

# Edge Imaging with Obscured Apertures Apodised by Amplitude Filters

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**Abstract.** The analytical studies were made for obscured apertures apodised with amplitude filter under the influence of defocus. The coherent edge-response of optical systems apodised with shaded aperture in the case of shrink aperture has been studied. The image quality assessment parameters have been evaluated by masking the aperture from the outside. Results are drawn for various obscured apertures and compared with and without apodisation. It is found that, this type of shading and shaping of apertures is found to be effective in enhancing the image quality and the resolving power of the optical imaging systems.

**Key words:** Edge Imaging, Coherence, Apodisation, Amplitude filters, Edge-ringing, Edge-gradient and Edge-shift.

## I. INTRODUCTION

A coherent optical system is linear in the complex field amplitude which means, that the resultant amplitude is the sum of the component amplitudes and the role of transfer function is played by the pupil function itself. The transfer function of optical system has a sharp cut-off in coherent illumination. An edge object has strong high frequency components. The cut-off of the coherent optical system is effectively at a low value as compared to Fourier spectrum of the sharp edge [1] and physical result is the unwanted edge ringing. The presence of these spurious fringes in the edge response and the apparent shift of the imaged edge lead to the difficulty in finding the location and the measurement of the edge. Studies on this subject indicate the importance of coherent imagery in areas like spatial filtering techniques and microscopy. Literature is rich in the studies on circular apertures, employing apodisation to reduce the ringing [2-10].

In this paper, we proposed to study the joint effects of apodisation and obscuration on the diffraction images of coherently illuminated straight edge. The edge-ringing, edge-shift and edge-gradient of the edge fringes have been evaluated for different values of apodisation using shrink apertures. For this we considered rotationally symmetric, diffraction-limited and defocused coherent optical system. These investigations have suggested the use of certain pupil functions in conjunction with optimal apodizers to assess the edge image quality [11-17].

## II. THEORY

The mathematical representation of amplitude transmission of an opaque straight edge is given by

$$\begin{aligned} A(u, v) &= 1 & \text{for } u \geq 0 \\ A(u, v) &= 0 & \text{for } u < 0 \end{aligned} \tag{1}$$

This indicates that the transmission function is discontinuous at  $u = 0$ . The Fourier transform for this equation gives the amplitude spectrum of the object and is given by [18]

$$a(x, y) = \frac{1}{2} \cdot \delta(y) \left[ \delta(x) + \frac{1}{i\pi x} \right] \tag{2}$$

where  $\delta(x)$  is the Dirac-delta function. The modified object amplitude spectrum at the exit pupil of the optical system is given by

$$a'(x, y) = a(x, y) \cdot f(x, y) \tag{3}$$

where  $f(x, y)$  is the pupil function of the optical system. For the given optical system the complex amplitude distribution in the image plane is given by the inverse Fourier transform of expression (3). Thus

$$A(u', v') = \iint_{\text{pupil}} a(x, y) \cdot f(x, y) \{ \exp 2\pi i (u'x + v'y) \} dx dy \tag{4}$$

The present work constitutes one-dimensional edge condition and hence, the general form of amplitude distribution is given by

$$A'(u', v') = \frac{1}{2} + \frac{1}{\pi} \int_0^1 f(x, 0) \frac{\sin(Zx)}{x} dx \tag{5}$$

where  $Z=2\pi u'$  and  $f(x, 0)$  is the coherent transfer function of the system. The coherent transfer function  $f(x, 0)$  in the current study is rotationally symmetric and satisfies the condition

$$f(x, 0) = f(-x, 0) \tag{6}$$

Pupil function for shaded aperture is

$$f(r) = 1 - \pi\beta r^2 \tag{7}$$

where  $r$  is the normalized distance of an arbitrary point on the pupil from its centre and  $\beta$  is the apodisation parameter. The term  $\beta$  controls the degree of non-uniformity of transmission over the pupil. The value of  $\beta= 0$ , corresponds to diffraction limited Airy system having uniform transmission of unity over the entire aperture.

