequence component of the stator. It is convenient to set \(\theta = 0\), so that the \(q\) axis is aligned with the \(a\) axis. Ignoring the zero sequence components, the transformation relations can be simplified using (3)-(4).

\[
V_{qs} = \frac{2}{5} V_a + V_c \cos(-2\pi/5) + V_d \cos(-4\pi/5) + V_e \cos(2\pi/5)
\]

(3)

\[
V_{ds} = \frac{2}{5} V_b \sin(-2\pi/5) + V_c \sin(-4\pi/5) + V_d \sin(2\pi/5)
\]

(4)

The two-phase \(d'\)-\(q'\) winding are transformed into the hypothetical windings mounted on the \(d\)-\(q\) axis, which rotate at synchronous speed \(\omega_r\) with respect to the \(d'\)-\(q'\) axis and the angle \(\theta_e = \omega_e t\) t. the voltage on the \(d'\)-\(q'\) axis can be converted into the \(d\)-\(q\) frame as follows[12]-[13]:

\[
\begin{bmatrix}
V_{q_e} \\
V_{d_e}
\end{bmatrix} = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
V_{q_s} \\
V_{d_s}
\end{bmatrix}
\]

(5)

For the two-phase machine, we need to represent both \(d'\)-\(q'\) and \(d\)-\(q\) circuit and their variables in a synchronously rotating \(d\)-\(q\) frame. The \(d\)-\(q\) frame voltage expressions are:

\[
V_{qs} = R_s i_{qs} + \frac{d}{dt} \psi_{qs} + \omega_e \psi_{qs} ; \quad V_{ds} = R_s i_{ds} + \frac{d}{dt} \psi_{ds} - \omega_e \psi_{qs}
\]

(6)

\[
V_{q_s} = R_s i_{q_s} + \frac{d}{dt} \psi_{q_s} - \omega_e \psi_{q_s} ; \quad V_{d_s} = R_s i_{d_s} + \frac{d}{dt} \psi_{d_s} - \omega_e \psi_{q_s}
\]

(7)

Since the rotor actually moves at speed \(\omega_r\), the \(d'\) axes fixed on the rotor which moves at a speed of \((\omega_e - \omega_r)\) relative to the synchronously rotating frame. The flux linkage expressions for \(d\) frame are:

\[
F_{qs} = \omega_b \psi_{qs} = X_{ls} i_{qs} + X_m (i_{qs} + i_{qr}) ; \quad F_{qr} = \omega_b \psi_{qr} = X_{lr} i_{qr} + X_m (i_{qs} + i_{qr})
\]

(8)

\[
F_{ds} = \omega_b \psi_{ds} = X_{ls} i_{ds} + X_m (i_{ds} + i_{dr}) ; \quad F_{dr} = \omega_b \psi_{dr} = X_{lr} i_{dr} + X_m (i_{ds} + i_{dr})
\]

(9)

\[
F_{qm} = \omega_b \psi_{qm} = X_{m} (i_{qs} + i_{qr}) ; \quad F_{dm} = \omega_b \psi_{dm} = X_{m} (i_{ds} + i_{dr})
\]

(10)

where \(X_{ls} = \omega_b L_{ls}\), \(X_{lr} = \omega_b L_{lr}\), and \(X_m = \omega_b L_m\)

(11)

By substituting (8)-(9) in (6)-(7), the voltage expressions can be written as:

\[
V_{qs} = R_s i_{qs} + \frac{1}{\omega_b} \frac{d}{dt} F_{qs} + \frac{\omega_e}{\omega_b} F_{ds} - \frac{\omega_e}{\omega_b} F_{qs}
\]

(12)

\[
0 = R_s i_{qr} + \frac{1}{\omega_b} \frac{d}{dt} F_{qr} + \frac{(\omega_e - \omega_r)}{\omega_b} F_{dr} - \frac{(\omega_e - \omega_r)}{\omega_b} F_{qr}
\]

(13)
By substituting (11) in (8)-(9), the flux linkage equations can be written as

\[ \frac{F_{qs}}{\omega_b} = \left[ (L_{ls} + L_m) i_{qs} + L_m i_{qr} \right] \]

\[ \frac{F_{ds}}{\omega_b} = \left[ (L_{ls} + L_m) i_{ds} + L_m i_{dr} \right] \]  \hspace{1cm} (14)

\[ \frac{F_{qr}}{\omega_b} = \left[ (L_{ls} + L_m) i_{qr} + L_m i_{qs} \right] \]

\[ \frac{F_{dr}}{\omega_b} = \left[ (L_{lr} + L_m) i_{dr} + L_m i_{dq} \right] \]  \hspace{1cm} (15)

