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New Flexible Weibull Distribution

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Abstract

In the present paper, a new function is suggested to develop a new lifetime model. The new model is proposed by considering the linear scheme of the two logarithms of cumulative hazard functions. The proposed model is known as New Flexible Weibull distribution, capable of modeling data with increasing or bathtub shaped failure rates and offers a greater distribution flexibility. Therefore, it can be useful to use an alternative model to many other ageing distributions, where, data modeling with increasing or bathtub shaped failure rates are of interest. A brief mathematical explanation for the reliability function is provided. The parameters of the proposed model are estimated by using the maximum likelihood method. To claim the workability of the proposed model, two illustrated examples are provided.

Keywords

Increasing; Bathtub shape; Ageing behaviour; Maximum likelihood estimates

I. INTRODUCTION

In reliability discipline, ageing distributions, such as Exponential, Gamma, Rayleigh, linear failure rate, lognormal or Weibull distribution are extensively used to model real phenomena. Of these ageing distributions, the Weibull distribution due to Waloddi Weibull is a prominent distribution to model lifetime data. The expression for the cumulative distribution function (CDF) of the Weibull distribution is given in (1).

$$G(z)=1-e^{-\beta z^{\alpha}}, \qquad z,\alpha,\beta>0. \tag{1}$$

Due to usefulness in reliability discipline, numerous extensions based on Weibull distribution have been introduced in the literature to model lifetime data. These extensions, includes Modified Weibull (MW) distribution due to Sarhan and Zaindin (2009), Kumaraswamy Weibull (KW) distribution by Cordeiro et al. (2010), Beta Weibull (BW) distribution studied by Famoye et al. (2005), Beta modified Weibull (BMW) distribution proposed by Silva et al. (2010), Generalized modified Weibull (GMW) distribution studied by Carrasco et al. (2008), Exponentiated modified Weibull extension (EMWE) distribution due to Sarhan and Apaloo (2013), On transmuted flexible Weibull extension (TFWEx) distribution of Ahmad and Hussain (2017), and Generalized Flexile Weibull extension (GFWEx) distribution proposed by Ahmad and Iqbal (2017), etc. For a brief review of these extensions one may refer to Murthy et al. (2003) and Pham and Lai (2007). A very small amount of the enormous applications of the Weibull model in reliability engineering including coatings by Almeida (1999), adhesive wear in metals by Queeshi and Sheikh (1997), pitting corrosion in pipes studied by Sheikh et al. (1990) and fracture strength of glass due to Keshevan et al. (1980). Gurvich et al. (1997) introduced a new class of lifetime distributions defined by the CDF

$$G(z)=1-e^{-\beta F(z)}, \qquad z,\beta>0. \tag{2}$$

where $F(z)$ is monotonically increasing function of z. It is a very useful technique to combine two survival functions and create a new function as:

$$S(z)=\gamma_1 S_1(z)+(1-\gamma_2)S_2(z),$$

Where $0<\gamma_1,\gamma_2<1$, this method of generating new functions is known as a mixture of distributions, or

$$S(z)=\gamma_1 S_1(z)+\psi S_2(z), \qquad \gamma_1,\psi>0. \tag{3}$$

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One can also generate a new function by combining two cumulative hazard functions as:

$$H(z) = \beta H_1(z) + \theta H_2(z), \tag{4}$$

In term of cumulative hazard function, the CDF can be written as

$$G(z) = 1 - e^{-H(z)}, \qquad z>0, \tag{5}$$

where $H(z)$ fulfils the conditions stated below

i. $H(z)$ is nonnegative and increasing function of z,

ii. $\lim_{z\to 0} H(z) \to 0$ and $\lim_{z\to\infty} H(z) \to \infty$.