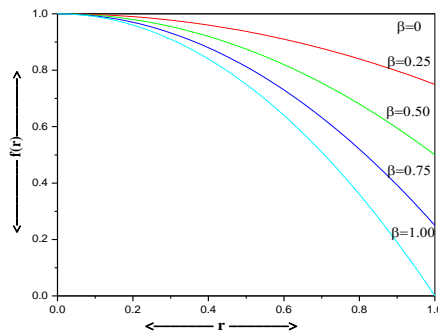


Fig.1 Pupil transmission curves for various values of  $\beta$

For this apodizer the amplitude transmittance decreases monotonically from the center towards the edges of the pupil. The above Fig.1 shows this phenomenon in greater detail. Higher spatial frequency components of the object are diffracted by a larger angle and hence these go predominantly through the edge of the aperture. As the pupil transmittance is decreased at the edges as compared to that of the center, due to apodization, the result is reduction in the higher spatial frequency components in the image. This manifests as partial or full suppression of the undesired optical side lobes or secondary maxima, which consequently enhances image features.

On introducing wave aberration such as defect-of-focus expression (5) takes the form

$$A'(u', v') = \frac{1}{2} + \frac{1}{\pi} \int_0^1 f(r) \exp(-i\phi_d \frac{x^2}{2}) \frac{\sin(Zx)}{x} dx \tag{8}$$

For the given shrink aperture shaded with the amplitude filter the expression (8) becomes,

$$A'(u', v') = \frac{1}{2} + \frac{1}{\pi} \int_0^\epsilon (1 - \pi\beta r^2) \exp(-i\phi_d \frac{x^2}{2}) \frac{\sin(Zx)}{x} dx \tag{9}$$

Where  $0 < \epsilon \leq 1$  is the radial obscuration parameter.

Now the intensity distribution of an edge image formed by an apodized optical system is given by the squared modulus of expression (9).

Thus

$$B(u') = B(Z) = |A'(u')|^2$$

$$B(Z) = |A'(Z)|^2 = \left| \frac{1}{2} + \frac{1}{\pi} \int_0^\epsilon (1 - \pi\beta r^2) \exp[-i(\phi_d \frac{x^2}{2})] \frac{\{\sin(Zx)\}}{x} dx \right|^2 \tag{10}$$

### III. RESULTS AND INTERPRETATION

The investigations on the effects of defocus and obscuration parameter on the images of edge objects formed by coherent optical systems apodised by the amplitude filter in the case of shrink aperture have been evaluated using the expressions (10) by employing Matlab7.8. The intensity distribution  $B(Z)$  in the images of straight edge objects has been obtained for different values of dimensionless diffraction variable  $Z$  varying from -3 to +20. The image quality assessment parameter such as edge-ringing, edge-shift and edge-gradient have been studied for various values of apodisation, defocus and obscuration parameters. The outer obscuration parameter of the aperture considered are  $\epsilon=1, 0.9, 0.8, 0.7$  and  $0.6$ . However the value  $\epsilon=1$  represents the circular aperture.

Fig.2a shows the intensity distribution profile of the straight edge for unapodised and aberration free optical system (Airy case) for both circular and shrink apertures. The edge ringing is pronounced and is insensitive to obscuration parameter  $\epsilon$  as the system is unapodised. Fig.2b illustrates the case where the defocus parameter  $\phi_d = \pi$  is introduced. For shrink apertures, the negative maximum amplitude increases with obscuration parameter  $\epsilon$  and hence the presence of ringing is much more pronounced. From the fig.2c it is observed that, at the defocused plane  $2\pi$ , by increasing the shrinking zone of the aperture the unwanted edge ringing has been reduced along with edge shift and improving the edge gradient. In Figs.2d, 2e&2f, the optical system is apodised by shaded apertures. We find increase in edge ringing with obscuration parameter  $\epsilon$  for  $\phi_d = 0$ ,

$\pi$  planes, but it decreases and increase in edge gradient at  $\phi_d=2\pi$  in the presence of apodisation. Hence this defocused plane may be designated as the optimum receiving image plane. It is evident that the apodised optical systems are more sensitive to aperture obscuration than unapodised ones. In Figs.2c & 2f, the image intensity distribution curves are almost similar and clearly seen that, at certain defocused planes, aperture shaping lowering the ringing effect even in the absence of shading, however, it is more effective along with aperture shading.

