

# Novel Estimation of MISO-OFDM System Using CDD

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**ABSTRACT:** In Orthogonal Frequency Division Multiplexing (OFDM) systems, the cyclic-delay diversity (CDD) scheme is a low-complexity means of increasing transmit diversity. Because it requires only one inverse discrete Fourier transform (IDFT) operation at the transmitter. The mean square error (MSE) of both the least square (LS) channel estimate and the minimum mean square error (MMSE) channel estimate are applied in the design of optimum pilot sequence for CDD MISO -OFDM system to enhance channel estimation performance. The simulation results confirm that the proposed pilot design minimizes the MSE of both the LS and the MMSE channel estimates in MISO-OFDM system and compared with the theoretical values.

**KEYWORDS**—OFDM, pilot sequence, cyclic delay diversity (CDD), channel estimation.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one of the most attractive transmission schemes for digital communications. It enables the frequency diversity by multi-path channels to be exploited by interleaving across sub-carriers and applying forward error correction (FEC) schemes. As a result, they provide an efficient means of improving the performance of multiple-input multiple-output (MIMO) networks. In general, the spatial diversity achieved by the multiple antennas in OFDM systems is exploited using either space-time block coding (STBC)[2,3] or cyclic-delay diversity (CDD) schemes[4-6].

CDD-OFDM increases the transmission diversity, in which the multiple-input single-output (MISO) channel is transformed into an equivalent single-

input single-output (SISO) channel with increased frequency selectivity. In other words, CDD-OFDM converts spatial diversity into frequency diversity[5-7]. Unlike the convolution inter-leaver with block delay, CDD scheme does not induce any rate loss. Compared to STBC-OFDM[8], CDD-OFDM has the advantage that only one inverse discrete Fourier transform (IDFT) operation is required at the transmitter. Furthermore, no additional decoders are required at the receiver end. Previous studies have reported that the pilots in SISO-OFDM and MIMO-OFDM systems should be equally powered and equally spaced.

Pilots transmitted from different antennas in MIMO-OFDM systems should be phase orthogonal to minimize the mean square error of the channel estimate. To achieve a phase orthogonal condition, pilot transmitted from different antenna must have different values in frequency domain. As a result, it needs  $N_T$  IDFT Operation to generate pilot sequence for each transmitter, where  $N_T$  is the number of transmitting antenna. In CDD OFDM system, only one IDFT operation is required at transmitter, regardless of the number of transmit antennas. As the transmitter architectures of traditional MIMO-OFDM and CDD-OFDM systems differ, the pilot sequences in MIMO-OFDM systems are inapplicable to CDD-OFDM systems.

The equally spaced pilot[9-13] sequence was used to estimate the equivalent SISO channel but this scheme requires a large number of pilots. In [8] the authors presented a pilot-aided method to estimate the  $N_T$  channels individually for CDD-OFDM systems, in which the MSE of the channel estimate was minimized under

the assumption that the channel remained static over  $N_T$  consecutive OFDM symbols, this restricts its application.

In this work, a pilot design for CDD OFDM system is implemented. There are two main contributions. First the pilot design criteria which minimize the MSE of both the least square (LS) channel estimate and the minimum mean square error (MMSE) channel estimate in CDD-OFDM systems are implemented. Second the simulated results are compared with theoretical results. The proposed design methodology enables the status of the channel to be estimated using one OFDM symbol.

The organization of this paper is as follows. Section II describes the system model. Section III explains the LS and MMSE channel estimation methods in MISO-OFDM systems, it also presents the optimal pilot design procedure. Section IV demonstrates the illustration of the pilot design procedure. Section V confirms the optimality of the proposed pilot design methodology by performing simulation experiments. Section VI provides some concluding remarks.