The probability density function (PDF) corresponding to (5) has the following expression

$$g(z) = h(z)\ e^{-H(z)}, \qquad z > 0.$$

The modified Weibull distributions introduced by Xie and Lai (1996), Sarhan and Zaindin (2009), Lemonte et al. (2014) and Almalki and Yaun (2013) belongs to the class stated in (5). Here in (5), the $H(z)$ is bounded. Conversely, in the present paper, we propose a new function trying to relax the boundary conditions, so, we use $log\ H(z)$ in place of $H(z)$. Because, it would be more interesting to use $log\ H(z)$ rather $H(z)$ in order to introduce a very flexible model. Hence, one can write (4) as

$$H(z) = H_1^{\beta}(z) \times H_2^{\theta}(z), \tag{6}$$

The expression provided in (6) can be written as

$$logH(z) = \beta logH_1(z) + \theta logH_2(z). \tag{7}$$

We use the mixture of the two logarithm of cumulative hazard functions, proposed as z^{γ} and z to introduce a new very flexible lifetime model. So, the expression given in (7), can be written in the form given below

$$H(z) = e^{\beta z^{\gamma} + \theta z}. \tag{8}$$

By using (8) in (5), one can easily get the CDF of the proposed distribution.

The proposed distribution is known as New Flexible Weibull (NFW) distribution and is able to model life time data with increasing or bathtub shaped failure rates. The present paper is designed as: Section 2, contains the definition and visual sketching of the proposed distribution. Section 3, covers the basic mathematical properties. Section 4, describe the ageing behaviour and different relationship of reliability properties of the model. Section 5, contains the estimation of the model parameters. Section 6, offers the analysis to real data sets. Finally, section 7, conclude the paper.

II. NEW FLEXIBLE WEIBULL DISTRIBUTION

The CDF of the NFW distribution is given by

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$$G(z;\gamma,\beta,\theta)=1-e^{-e^{(\beta z^{\gamma}+\theta z)}}, \qquad z,\gamma,\beta,\theta>0. \tag{9}$$

The PDF corresponding to (9) is given by

$$g(z;\gamma,\beta,\theta)=(\gamma\beta z^{\gamma-1}+\theta)e^{(\beta z^{\gamma}+\theta z)}\,e^{-e^{(\beta z^{\gamma}+\theta z)}}.$$

The survival function (SF) of the NFW distribution is

$$S(z;\gamma,\beta,\theta)=e^{-e^{(\beta z^{\gamma}+\theta z)}},$$

with HF

$$h(z;\gamma,\beta,\theta)=(\gamma\beta z^{\gamma-1}+\theta)e^{(\beta z^{\gamma}+\theta z)}\,. \tag{10}$$

The figure 1 & figure 2, displays the HFs of the NFW distribution for different values of parameters.

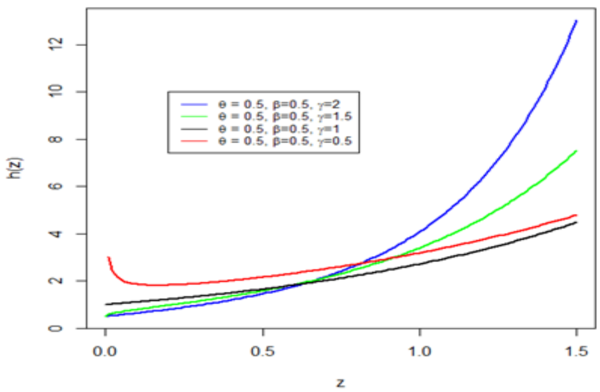

Figure 1: HF of the NFW distribution, for different values of parameters.

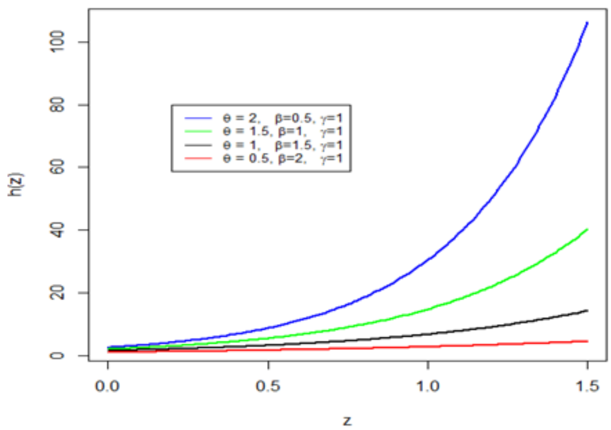

Figure 2: HF of the NFW distribution, for different values of parameters.

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III. BASIC PROPERTIES

This section of the paper covers the basic statistical properties of the NFW distribution.

