HYDROMAGNETIC FLOW PAST A PARABOLIC STARTED VERTICAL PLATE IN THE PRESENCE OF HOMOGENEOUS CHEMICAL REACTION OF FIRST ORDER

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ABSTRACT: Laplace transform solution of unsteady flow past a parabolic starting motion of an infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of homogeneous chemical reaction of first order has been analyzed. The plate temperature is raised linearly with time and the concentration level near the plate is raised uniformly. The solutions for the velocity, temperature and concentration fields are studied for the different physical parameter. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the chemical reaction parameter as well as magnetic field parameter.

Keywords: parabolic, homogeneous, heat and mass transfer, chemical reaction, first order, vertical plate, magnetic field.

I. INTRODUCTION

The study of simultaneous heat and mass transfer in the presence of MHD plays an important role in petroleum industries, geophysics and in astrophysics. It also finds applications in many engineering problems such as magneto hydrodynamic generator, plasma studies, in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in the movement of earth’s core. It has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics.

Chemical reactions can be divided in to two groups. They are (i) Homogeneous and (ii) Heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and
homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration of only one reactant and is independent of others. Decomposition of nitrogen pentoxide in the gas phase as well in an organic solvent like $CCl_4$, conversion of N-chloroacetanilide into p- chloroacetanilide, hydrolysis of methyl acetate and inversion of cane sugar. The radioactive disintegration of unstable nuclei are the best examples of first order reactions.

Chambre and Young [3] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al [4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al [5]. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Heat transfer effects on impulsively started an infinite vertical plate in the presence of magnetic field was studied by Soundalgekar et al [8]. Mass transfer effects on MHD flow past an impulsively started an infinite isothermal vertical plate with uniform mass diffusion studied by Soundalgekar et al [9]. Rajesh Kumar et al [7] have studied exact solution of hydromagnetic flow on moving vertical surface with prescribed uniform heat flux. Recently, Muthucumarswamy et al [6] studied MHD effects on accelerated isothermal vertical plate with uniform mass diffusion using Laplace transform method.

Agrawal et al [1] studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al [2] further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

It is proposed to study the effects of on flow past an infinite isothermal vertical plate subjected to parabolic motion with uniform mass diffusion, in the presence of magnetic field and chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique and the resultant solutions are in terms of exponential and complementary error function.

II. MATHEMATICAL FORMULATION

The unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and uniform diffusion, in the presence of chemical reaction of first order has been considered. The $x$-axis is taken along the plate in the vertically upward direction and the $y$-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature $T_\infty$ and
At time $t' > 0$, the plate is started with a velocity $u = u_0 t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to $T_w$ and the concentration level near the plate are also raised to $C'_w$. A chemically reactive species which transforms according to a simple reaction involving the concentration is emitted from the plate and diffuses into the fluid. The plate is also subjected to a uniform magnetic field of strength $B_0$. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq’s approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2_0}{\rho} u$$

(1)

$$\rho C' \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2}$$

(2)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - k_i (C' - C'_\infty)$$

(3)

With the following initial and boundary conditions:

$$u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y, t' \leq 0$$

$$t' > 0: \quad u = u_0 t'^2, \quad T = T_\infty + (T_w - T_\infty)A t', \quad C' = C'_w \quad \text{at} \quad y = 0$$

$$u \to 0, \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty$$

(4)

On introducing the following non-dimensional quantities:

$$U = u \left(\frac{u_0}{v^2}\right)^{\frac{1}{3}}, \quad t = \left(\frac{u_0^2}{v}\right)^{\frac{1}{3}} t', \quad Y = y \left(\frac{u_0}{v^2}\right)^{\frac{1}{3}}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T_w - T_\infty)}{(v u_0)^{\frac{2}{3}}}, \quad Gc = \frac{g\beta(C'_w - C'_\infty)}{(v u_0)^{\frac{2}{3}}}, \quad K = K_i \left(\frac{v}{u_0^2}\right)^{\frac{1}{3}}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D}$$

(5)

The equations (1) to (3) reduces to the following dimensionless form:
\[
\begin{align*}
\frac{\partial U}{\partial t} &= Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \\
\frac{\partial \theta}{\partial t} &= \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \\
\frac{\partial C}{\partial t} &= \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC 
\end{align*}
\]