Fig.3a shows the variation of the edge- ringing with obscuration parameter  $\epsilon$  for different defocus planes without apodisation. The magnitude of edge-ringing is found to be almost constant for all the values of  $\epsilon$  as the system is aberration-free and unapodised, i.e., for Airy pupil. For the plane  $\phi_d=\pi$  edge-ringing is increasing with  $\epsilon$  but it is in decreasing trend and attain the minimum (0.13946) value at  $\epsilon=0.7$  for  $\phi_d=2\pi$ . The curve for  $\phi_d=2\pi$  cuts the curve for  $\phi_d=0$  at the value of  $\epsilon$  around 0.875 and cuts the curve for  $\phi_d=\pi$  at the value of  $\epsilon$  around 0.82. For these values the edge-ringing is almost the same. Fig.3b shows the variation of the edge- ringing with obscuration parameter  $\epsilon$  for different defocus planes in the presence of apodisation. The edge-ringing is much more pronounced with  $\epsilon$  varies from 1(circular aperture) to 0.6(shrunk aperture) for the planes  $\phi_d=0, \pi$ . But it attains much lower value for  $\phi_d=2\pi$  at  $\epsilon=0.7$ , however it is in increasing trend for  $\epsilon<0.7$ .

Figs.3c and 3d illustrate the variation of edge- shift with radial obscuration  $\epsilon$  for various values of  $\phi_d$  without and with apodisation respectively. The minimum shift occurs for the circular aperture ( $\epsilon=1$ ) and is independent of  $\epsilon$  for airy pupils. The edge-shift is increasing in the defocused optical systems for  $\epsilon=1, 0.9&0.8$  even in the presence of apodisation. However it is in decreasing trend for  $\epsilon\leq 0.7$  for both unapodised and apodised systems at certain defocused plane  $\phi_d=2\pi$ . The minimum shift occurs for the circular aperture ( $\epsilon=1$ ) and is dependent of  $\epsilon$  for airy pupil and shaded pupils. The edge-shift is increasing in the defocused optical systems as the obscuration zone increases for both unapodised and apodised optical systems.

Fig.3e and 3f show the fall of edge-gradient with obscuration ( $\epsilon=1, 0.9, 0.8, 0.7&0.6$ ) regardless of the values of apodisation for  $\phi_d=0, \pi$ . But it rises with obstructing zone of the aperture from 1(circular aperture) 0.7(shrink aperture) at  $\phi_d=2\pi$ . This increase in edge gradient improves the degraded edge images.

#### IV. CONCLUSIONS

The important conclusions of the investigations on the effects of defocus and aperture shaping on the images of straight edge objects formed by the coherent optical systems apodised with the shaded aperture filter are summarized as:

- i. The unapodized optical systems are less sensitive to aperture shaping in the absence of defocus than the apodised ones.
- ii. The unwanted edge ringing is found to reduce even in the presence of defocus with the aperture obscuration.
- iii. Apodisation along with aperture shaping is useful in improving the performance of defocused coherent optical systems.
- iv. For  $\phi_d=2\pi$  the edge ringing shows a decreasing trend and there is an improvement in edge gradient for both unapodised and apodised optical systems with aperture shaping. Hence at this defocused plane the optical system may be considered to be optimum in edge imaging.

Hence this type of aperture shading and shaping is found to be very effective in improving the image quality and resolution capabilities of the optical imaging systems for coherent edge imaging.