3.1. Quantile and Median

The expression for the q^{th} quantile z_q of the NFW model is given by

$$\beta z_q^{\gamma}+\theta z_q-\log\left\{-\log\left(1-q\right)\right\}=0. \tag{11}$$

Using $q=0.50$, in (11), one can easily find the median of the NFW distribution. Also, putting $q=0.25$, and $q=0.75$, in (11), one may get the 1st and 3rd quartiles of the NFW distribution, respectively.

3.2. Generation of Random Numbers

The expression for generating random numbers from NFW distribution is given by

$$\beta z^{\gamma}+\theta z-\log\left\{-\log\left(1-R\right)\right\}=0, \qquad R\sim U\left(0,1\right).$$

IV. AGEING BEHAVIOUR

If $\lim_{z\to\infty} h\left(z;\gamma,\beta,\theta\right)=\infty,$ the hazard function is said to be increasing.

$$\lim_{z\to\infty} h\left(z;\gamma,\beta,\theta\right)=\infty,=\lim_{z\to\infty}\left(\gamma\beta z^{\gamma-1}+\theta\right)e^{\left(\beta z^{\gamma}+\theta z\right)}.$$

$$\lim_{z\to\infty} h\left(z;\gamma,\beta,\theta\right)=\infty. \tag{12}$$

The rest of this section is further subdivided into subsections, in which we consider some possible relationship between reliability properties.

4.1. Increasing Failure Rate

The hazard function is said to be increasing, if the first derivative of HF provides a positive value for all $z\geq 0$. i.e. $h'\left(z\right)\geq 0$. By getting the first derivative of $h\left(z;\gamma,\beta,\theta\right)$, one can derive the following expressions

$$h'\left(z;\gamma,\beta,\theta\right)=\left\{\left(\gamma\beta z^{\gamma-1}+\theta\right)^2+\gamma\left(\gamma-1\right)\beta z^{\gamma-2}\right\}e^{\left(\beta z^{\gamma}+\theta z\right)}>0$$

$$\left(\gamma\beta z^{\gamma-1}+\theta\right)^2+\gamma\left(\gamma-1\right)\beta z^{\gamma-2}>0$$

Let $\beta=\eta\theta$. Then

$$\theta^2\left(\eta\gamma z^{\gamma-1}+1\right)^2+\theta\eta\gamma\left(\gamma-1\right)z^{\gamma-2}>0$$

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$$\left(\eta\gamma z^{\gamma-1}+1\right)+\left\{\frac{\eta\gamma\left(\gamma-1\right)z^{\gamma-2}}{\theta}\right\}^{1/2}>0, \quad \left(\text{note that z},\sigma>0\right), \text{so}$$

$$W\left(z\right)=\left(\eta\gamma z^{\gamma-1}+1\right)+\left\{\frac{\eta\gamma\left(\gamma-1\right)z^{\gamma-2}}{\theta}\right\}^{1/2}>0. \qquad (13)$$

Using the result provided in (12) and in (13), it is detected that for the NFW distribution, modeling data with decreasing failure rate is impossible.

4.2. Survival Function

The SF gives the probability that a particular entity will survive afterwards a definite time unit. The SF play an significant role in biomedical and reliability analysis, for example, in biomedical analysis: it states the further survival time of a patient outside a definite time, in engineering reliability: it states extra performance (or additional life) of an electronic component beyond a definite time, as mentioned earlier the SF of the NFW distribution is given by

$$S\left(z;\gamma,\beta,\theta\right)=e^{-e^{\left(\beta z^{\gamma}+\theta z\right)}}.$$

In term of SF, HF and CHF, the CDF and PDF of NFW distribution can be expressed as

$$G\left(z;\gamma,\beta,\theta\right)=1-S\left(z;\gamma,\beta,\theta\right)$$

$$G\left(z;\gamma,\beta,\theta\right)=1-e^{-H\left(z;\gamma,\beta,\theta\right)}$$

$$G\left(z;\gamma,\beta,\theta\right)=1-e^{-\int_0^z h\left(z;\gamma,\beta,\theta\right)dz}.$$