## II.SYSTEM MODEL

This paper focuses on the channel estimation problem in MISO OFDM systems. The MISO-OFDM system comprise  $N$  sub carriers,  $N_T$  transmit antennas, and a single receive antenna. The analysis and findings are equally applicable to MIMO OFDM systems. Given  $u^{th}$  frequency domain sequence  $X_u$ , the  $u^{th}$  time domain sequence has the form  $x_u = F^H X_u$ , where  $F$  is an  $N \times N$  DFT matrix. The  $u^{th}$  time domain transmitted signal is  $x_{u,nT}, nT \in \{0, 1 \dots N_T - 1\}$ .

$$x_{u,nT} = \frac{1}{\sqrt{N_T}} C^{d_{u,nT}} x_u \quad (1)$$

Where,

$$C = \begin{bmatrix} 0 & \dots & \dots & \dots & 1 \\ 1 & 0 & \dots & \dots & 0 \\ \vdots & 1 & \dots & \dots & \vdots \\ \vdots & 0 & 1 & \dots & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \dots & \dots & \dots & 1 & 0 \end{bmatrix}, \quad (2)$$

$d_{u,nT} \in \{0, 1 \dots N-1\}$ ,  $x_{u,nT}$  is the cyclically delayed form of the original time domain sequence  $x_u$ . Since the cyclic delay induces a linear phase shift in the dual domain, the frequency domain signal,  $X_{u,nT} = F X_{u,nT}$ .

Let the channel impulse response of  $n_T^{th}$  transmit antenna be denoted as  $\mathbf{h}_{n_T} = [h_{n_T}(0), h_{n_T}(1) \dots h_{n_T}(L-1)]^T$ , which is an  $L \times 1$  column vector, where  $L$  is the channel length. Channel impulse response  $h_{n_T}(l) \in \{0, 1 \dots L-1\}$ , are independent complex Gaussian random variables with

zero mean and a variance  $\sigma_{h_{n_T}(l)}^2$ . The channel frequency response of the  $n_T^{th}$  antenna,  $H_{n_T}$  can be expressed in the form of  $H_{n_T} = \sqrt{N} F_L \mathbf{h}_{n_T}$ , where  $F_L$  is an  $N \times L$  sub matrix of the normalized DFT matrix  $F$  (i.e.  $F_L$  contains the first  $L$  columns of  $F$ )

If the cyclic prefix is added and removed correctly, and then the  $u^{th}$  received signal is,

$$Y_u = \sum_{n_T=0}^{N_T-1} [diag(X_{u,n_T}) \cdot H_{n_T}] + W_u \quad (3)$$

Where,  $W_u$  is an  $N \times 1$  column vector representing frequency domain additive white Gaussian noise (AWGN) with a variance  $\sigma^2$ . The optimal pilot design involves the pilot pattern, the pilot locations and cyclic delays of various OFDM symbol in each antenna.

## III.CHANNEL ESTIMATION IN MISO OFDM SYSTEMS

It is assumed that channels remain static for  $U$  consecutive OFDM symbols and receiver collects a total of  $P$  pilot symbols from these OFDM symbols. The bandwidth efficiency depends upon the channel coherence time. It is assumed that  $p^{th}$  pilot symbol is located on  $n_p^{th}$  subcarrier of  $u_p^{th}$  OFDM symbol.

$$Y_{u_p}(n_p) = \frac{1}{\sqrt{N_T}} \cdot X_{u_p} \cdot \left[ \sum_{n_T=0}^{N_T-1} \gamma_{p,n_T} \cdot \sqrt{N} \cdot F_L(n_p) \mathbf{h}_{n_T} \right] + W_{u_p}(n_p) \quad (4)$$

$$= \sqrt{N} \cdot \Psi_p \cdot \tilde{\mathbf{h}} + W_{u_p}(n_p) \quad (5)$$

Where,  $X_{u_p}$ ,  $Y_{u_p}$  and  $W_{u_p}$  denote the  $u_p^{th}$  transmitted OFDM symbol,  $u_p^{th}$  received OFDM symbol and AWGN noise respectively.

$$\gamma_{p,n_T} = \exp\left\{j2\pi n_T d_{u,n_T} / N\right\} \quad (6)$$

Where,  $d_{u,n_T}$  is the cyclic delay of the  $u_p^{th}$  OFDM symbol of  $n_T^{th}$  transmit antenna.  $F_L(n_p)$  denotes the  $n_p^{th}$  row of  $F_L$ .

$\Psi_p \equiv X_{u_p}(n_p) \cdot [\gamma_{p,0} \cdot F_L(n_p), \gamma_{p,1} \cdot F_L(n_p), \dots, \gamma_{p,N-1} \cdot F_L(n_p)]$  is a  $1 \times \tilde{L}$  row vector, and  $\tilde{L} \equiv N_T \cdot L$ ,  $\tilde{\mathbf{h}} \equiv \frac{1}{\sqrt{N_T}} \cdot [h_0^T, h_1^T, \dots, h_{N_T-1}^T]^T$ , is an  $\tilde{L} \times 1$  column vector.