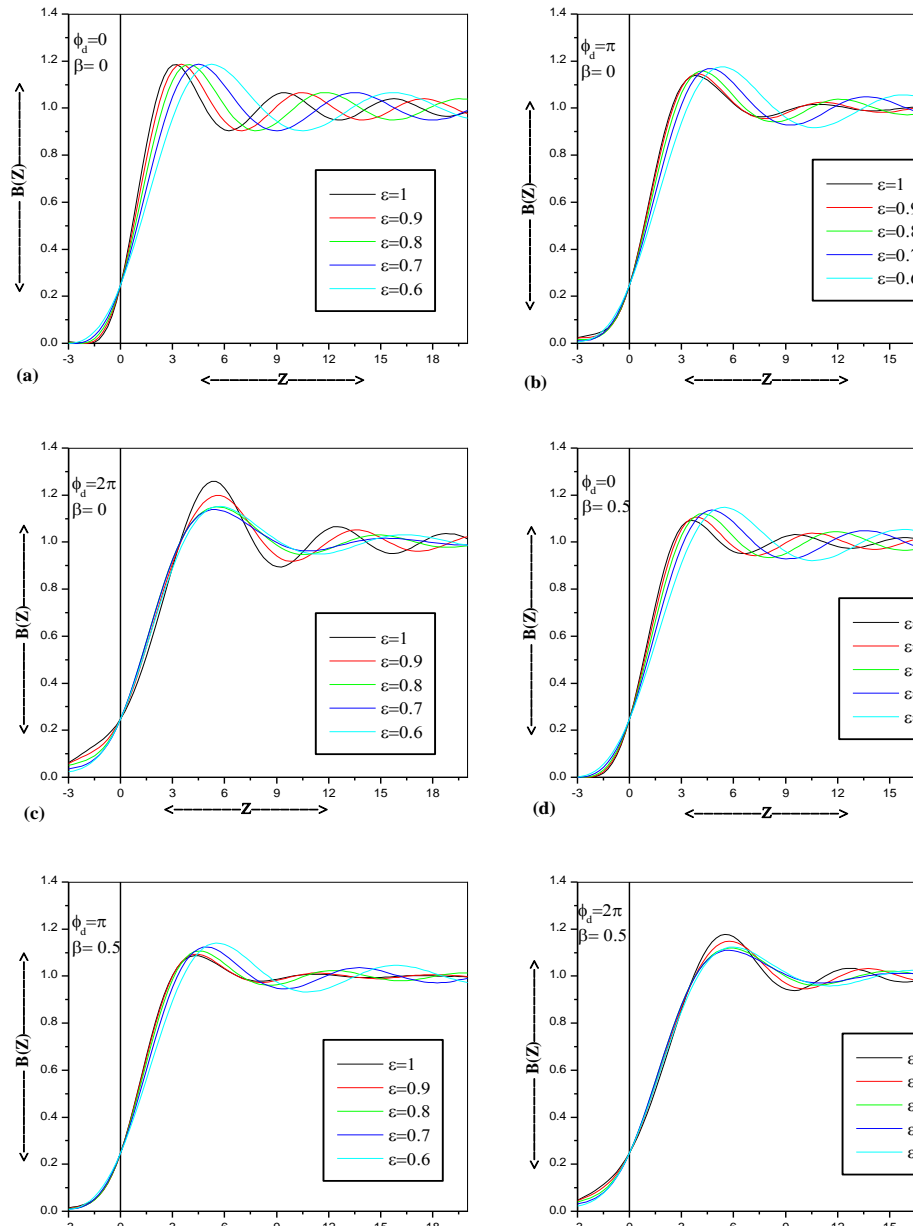


Fig.2 The intensity profiles for un-apodised and apodised pupil functions under the influence of defocus and obscuration