By substituting (14) in (12), we get (16)-(17).

\[ \frac{d (L_{ls} + L_m) i_{qs}}{dt} = V_{qs} - L_m \frac{d i_{qr}}{dt} - R_s i_{qr} + \omega_e \left[ (L_{ls} + L_m) i_{ds} + L_m i_{dr} \right] \] \hspace{1cm} (16)

\[ i_{qs} = \int \left[ V_{qs} - L_m \frac{d i_{qr}}{dt} - R_s i_{qr} - \omega_e \left[ (L_{ls} + L_m) i_{ds} + L_m i_{dr} \right] \right] dt \] \hspace{1cm} (17)

Similarly, we can write remaining current equations as given in (18)-(20).

\[ i_{ds} = \int \left[ V_{ds} - L_m \frac{d i_{dr}}{dt} - R_s i_{dr} + \omega_e \left[ (L_{ls} + L_m) i_{qs} + L_m i_{qr} \right] \right] dt \] \hspace{1cm} (18)

\[ i_{qr} = \int \left[ V_{qr} - L_m \frac{d i_{rq}}{dt} - R_s i_{rq} - \omega_e \left[ (L_{ls} + L_m) i_{ds} + L_m i_{dr} \right] \right] dt \] \hspace{1cm} (19)

\[ i_{dr} = \int \left[ V_{dr} - L_m \frac{d i_{rd}}{dt} - R_s i_{rd} + \omega_e \left[ (L_{ls} + L_m) i_{qs} + L_m i_{qr} \right] \right] dt \] \hspace{1cm} (20)

Now, the flux linkage expressions in terms of currents can be written as follows:

\[ \Psi_{qs} = L_{ls} i_{qs} + L_m (i_{qs} + i_{qr}) \]

\[ \Psi_{dr} = L_{ls} i_{ds} + L_m (i_{ds} + i_{dr}) \]  \hspace{1cm} (21)

\[ \Psi_{qr} = L_{lr} i_{qr} + L_m (i_{qr} + i_{qs}) \]

\[ \Psi_{dr} = L_{lr} i_{dr} + L_m (i_{dr} + i_{qr}) \]  \hspace{1cm} (22)

\[ \Psi_{gm} = L_m (i_{qs} + i_{qr}) \]

\[ \Psi_{dr} = L_m (i_{ds} + i_{dr}) \]  \hspace{1cm} (23)

Now, the torque expression can be written as follows:

\[ T_e = \frac{5}{2} \frac{P}{2} \frac{L_m}{(L_m + L_{lr})} (\Psi_{qs} - \Psi_{qr} i_{ds}) \] \hspace{1cm} (24)

The speed and torque are given by the following relation:

\[ J \frac{d \omega_r}{dt} + B \omega_r = T_e - T_L \] \hspace{1cm} (25)

### III. CONTROLLER OF FIVE-PHASE INDUCTION-MOTOR (INDIRECT VECTOR CONTROL METHOD)

Fig. 1 shows the complete block diagram of indirect vector control method.
A. dq axis rotating current:
The five-phase currents can be resolved into \( i_{ds} \) and \( i_{qs} \) components and can be represented in the matrix form from (2).

B. Flux \( \Phi_{ir} \):
The flux produced in the stator winding is given by

\[
\Phi_{ir} = \frac{L_m i_{ds}^*}{(1 + Ts)}
\]
where \( T = L_r / R_r \).

C. Rotor position:
Now, the rotor position \( \theta = \int (\omega_r + \omega_m) \)
where, \( \omega_m \) is mechanical speed and \( \omega_r \) is rotor frequency (rad/sec)

\[
\omega_r = \frac{L_m i_{ds}^* R_r}{(L_r + \Phi_{ir}^*)}
\]

D. Fundamental reference torque (\( T_e^* \)):
The fundamental reference torque (\( T_e^* \)) is obtained by using proportional plus integral (PI) controller whose input is the error between the actual and reference speed.

E. Synchronous rotating dq-axis current:
The dq-axis synchronous rotating currents are given by

\[
i_{ds}^* = \frac{\Phi_{ir}^*}{L_{rd}} \quad ; \quad i_{qs}^* = \frac{2}{5P} \frac{L_r}{L_{rq}} T_e^*
\]
where \( \Phi_{ir}^* \) is constant flux, \( i_{ds}^* \) and \( i_{qs}^* \) are d and q axis synchronous rotating reference currents respectively.