Now PDF of FWEx distribution can be expressed as

$$g\left(z;\gamma,\beta,\theta\right)=\frac{d}{dz}\left\{G\left(z;\gamma,\beta,\theta\right)\right\}$$

$$g\left(z;\gamma,\beta,\theta\right)=\frac{d}{dz}\left\{1-e^{-\int_0^z h\left(z;\gamma,\beta,\theta\right)dz}\right\}$$

$$g\left(z;\gamma,\beta,\theta\right)=h\left(z;\gamma,\beta,\theta\right)e^{-\int_0^z h\left(z;\gamma,\beta,\theta\right)dz}$$

or

$$g\left(z;\gamma,\beta,\theta\right)=h\left(z;\gamma,\beta,\theta\right)e^{-H\left(z;\gamma,\beta,\theta\right)}$$

or

$$g\left(z;\gamma,\beta,\theta\right)=h\left(z;\gamma,\beta,\theta\right)S\left(z;\gamma,\beta,\theta\right).$$

In the theorem 1, we show that the function $S\left(z;\gamma,\beta,\theta\right).$ is a proper SF.

Theorem 1: The function $S\left(z;\gamma,\beta,\theta\right)$ is said to be a proper SF if and only if, it satisfies the following two properties:

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i. $\lim_{z\to 0} S(z;\gamma,\beta,\theta)=1.$

ii. $\lim_{z\to\infty} S(z;\gamma,\beta,\theta)=0.$

iii.

Proof of Theorem 1:

By definition,

$$G(z;\gamma,\beta,\theta)=\int_0^z g(z;\gamma,\beta,\theta)dz$$

$$S(z;\gamma,\beta,\theta)=1-\int_0^z g(z;\gamma,\beta,\theta)dz$$

$$S(z;\gamma,\beta,\theta)=\int_z^\infty g(z;\gamma,\beta,\theta)\,dz \tag{14}$$

If $z\to 0$ then from (14)

$$\lim_{z\to 0} S(z;\gamma,\beta,\theta)=\int_0^\infty g(z;\gamma,\beta,\theta)=1. \tag{15}$$

The result given in (15) can easily be verified, as density function over the entire range always integrates to one.

If $z\to\infty$ then using (14)

$$\lim_{z\to\infty} S(z;\gamma,\beta,\theta)=\lim_{z\to\infty}\int_z^\infty g(z;\gamma,\beta,\theta)=0. \tag{16}$$

4.3. Hazard Function

The HF (also known as failure rate function) gives the probability of failure of a particular entity conditioned that the entity has survived upto a definite time. As mentioned earlier, the HF of the NFW distribution is given by

$$h(z;\gamma,\beta,\theta)=\left(\gamma\beta z^{\gamma-1}+\theta\right)e^{\left(\beta z^{\gamma}\ +\ \theta z\right)}\ . \tag{17}$$

In the theorem 2, we show that the function $h(z;\gamma,\beta,\theta).$ is a proper HF.

Theorem 2: The function $(z;\gamma,\beta,\theta)\ h(z;\gamma,\beta,\theta)$ is said to be a proper HF if and only if, it satisfies the following two properties:

i. $h(z;\gamma,\beta,\theta)\geq 0\quad \forall z\geq 0.$

ii. $\int_0^\infty h(z;\gamma,\beta,\theta)\,dz=\infty.$

iii.

Proof of Theorem 2

i. The first property always hold since, $g(z;\gamma,\beta,\theta)\geq 0$ and $S(z;\gamma,\beta,\theta)>0,$ so

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$$h(z;\gamma,\beta,\theta)=\frac{g(z;\gamma,\beta,\theta)}{S(z;\gamma,\beta,\theta)}\geq 0.$$

ii. The second property can be proved as follow

iii.

$$\int_0^{\infty} h(z;\gamma,\beta,\theta)\,dz=\int_0^{\infty}\frac{g(z;\gamma,\beta,\theta)}{S(z;\gamma,\beta,\theta)}\,dz$$

$$\int_0^{\infty} h(z;\gamma,\beta,\theta)\,dz=\int_0^{\infty}\frac{h(z;\gamma,\beta,\theta)e^{-H(z;\gamma,\beta,\theta)}}{S(z;\gamma,\beta,\theta)}\,dz$$

$$\int_0^{\infty} h(z;\gamma,\beta,\theta)\,dz=\int_0^{\infty}\left\{-\frac{d}{dz}\left\{\log\left(S(z;\gamma,\beta,\theta)\right)\right\}\right\}dz$$

$$\int_0^{\infty} h(z;\gamma,\beta,\theta)\,dz=-\log\left(S(z;\gamma,\beta,\theta)\right)\Big|_0^{\infty}$$