The corresponding initial and boundary conditions in dimensionless form are as follows:

\[ U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \leq 0 \]

\[ t > 0: \quad U = t^2, \quad \theta = t, \quad C = 1 \quad \text{at } Y = 0 \]

\[ U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as } Y \to \infty \]

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

\[
\theta = t \left[ 1 + 2\eta^2 Pr \text{erfc} (\eta \sqrt{Pr}) - \frac{2\eta \sqrt{Pr}}{\sqrt{\pi}} \exp(-\eta^2 Pr) \right] 
\]

\[
C = \frac{1}{2} \left[ \exp(2\eta \sqrt{KtSc}) \text{erfc} (\eta \sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta \sqrt{KtSc}) \text{erfc} (\eta \sqrt{Sc} - \sqrt{Kt}) \right] 
\]

\[
U = \left( \eta^2 + M(t + 2ac) \right)t + 2(c + d)M \left[ \exp(2\eta \sqrt{Mt}) \text{erfc} (\eta + \sqrt{Mt}) + \exp(-2\eta \sqrt{Mt}) \text{erfc} (\eta - \sqrt{Mt}) \right] 
\]

\[
+ \left( \frac{\eta \sqrt{t}(1 - 4M(t + ac))}{4M^{3/2}} \right) \left[ \exp(-2\eta \sqrt{Mt}) \text{erfc} (\eta - \sqrt{Mt}) - \exp(2\eta \sqrt{Mt}) \text{erfc} (\eta + \sqrt{Mt}) \right] 
\]

\[
- \frac{\eta}{M \sqrt{\pi}} \exp(-(\eta^2 + Mt + \frac{c \exp(at) \exp(2\eta \sqrt{Pr} at) \text{erfc} (\eta \sqrt{Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{Pr} at) \text{erfc} (\eta \sqrt{Pr} - \sqrt{at})}{c \exp(at) \exp(2\eta \sqrt{(M + at)} t) \text{erfc} (\eta + \sqrt{(M + at)} t) + \exp(-2\eta \sqrt{(M + at)} t) \text{erfc} (\eta - \sqrt{(M + at)} t)} 
\]

\[
- d \exp(bt) \exp(2\eta \sqrt{(M + bt)} t) \text{erfc} (\eta + \sqrt{(M + bt)} t) + \exp(-2\eta \sqrt{(M + bt)} t) \text{erfc} (\eta - \sqrt{(M + bt)} t) \right]
\]
III. RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters \( K, M, Pr, Gr, Gc, Sc \) and \( t \) upon the nature of the flow and transport. The value of the Schmidt number \( Sc \) is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number \( Pr \) are chosen such that they represent water \( (Pr = 7.0) \). The numerical values of the velocity are computed for different physical parameters like chemical reaction parameter, magnetic field parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 illustrates the effect of the concentration profiles for different values of the chemical reaction parameter \( (K = 0.2,2,5,10) \) at \( t = 0.4 \). The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing chemical reaction parameter.

The velocity profiles for different values of the chemical reaction parameter \( (K = 2,5,10) \), \( Gr = 5 = Gc, Pr = 7, M = 2 \) and \( t = 0.4 \) are shown in figure 2. It is observed that the velocity increases with decreasing chemical reaction parameter.

Figure 3 demonstrates the effect of velocity for different values of the magnetic field parameter \( (M = 1.4,1.6,2) \), \( Gr = 5 = Gc, Pr = 7, K = 2 \) and \( t = 0.4 \). It was observed that the velocity increases with decreasing values of the magnetic field parameter. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

Figure 4 demonstrates the effects of different thermal Grashof number \( (Gr = 2, 5) \), mass Grashof number \( (Gc = 5, 10) \), \( K = 2, M = 2 \) and \( Pr = 7 \) on the velocity at \( t = 0.2 \). It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.
The velocity profiles for different values of the time \( t = 0.2, 0.4, 0.6, 0.8 \), \( Gr = 5 = Gc \), \( K = 2 \) and \( M = 2 \) are presented in figure 5. The trend shows that the velocity increases with increasing values of the time \( t \). The effect of velocity profiles for different values of the Schmidt number \( (Sc = 0.16, 0.3, 0.6, 2.01) \), \( Gr = 5 = Gc \), \( Pr = 7 \), \( M = 2 \) and \( t = 0.6 \) are shown in figure 6. It is observed that the velocity increases with decreasing values of the Schmidt number.

