

Numerical Simulation of Two Dimensional Incompressible Viscous Flows in a Lid-Driven Cavity

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ABSTRACT: The present work shows the prediction in Cartesian using the stream function- vorticity formulation for the Lid Driven Cavity. The governing equations are discretised and the variation of flow is analysed by using the commercial CFD software. Contour of Variation of Local skin Friction coefficient along left and right wall of the Cavity is shown. Variation of local skin friction coefficient along top and bottom wall of the cavity is plotted. Flow simulation for Reynold number 100 is taken with a square cavity the top lid is moving and the other side is stationary

KEYWORDS: CFD, stream function, vorticity, lid driven cavity, skin friction coefficient.

I. INTRODUCTION

Due to the development in the CAD/CAE technology, now it is possible to analyse the fluid flow problem, in general any engineering problems. Now a days the CFD approach is becoming popular has come in this way. In the present work of flow analysis inside a square cavity with the top lid moving with low velocity is analyzed and postprocessor velocity contour is obtained with commercial software. Initially the geometry is created and then the fluid domain is discretized by mesh. In the solver and postprocessor its solutions and result are obtained. Basically a theory of CFD- and stream function-vorticity approach is used and new software tool are applied. This technique may be used for designing of any fluid equipment by using the advantage of CAE.

These phenomena are governed by set of partial differential equations which in most cases have no analytical solution. In addition to the governing equations, we also need the boundary and initial conditions, material properties, and geometrical details in order to completely describe the problem.

II.GOVERNING EQUATIONS AND NUMERICAL PROCEDURE

2.1Continuity equation

Physical principle:- Law of conservation of mass.

$$\frac{\partial}{\partial t} \iiint_V \rho \, dV + \iint_S \rho \, \mathbf{v} \cdot d\mathbf{S} = 0$$

2. 2 Momentum equation

Physical principle: - Newton's second law (F=ma)

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u V) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v V) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho f_y$$

$$\frac{\partial(\rho w)}{\partial t} + \nabla \cdot (\rho w V) = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho f_z$$

Equations are the momentum equations in x, y and z direction respectively in conservation form.

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho f_y$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho f_z$$

2.3 Discretisation methods

The discretisation methods i.e. the numerical methods for solving PDEs include the finite difference methods (FDM), and the finite volume method (FVM) is used.

2.4 Stream Function Formulation

Flow prediction in Cartesian using the stream function- vorticity formulation

For two dimensional steady incompressible flow in a rectangular geometry, the governing equations for stream function and vortices are the following:

(i) Definition of vorticity:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

Where ψ is the stream function and ω is the vorticity

(ii) Convective – diffusive transport equation for vorticity

$$\left(u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \right) = \nu \left\{ \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right\}$$

Where the velocity components u and v are given by

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$$

The boundary conditions for the lid-driven cavity problem are

$u = 0, v = 0$ at $x = 0$; $u = 0, v = 0$ at $y = 0$;

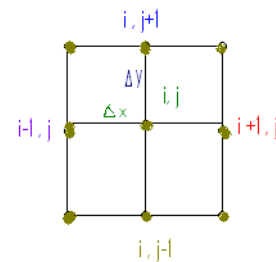
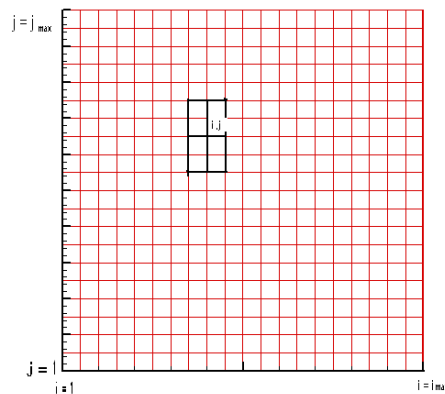
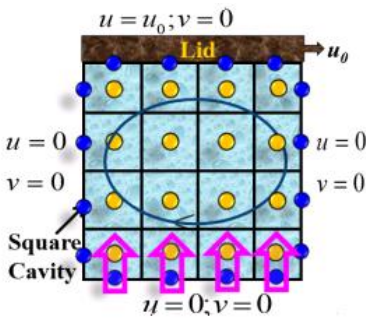
$u = 0, v = 0$ at $x = L$; $u = U_0, v = 0$ at $y = H$;

Where U_0 is the velocity of the lid.