The  $P$  received pilot symbols is  $\tilde{\mathbf{Y}} \equiv [Y_{u_0}(n_0), Y_{u_1}(n_1), \dots, Y_{u_{p-1}}(n_{p-1})]^T$ ,  $P \times 1$  column vector.

In other form  $\tilde{Y}$  can be written in terms of  $\tilde{\Psi}$  as

$$\tilde{Y} = \sqrt{N} \cdot \tilde{\Psi} \cdot \tilde{h} + \tilde{W} \quad (7)$$

Where  $\tilde{\Psi} \equiv \begin{bmatrix} \Psi_0 \\ \Psi_1 \\ \vdots \\ \Psi_{p-1} \end{bmatrix}$ , is a  $P \times \tilde{L}$  matrix. Also

$$\tilde{\Psi} = [A_0 \tilde{F}_L, A_1 \tilde{F}_L, \dots, A_{N_T-1} \tilde{F}_L] \quad (8)$$

Where,  $A_{n_T} \equiv$

$$\begin{bmatrix} X_{u_0}(n_0) \gamma_{0,n_T} & & \\ & X_{u_1}(n_1) \gamma_{1,n_T} & \\ & & \ddots \\ & & & X_{u_p}(n_p) \gamma_{p,n_T} \end{bmatrix} \quad (9)$$

is an  $P \times P$  matrix.  $\tilde{F}_L \equiv [F_L(n_0)^T, F_L(n_1)^T, \dots, F_L(n_{p-1})^T]^T$ , is a  $P \times L$  matrix.  $\tilde{W}$  is a  $P \times 1$  column vector of AWGN.

#### A Analysis of LS channel estimate

The LS channel estimate is given as  $\hat{h}_{LS} \equiv \tilde{\Psi}^+ \tilde{Y} = \tilde{h} + \tilde{\Psi}^+ \tilde{W}$ , where  $\tilde{\Psi}^+ \equiv [\tilde{\Psi}^H \tilde{\Psi}]^{-1} \tilde{\Psi}^H$ .

The MSE of the channel estimate is given by,

$$\sigma_{LS}^2 \equiv \text{tr} \left\{ \tilde{\Psi}^+ E[\tilde{W} \tilde{W}^H] \cdot [\tilde{\Psi}^+]^H \right\} \quad (10)$$

where  $E[\tilde{W} \tilde{W}^H] = \sigma_W^2 I_p$ ,  $I_p$  is a  $P \times P$  identity matrix.

The lower bound of the MSE of the LS channel estimate is given by

$$\sigma_{LS}^2 = \sigma_W^2 \cdot \text{tr} \{ [\rho I_L]^{-1} \} \quad (11)$$

Thus the pilot symbols that satisfies the condition  $\tilde{\Psi}^H \tilde{\Psi} = \rho I_L$  minimizes the MSE of the LS channel estimate.

#### B Analysis of MMSE channel estimate

The MMSE channel estimate is given as  $\hat{h}_{MMSE} \equiv R_{\tilde{h}\tilde{Y}} R_Y^{-1} \tilde{Y}$ , where  $R_{\tilde{h}\tilde{Y}} = E\{\tilde{h} \tilde{Y}^H\}$  and  $R_Y = E\{\tilde{Y} \tilde{Y}^H\}$ .

The MMSE of the channel estimate is given by

$$\sigma_{MMSE}^2 \equiv \text{tr} \left[ \left( R_{\tilde{h}}^{-1} + \frac{1}{\sigma^2} \tilde{\Psi}^H \tilde{\Psi} \right)^{-1} \right] \quad (12)$$

where  $R_{\tilde{h}} = E\{\tilde{h} \tilde{h}^H\}$ .