**IV. CONCLUSION**

An exact solution of MHD flow past a parabolic starting motion of the infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration and the velocity fields for different physical parameters like chemical reaction parameter, magnetic field parameter, thermal Grashof number, mass Grashof number and \( t \) are studied graphically. The conclusions of the study are as follows:

(i) The velocity increases with increasing thermal Grashof number or mass Grashof number and time \( t \) in the presence of magnetic field parameter. But the trend is just reversed with respect to the chemical reaction parameter or magnetic field parameter.

(ii) The temperature of the plate increases with decreasing values of the Prandtl number.

(iii) The plate concentration increases with decreasing values of the chemical reaction parameter.

**REFERENCES**


**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Constants</td>
</tr>
<tr>
<td>C'</td>
<td>species concentration in the fluid $kgm^{-3}$</td>
</tr>
<tr>
<td>C</td>
<td>dimensionless concentration</td>
</tr>
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<td>$C_p$</td>
<td>specific heat at constant pressure $J. kg^{-1}. k$</td>
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<tr>
<td>D</td>
<td>mass diffusion coefficient $m^2.s^{-1}$</td>
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<tr>
<td>$G_c$</td>
<td>mass Grashof number</td>
</tr>
<tr>
<td>$Gr$</td>
<td>thermal Grashof number</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity $m.s^{-2}$</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity $W.m^{-1}.K^{-1}$</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
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<td>$Sc$</td>
<td>Schmidt number</td>
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<tr>
<td>$T$</td>
<td>temperature of the fluid near the plate $K$</td>
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<tr>
<td>$t'$</td>
<td>time $s$</td>
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<tr>
<td>$u$</td>
<td>velocity of the fluid in the $x'$-direction $m.s^{-1}$</td>
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<td>$u_0$</td>
<td>velocity of the plate $m.s^{-1}$</td>
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<tr>
<td>$u$</td>
<td>dimensionless velocity</td>
</tr>
<tr>
<td>$y$</td>
<td>coordinate axis normal to the plate $m$</td>
</tr>
<tr>
<td>$Y$</td>
<td>dimensionless coordinate axis normal to the plate</td>
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**Greek symbols**

<table>
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<tr>
<td>$\beta$</td>
<td>volumetric coefficient of thermal expansion $K^{-1}$</td>
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<tr>
<td>$\beta^*$</td>
<td>volumetric coefficient of expansion with concentration $K^{-1}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>coefficient of viscosity $Ra.s$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity $m^2.s^{-1}$</td>
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<td>$\rho$</td>
<td>density of the fluid $kg.m^{-3}$</td>
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<td>$\tau$</td>
<td>dimensionless skin-friction $kg.m^{-1}.s^2$</td>
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<td>$\theta$</td>
<td>dimensionless temperature</td>
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<tr>
<td>$\eta$</td>
<td>similarity parameter</td>
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<td>$erfc$</td>
<td>complementary error function</td>
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**Subscripts**

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<td>$w$</td>
<td>conditions at the wall</td>
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free stream conditions
Fig. 2 Velocity profiles for different K

- U
- K
- 2
- 5
- 10

0 0.5 1 1.5 2 2.5

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Fig. 3  Velocity profiles for different M
Fig. 4  Velocity profiles for different Gr & Gc
Fig. 5  Velocity profiles for different $t$
BIOGRAPHY

Dr R Muthucumaraswamy, received his B.Sc. degree in Mathematics, from Gurunanank College, University of Madras in 1985, M.Sc. degree in Applied Mathematics from Madras Institute of Technology, Anna University in 1987, M.Phil. degree in Mathematics from Pachaiyappas’ College, University of Madras in 1991 and Ph.D. degree in Mathematics from Anna University in 2001. His area of specialization is Theoretical and Computational Fluid Dynamics. He published 202 papers in National/International journals and in conferences. He completed two funded projects from Defence Research and Developmental Organization in 2009 and 2012. He received best teacher award in the year 2001. He is also a member of Indian society of heat and mass transfer, Indian society for Technical Education, Federation of science clubs of Tamil Nadu. He received best Mathematician award in the year of Mathematics 2012. Presently, he is Professor and Head, Department of Applied Mathematics, Sri Venkateswara College of Engineering, Irungattukottai (Sriperumbudur Taluk), India.

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