$$\int_0^{\infty} h(z;\gamma,\beta,\theta)\,dz=\log\left(S(0;\gamma,\beta,\theta)\right)-\log\left(S(\infty;\gamma,\beta,\theta)\right)$$

$$\int_0^{\infty} h(z;\gamma,\beta,\theta)\,dz=\log(1)-\log(0)$$

$$\int_0^{\infty} h(z;\gamma,\beta,\theta)\,dz=\infty.$$

In term of SF, the HF of NFW distribution can be expressed as

$$h(z;\gamma,\beta,\theta)=\frac{g(z;\gamma,\beta,\theta)}{S(z;\gamma,\beta,\theta)} \tag{18}$$

$$h(z;\gamma,\beta,\theta)=\frac{1}{S(z;\gamma,\beta,\theta)}\left\{\frac{d}{dz}\left(G(z;\gamma,\beta,\theta)\right)\right\}$$

$$h(z;\gamma,\beta,\theta)=\frac{1}{S(z;\gamma,\beta,\theta)}\left\{-\frac{d}{dz}\left(S(z;\gamma,\beta,\theta)\right)\right\}$$

$$h(z;\gamma,\beta,\theta)=-\frac{d}{dz}\left\{\log\left(S(z;\gamma,\beta,\theta)\right)\right\}.$$

$$h(z;\gamma,\beta,\theta)=-\frac{d}{dz}\left\{\log\left(e^{-e^{\left(\beta z^{\gamma}+\theta z\right)}}\right)\right\}.$$

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$$h(z;\gamma,\beta,\theta)=\left(\gamma\beta z^{\gamma-1}+\theta\right)e^{\left(\beta z^{\gamma}+\theta z\right)}.$$

Using (18), the following important results can be derived

I) If $z\to 0$ then $\lim_{z\to 0} S(z;\gamma,\beta,\theta)=1,$ so then (18) will be

$$h(z;\gamma,\beta,\theta)=g(z;\gamma,\beta,\theta)\geq 0.$$

II) If $z\to\infty$ then $\lim_{z\to\infty} S(z;\gamma,\beta,\theta)=0,$ so then failure rate will be very high.

4.4. Cumulative Hazard Function

The HF is not always constant and may changes with time, so the CHF can be used to check whether the HF is changing or not. The CHF of the NFW distribution can be obtained as

$$H(z;\gamma,\beta,\theta)=\int_0^z h(z;\gamma,\beta,\theta)\,dz,$$

Using the HF of NFW distribution in above equation

$$H(z;\gamma,\beta,\theta)=\int_0^z\left(\gamma\beta z^{\gamma-1}+\theta\right)e^{\left(\beta z^{\gamma}+\theta z\right)}dz,$$

On solving, the following expression is obtained

$$H(z;\gamma,\beta,\theta)=e^{\left(\beta z^{\gamma}+\theta z\right)}.$$

In term of SF and CDF, the CHF of NFW distribution can be expressed as

$$S(z;\gamma,\beta,\theta)=e^{-H(z;\gamma,\beta,\theta)}$$

$$H(z;\gamma,\beta,\theta)=-\log\left\{S(z;\gamma,\beta,\theta)\right\}$$

$$H(z;\gamma,\beta,\theta)=-\log\left\{1-G(z;\gamma,\beta,\theta)\right\}.$$

4.5. Reversed Hazard Function

The reversed hazard function (RHF) plays a prominent role in reliability and health studies. Anderson et al. (1993) revealed that the RHF plays the same role in the analysis of left-censored data as the HF plays in the analysis of right-censored data.
The RHF of NFW distribution can be obtained as

$$r(z;\gamma,\beta,\theta)=\frac{g(z;\gamma,\beta,\theta)}{G(z;\gamma,\beta,\theta)}$$

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$$r(z;\gamma,\beta,\theta)=\frac{h(z;\gamma,\beta,\theta)S(z;\gamma,\beta,\theta)}{1-S(z;\gamma,\beta,\theta)}$$

$$r(z;\gamma,\beta,\theta)=\frac{h(z;\gamma,\beta,\theta)}{\left\{\frac{1}{S(z;\gamma,\beta,\theta)}\right\}-1}$$

$$r(z;\gamma,\beta,\theta)=\frac{h(z;\gamma,\beta,\theta)}{\left\{\frac{1}{e^{-H(z;\gamma,\beta,\theta)}}\right\}-1}$$

$$r(z;\gamma,\beta,\theta)=\frac{h(z;\gamma,\beta,\theta)}{\left\{\frac{1}{e^{-\left\{\int_0^z h(z;\gamma,\beta,\theta)dz\right\}}}\right\}-1}. \quad (19)$$

From (19) it is will-clear that, as $S(z;\gamma,\beta,\theta)$ is decreasing, then the denominator in (19) is increasing result in decreasing the RHF.