The lower bound of the MMSE of the given by,  $\sigma_{MMSE}^2 \equiv$

$$\text{tr} \left[ \left( R_{\tilde{h}}^{-1} + \frac{1}{\sigma^2} \tilde{\Psi}^H \tilde{\Psi} \right)^{-1} \right] = \sum_{l=0}^{L-1} \frac{\sigma_{\tilde{h}(l)}^2 \sigma^2}{\sigma^2 + \sigma_{\tilde{h}(l)}^2 \rho} \quad (13)$$

#### C. Proposed pilot design procedure

The pilot design procedure has the following steps

##### Step 1: Determination of U

P pilot symbols are scattered over U consecutive OFDM symbols. The channel has to remain static for minimum of 1 OFDM symbol to a maximum of channel coherence time.

##### Step 2: Determination of number of pilot symbols

The pilot symbols has to be partitioned into groups, with equal number of pilots in each group i.e.  $P=Q \cdot N_Q$ , where  $N_Q \geq L$  and  $Q \geq N_T$ .

##### Step 3: Determination of pilot pattern

$$X_{u_{(q,\tau)}}(n_{(q,\tau)}) = \left( \frac{\rho}{P} \right) \exp(j\theta_{(q,\tau)})$$

is the form of the  $\tau^{th}$  pilot of the  $q^{th}$  group. For simplicity in this design  $\theta_{(q,\tau)}$  is set to 0. thus the pilot pattern becomes

The  $\tau^{th}$  pilot of the  $q^{th}$  group has the form  $X_{u_{(q,\tau)}}(n_{(q,\tau)}) = \left( \frac{\rho}{P} \right)$  (14)

##### Step 4: Determination of pilot locations and cyclic delays

$d_{u_{q,n_T}} = \lambda_q \cdot n_T \cdot N_Q$  is the cyclic delay of the  $u_q^{th}$  OFDM symbol at the  $n_T^{th}$  antenna. Cyclic delay coefficient  $\lambda_q$  is important for the determination of the cyclic delay. The subcarrier index and the OFDM symbol index are the parameters necessary for the determination of the pilot location. The subcarrier index  $n_{(q,\tau)}$  is  $n_{(q,\tau)} = t_q + \tau T$ , where T the subcarrier spacing is given by  $T = N/N_Q$  and  $t_q \in \{1, 2, \dots, T-1\}$ .

The pilot subcarrier index, OFDM symbol index, cyclic delay coefficient has to be jointly determined using the formula,

$$t_q \cdot \lambda_q = \frac{N(q+1)}{P}, \lambda_q \in \{1, 2, \dots, \frac{N-1}{(N_T-1) \cdot N_Q}\} \quad (15)$$

The cyclic delay may vary from one OFDM symbol to another for a given antenna, i.e. an OFDM symbol transmitted from a given antenna cannot have two different cyclic delays. The criteria for determining the OFDM symbol index are,

1. Each pilot group is associated with a cyclic delay coefficient  $\lambda_q$ . the value of  $\lambda_q$  may be repeated among different groups.
2. Pilots with different values of  $\lambda_q$  are transmitted through different OFDM symbols.
3. The pilots having the same value of  $\lambda_q$  may or may not belong to the same pilot group.

Therefore the OFDM symbol index of a given pilot is not fixed, but has certain flexibility.

#### IV. ILLUSTRATION OF PILOT DESIGN PROCEDURE

The pilot design for  $N_T=4$  transmit antennas,  $N=128$  subcarriers, multipath channel length  $L=8$ , results in minimum number of pilot symbols as  $P=32$ . This is obtained by setting the number of pilot groups equal to

$Q \geq 4 (Q \geq N_T)$  and the number of pilots in each group  $N_Q = 8 (N_Q \geq L)$ . Thus the pilot symbol pattern is  $\rho/P = \rho/32$ .

The channel estimation for one OFDM symbol is desirable for a time varying channel. All the pilot symbols having the same value of  $\lambda_q$  has to be allocated to a single OFDM symbol,  $\lambda_q$  i.e.  $\lambda_q = \lambda \in \{1, 2, \dots, \lfloor \frac{N-1}{N_T-1} \cdot N_Q \rfloor\}$ ,  $q=0, 1, \dots, Q-1$ . The subcarrier index of the  $\tau^{th}$  pilot within the  $q^{th}$  pilot group is  $n_{(q,\tau)} = t_q + \tau T$ , where  $t_q = N \cdot (q+1) / (\lambda \cdot P)$ . The cyclic delay can be computed as  $d_{u_q, n_T} = \lambda \cdot n_T \cdot N_Q$  for  $\lambda_q = \lambda$  such that  $t_q \in \{1, 2, \dots, T-1\}$ .