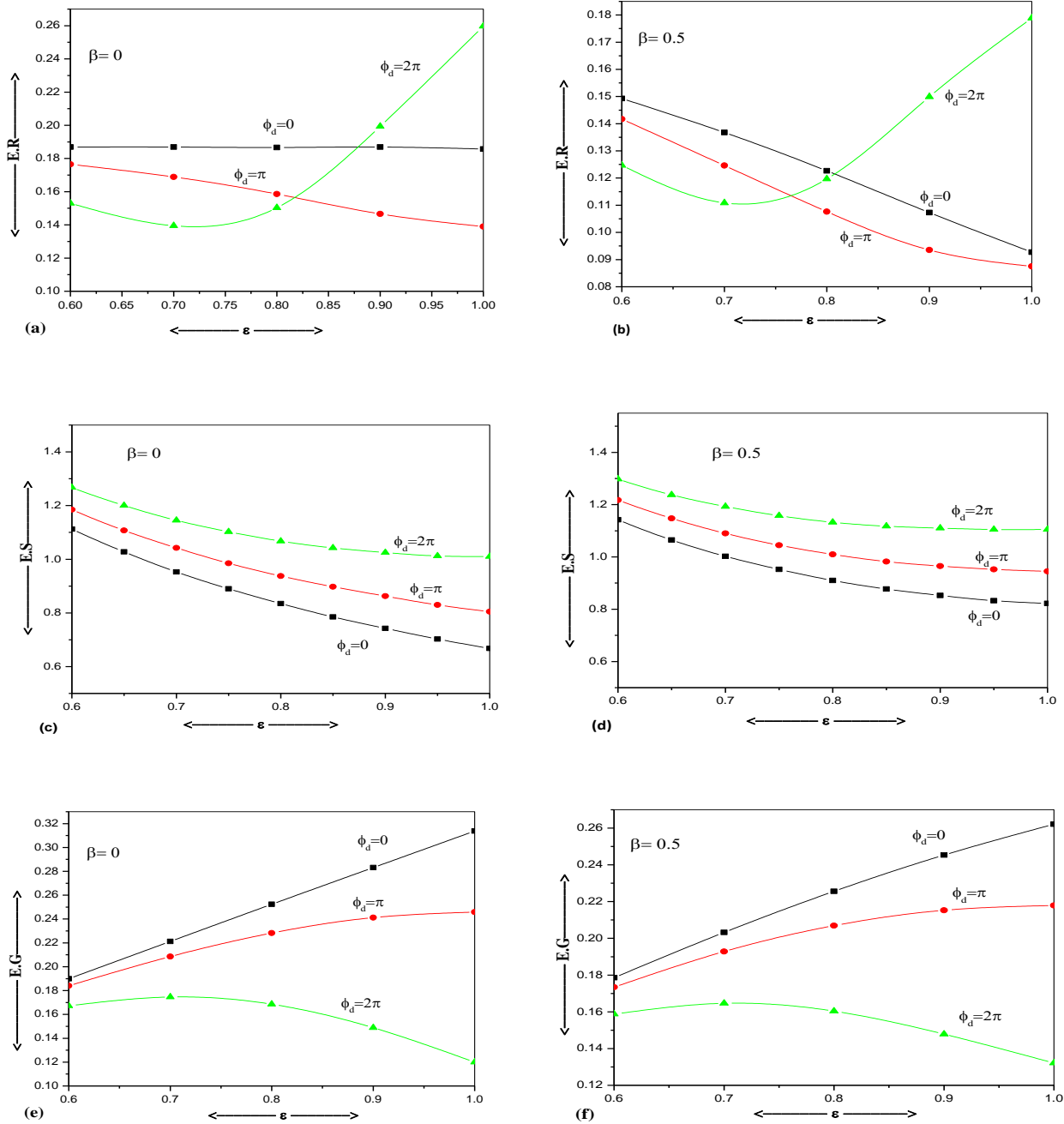


Fig.3 Shows the effect of Obscuration on Edge ringing (a & b), Edge shifting (c & d) and Edge gradient (e & f) at different defocused planes

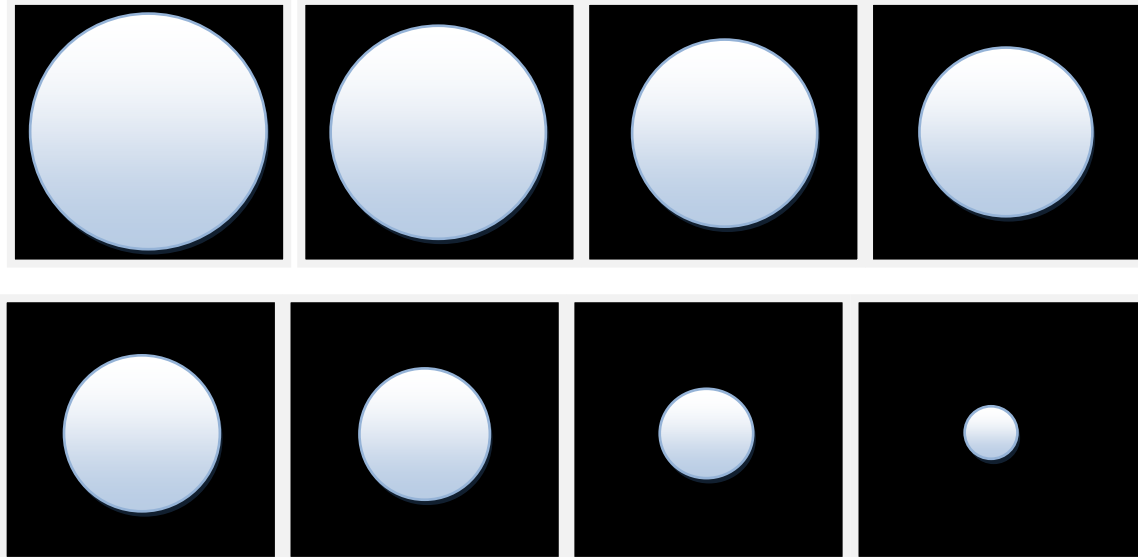


Fig.4 Shows shaded Circular aperture of unit radius (first) and various Shrunk apertures

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