$$\frac{\psi_{i+1,j} + \psi_{i-1,j} - 2\psi_{i,j}}{\Delta x^2} + \frac{\psi_{i,j+1} + \psi_{i,j-1} - 2\psi_{i,j}}{\Delta y^2} = -\omega_{i,j}$$

$$u_{i,j} \left[\frac{\omega_{i+1,j} - \omega_{i-1,j}}{2\Delta x} \right] + v_{i,j} \left[\frac{\omega_{i,j+1} - \omega_{i,j-1}}{2\Delta y} \right] =$$

$$V \left\{ \frac{\omega_{i+1,j} + \omega_{i-1,j} - 2\omega_{i,j}}{\Delta x^2} + \frac{\omega_{i,j+1} + \omega_{i,j-1} - 2\omega_{i,j}}{\Delta y^2} \right\}$$



III. RESULT

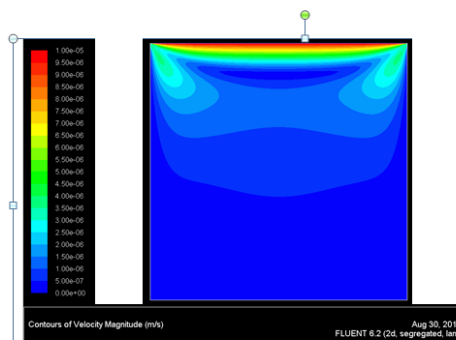


Fig.1. Velocity Contour-1

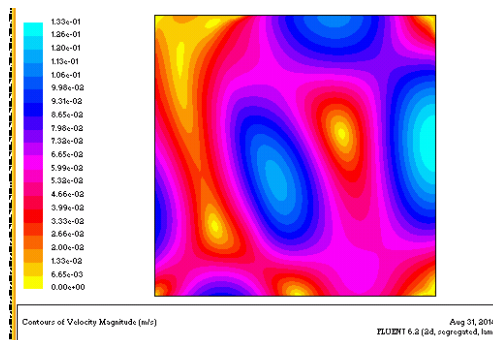


Fig.2. Velocity contour-2

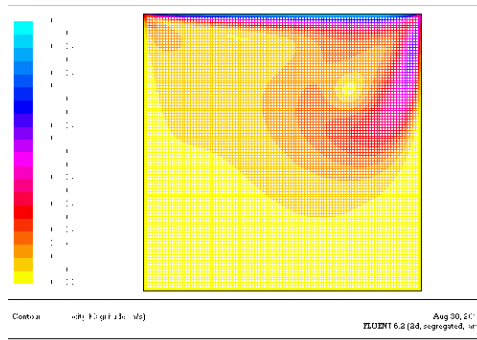


Fig.3 .Velocity contour-3

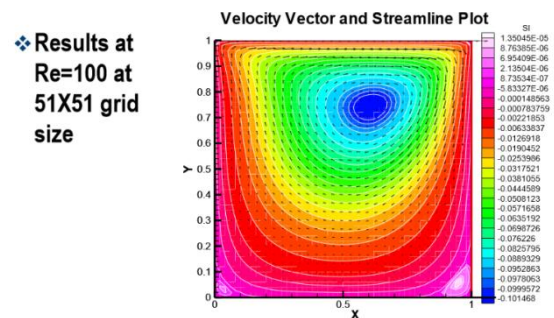


Fig.4. Velocity vector and stream line plot

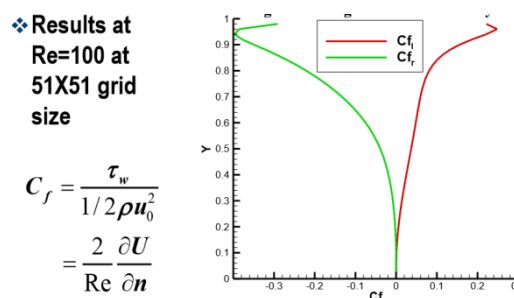


Fig.5. Variation of Local skin Friction coefficient along left and right wall of the Cavity

IV. CONCLUSION

In this present work the prediction in Cartesian using the stream function- vorticity formulation for the Lid Driven Cavity has been shown. On either side of the wall of the cavity the Local skin Friction coefficient varies in the same manner and found maximum between 0.9 and 1 on vertical and minimum at the vertically bottom.

Notations:

P- pressure

μ - viscosity

τ - viscous stress

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U, V, W - free stream mean velocity

 u', v', w' - instantaneous velocity ρ - density

f - body forces

 ψ - stream function and ω - vorticity**REFERENCES**

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