Therefore, the pilot design for  $U=1$  can be assumed as,  $\lambda_q = \lambda, \lambda \in \{1, 2, 3, 4, 5\}$ . Since  $\lambda$  has to be selected with the condition that  $t_q \in \{1, 2, \dots, T-1\}$ . Arbitrarily selecting  $\lambda=2$ , the cyclic delay of the  $n_T^{th}$  transmit antenna is given as  $d_{0, n_T} = 2 \cdot N_Q \cdot n_T = 16 \cdot n_T$ . The subcarrier indexes are given as  $n_{(q,\tau)} = 2(q+1) + 16\tau$ , where  $q \in \{0, 1, 2, 3\}$  and  $\tau = \{0, 1, 2, \dots, 7\}$ .

V. SIMULATION RESULTS AND DISCUSSION

The simulation considers a MISO system, with the CDD-OFDM system which comprises one receive antenna,  $N_T = 4$  transmit antennas,  $N = 128$  sub-carriers, the multi-path channel is assumed to be Rayleigh-distributed, which has a length of  $L = 8$ . The number of pilot groups and the number of pilots per group are specified as  $Q = 4$  and  $N_Q = 8$ , respectively, giving a pilot sequence of length  $P = 32$ .

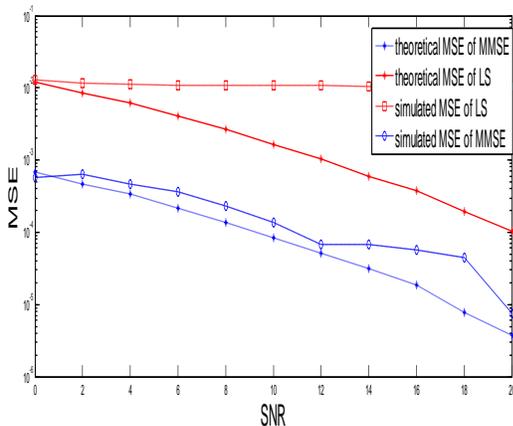


Fig. 1. MSE of LS and MMSE channel estimates for MISO-OFDM system. ( $L = 8, U = 1, N_T = 4, Q = 4, N_Q = 8$ ).

The Fig.1 shows the MSEs of the LS and MMSE channel estimates obtained using the proposed pilot sequence. As expected, the pilot sequence achieves nearly the MSE lower-bound for both the LS channel estimate and the MMSE channel estimate.

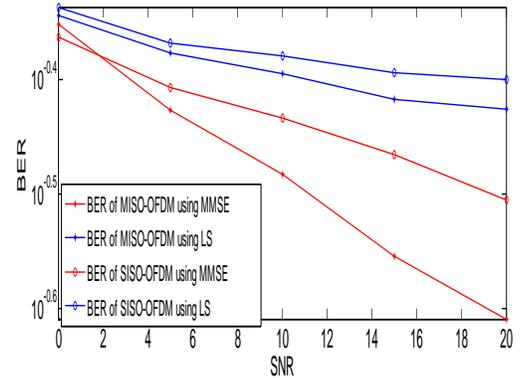


Fig. 2. BER performance of CDD-OFDM and SISO-OFDM systems

The Fig. 2 estimate the BER performance of CDD-OFDM and SISO-OFDM systems, where the channel length is assumed to be  $L = 8$  and is stationary during an OFDM symbol period. The channel is estimated using both LS and MMSE channel estimation.

VI. CONCLUSION

The pilot design criteria which minimize the MSE of both the LS channel estimate and the MMSE channel estimate in MISO-OFDM systems is implemented using CDD. In our proposed design, the pilots have the same amplitude and are equally partitioned into a number of pilot groups. The pilots in each group are also equally-spaced in the frequency domain. The pilot locations and cyclic-delay coefficient of each pilot group has been determined by this method. The optimal pilot sequence can be obtained from the proposed method when the channel remains static for only one OFDM symbol. The simulation results have confirmed that the proposed pilot structure achieves the MSE lower-bound for both the LS channel estimate and the MMSE channel estimate for MISO-OFDM using CDD.

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