V. ESTIMATION

This section of the paper deals with estimation of the model parameters using maximum likelihood (ML) procedure.

5.1. Maximum likelihood estimation

Let $Z_1, Z_2, \dots Z_k$ are randomly sampled from NFWD$(z;\gamma,\beta,\theta)$, the corresponding likelihood function of this sample is

$$\ln L=\sum_{i=0}^{k}\log\left(\gamma\beta z_i^{\gamma-1}+\theta\right)+\sum_{i=0}^{k}\left(\beta z_i^{\gamma}+\theta z_i\right)-\sum_{i=0}^{k}e^{\left(\beta z_i^{\gamma}+\theta z_i\right)}. \quad (20)$$

By deriving the partial derivatives of the expression given in (20) on parameter, and then equating the result equal to zero,

$$\frac{d\ln L}{d\beta}=\sum_{i=0}^{k}\frac{\gamma z_i^{\gamma-1}}{\left(\gamma\beta z_i^{\gamma-1}+\theta\right)}+\sum_{i=0}^{k}z_i^{\gamma}-\sum_{i=0}^{k}z_i^{\gamma}e^{\left(\beta z_i^{\gamma}+\theta z_i\right)}. \quad (21)$$

$$\frac{d\ln L}{d\gamma}=\beta\sum_{i=0}^{k}\frac{\left(\gamma z_i^{\gamma-1}\log(z_i)+z_i^{\gamma-1}\right)}{\left(\gamma\beta z_i^{\gamma-1}+\theta\right)}+\beta\sum_{i=0}^{k}z_i^{\gamma}\log(z_i)-\sum_{i=0}^{k}z_i^{\gamma}\log(z_i)e^{\left(\beta z_i^{\gamma}+\theta z_i\right)}. \quad (22)$$

$$\frac{d\ln L}{d\theta}=\sum_{i=0}^{k}\frac{1}{\left(\gamma\beta z_i^{\gamma-1}+\theta\right)}+\sum_{i=0}^{k}z_i-\sum_{i=0}^{k}z_i e^{\left(\beta z_i^{\gamma}+\theta z_i\right)}. \quad (23)$$

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It is observed that, the expressions given in (21)-(23) do not provides solution in closed forms; so, the estimates of the parameters can be determined numerically by utilizing iterating procedure. We used "SANN" algorithm in R language to estimate the parameters numerically.

VI. APPLICATIONS

In this section, two applications to real data sets are studied. The data are taken from taken from reliability analysis and the goodness of fit results of the proposed model are compared with three other well-known lifetime distributions such as flexible Weibull extension (FWEx), modified Weibull (MW) and inverse flexible Weibull extension (IFWEx) distributions. The investigative tools including Cramer-von-Misses (CM) test statistics, Anderson–Darling (AD) test statistic, Kolmogorov–Smirnov (K-S) test statistic, Akaike's Information Criterion (AIC), Hannan-Quinn information criterion (HQIC), Bayesian information criterion (BIC) and log likelihood $l(.,z)$, where $l(.,z)$ represents the log-likelihood function calculated at the maximum likelihood estimates are measured. On behalf of these measures it is perceived that the suggested model offers greater flexibility.