F. Rotating to stationary abcde conversion:
Now, the rotating currents are transformed into stationary five-phase currents (\( i_{abcde}^* \)) by taking inverse of (2).
Hysteresis Control PWM:

With the use of hysteresis control PWM method, actual and reference five-phase currents are compared and the gate pulses are generated to control the five-phase induction motor [5]. Fig. 2 shows the block diagram of hysteresis control PWM.

Fig. 2 Block diagram of Hysteresis Control PWM

Fig. 3 shows the simulation of five-phase voltages from the gate pulses. These five-phase voltages applied to the stator winding of five-phase induction motor.

Fig. 3 Five-phase voltages from the gate pulses

IV. SIMULATION RESULTS AND DISCUSSIONS

Five-phase induction motor is simulated in MATLAB software for different load conditions. No-load speed, current and torque waveforms are shown in Fig. 4, 6 and 8 respectively. Load torque of 1 Nm and 5 Nm are applied to the motor and the speed, current and torque responses of the motor are shown in Fig. 5, 7 and 9 respectively.

Fig. 4 shows the dynamic simulation result of no-load speed of five-phase induction motor.

Fig. 4 No-load speed
Fig. 5 (a) and (b) show the dynamic simulation result of speed at load torque $T_L = 1$ Nm and $T_L = 5$ Nm respectively.

![Fig. 5 Speed response at (a) $T_L = 1$ Nm (b) $T_L = 5$ Nm](image)

It is observed from Fig. 4 and 5 that the motor achieve rated speed of 160 rad/sec at around 1 sec and also achieve steady state speed for different load with approximate zero steady state error. The speed of five-phase induction motor is 160 rad/sec for the entire load and the response of the motor is under damped.

Fig. 6 shows the dynamic simulation result of no-load torque of five-phase induction motor.

![Fig. 6 No-load torque](image)

Fig. 7 (a) and (b) show the dynamic simulation result of torque response at load torque $T_L = 1$ Nm and $T_L = 5$ Nm respectively.

![Fig. 7 Torque response at (a) $T_L = 1$ Nm (b) $T_L = 5$ Nm](image)

From Figs. 6 and 7, we can see that the maximum torque developed in five-phase induction motor is varied with the load torque variations but these variations are very less. The steady state torque is almost same for entire load.

Fig. 8 shows the dynamic simulation result of no-load current of five-phase induction motor.
Fig. 8 No-load current

Fig. 9 (a) and (b) show the dynamic simulation result of current response at load torque $T_L = 1 \text{ N-m}$ and $T_L = 5 \text{ Nm}$ respectively.

Fig. 9 Current response at (a) $T_L = 1 \text{ Nm}$ (b) $T_L = 5 \text{ Nm}$

The variations in transient as well as steady state current are less with the change in load torque of five-phase induction motor.

The effect of load torque variations in motor performance can be seen in Table 1.

<table>
<thead>
<tr>
<th>Load (Nm)</th>
<th>Speed (rad/sec)</th>
<th>Maximum Torque (Nm)</th>
<th>Transient Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_L = 0$</td>
<td>160.00</td>
<td>277.84</td>
<td>180.52</td>
</tr>
<tr>
<td>$T_L = 1$</td>
<td>160.00</td>
<td>278.74</td>
<td>180.79</td>
</tr>
<tr>
<td>$T_L = 5$</td>
<td>160.00</td>
<td>282.20</td>
<td>182.01</td>
</tr>
</tbody>
</table>

Table 1 shows the comparative simulation results when different load torque applied to five-phase induction motor. The steady state speed of the motor is constant for entire load. The variations in maximum torque and current are less during load torque variations.

V. CONCLUSION

The speed of five-phase induction motor can be controlled using different types of methods such as $V/f$ method, vector control method etc. The dynamic modeling of five-phase induction motor is done in step by step approach in this paper. The speed control of the five-phase induction motor is achieved by indirect vector control method. From the simulation result, we can conclude that the actual speed successfully tracks the reference speed in both transient as well as steady state conditions. The steady state error in speed is almost zero. The variations in actual torque and in actual
current are very less in both the transient as well as steady state part even if change in load torque. The simulation results prove the effectiveness of the controller design.

APPENDIX

Table 2 parameters of five-phase induction motor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>7.5 hp</td>
</tr>
<tr>
<td>Voltage</td>
<td>400 Volt</td>
</tr>
<tr>
<td>Poles</td>
<td>4</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>0.22 Ω</td>
</tr>
<tr>
<td>Rotor Resistance</td>
<td>0.16</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>151.5 mH</td>
</tr>
<tr>
<td>Mechanical motion inertia</td>
<td>0.04 Kg-m²</td>
</tr>
</tbody>
</table>

REFERENCES