Example: 1

The first data set signifies the failure times of 84 Aircraft Windshield taken from Tahir et al. (2015). The failure times are as: 0.040, 1.866, 2.385, 3.443, 0.301, 1.876, 2.481, 3.467, 0.309, 1.899, 2.610, 3.478, 0.557, 1.911, 2.625, 3.578, 0.943, 1.912, 2.632, 3.595, 1.070, 1.914, 2.646, 3.699, 1.124, 1.981, 2.661, 3.779,1.248, 2.010, 2.688, 3.924, 1.281, 2.038, 2.82,3, 4.035, 1.281, 2.085, 2.890, 4.121, 1.303, 2.089, 2.902, 4.167, 1.432, 2.097, 2.934, 4.240, 1.480, 2.135, 2.962, 4.255, 1.505, 2.154, 2.964, 4.278, 1.506, 2.190, 3.000, 4.305, 1.568, 2.194, 3.103, 4.376, 1.615, 2.223, 3.114, 4.449, 1.619, 2.224, 3.117, 4.485, 1.652, 2.229, 3.166, 4.570, 1.652, 2.300, 3.344, 4.602, 1.757, 2.324, 3.376 and 4.663. The final results of the goodness of fit corresponding to the data given in example 1, are summarized in table 1 and 2.

Dist.	Max. Likelihood Estimates	AD	CM	KS	-log
NFW	$\hat{\beta}$ =0.246, $\hat{\theta}$ =2.867, $\hat{\gamma}$ =0.709	0.9302	0.1488	0.100	128.885
FWEx	$\hat{\beta}$ = 0.307 , $\hat{\theta}$ = 1.396	5.575	0.897	0.320	175.828
IFWEx	$\hat{\beta}$ =0.0643, $\hat{\theta}$ =0.498	1.820	0.225	0.4857	187.242
MW	$\hat{\beta}$ =2.798, $\hat{\theta}$ =0.044, $\hat{\gamma}$ =1.0260	0.5267	0.0567	0.666	275.07

Table 1: Goodness of fit results for NFW, FWEx, IFWEx and MW.

Dist.	AIC	BIC	CAIC	HQIC
NFW	263.771	271.099	264.067	266.719
FWEx	355.655	360.5412	355.802	357.620
IFWEx	378.48	383.371	378.632	380.450
MW	556.155	563.483	556.451	559.1026

Table 2: Goodness of fit results for NFW, FWEx, IFWEx and MW.

Example 2

The second data set taken from Khan and Jan (2016), signifies the times of failure for a sample of thirty devices. The times are 2.75, 0.13, 1.47, 0.23, 1.81, 0.30, 0.65, 0.10, 3.00, 1.73, 1.06, 3.00, 3.00, 2.12, 3.00, 3.00, 3.00, 0.02, 2.61, 2.93, 0.88, 2.47, 0.28, 1.43, 3.00, 0.23, 3.00, 0.80, 2.45 and 2.66. The final results of the goodness of fit corresponding to the data given in example 1, are summarized in table 3 and 4.

Dist.	Max. Likelihood Estimates	AD	CM	KS	-log
NFW	$\hat{\beta}$ =0.206, $\hat{\theta}$ =1.903, $\hat{\gamma}$ =0.742	1.287	0.195	0.185	39.168
FWEx	$\hat{\beta}$ =0.3283, $\hat{\theta}$ =0.1610	2.060	0.326	0.3937	53.6561

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IFWEx	$\hat{\beta}=0.029$, $\hat{\theta}=0.621$	1.600	0.242	0.2907	187.242
MW	$\hat{\beta}=4.797$, $\hat{\theta}=0.0082$, $\hat{\gamma}=1.24672$	1.3077	0.200	0.4727	75.951

Table 3: Goodness of fit results for NFW, FWEx, IFWEx and MW.

Dist.	AIC	BIC	CAIC	HQIC
NFW	84.337	88.541	85.260	85.682
FWEx	111.3122	114.114	111.756	112.208
IFWEx	106.341	109.144	106.786	107.2381
MW	157.90	162.106	158.826	159.247

Table 4: Goodness of fit results for NFW, FWEx, IFWEx and MW.

VII. CONCLUSION

In this article, a new life distribution titled as New Flexible Weibull Distribution is proposed by considering a linear system of the two logarithms of cumulative hazard functions. The suggested model offers greater distribution flexibility and is cable of modeling lifetime data with increasing and bathtub shaped failure rates. The ageing behaviour of the failure rate function, relationship between reliability properties along with estimation of parameters using maximum likelihood procedure are discussed. The suggested modal is illustrated by means of discussing two real data sets, and the final result of the New Flexible Weibull distribution were found reliable, compared with that of three other existing lifetime distributions.

We are quite hopeful that the proposed distribution will serve as one of the most prominent lifetime distributions and will attract a wide range of applications in biomedical analysis and reliability engineering.

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