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On Designing Skip- Lot Sampling System

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ABSTRACT: This paper deals with designing of a new skip-lot sampling system having two different reference sampling plans with same sample size and two different acceptance numbers. The operating procedure of the new system and the derivation of performance measures using Markov-chain formulation are given. The designing of the system for two specified points on the OC curve and AOQL of the developed plan are given. The effect of system parameters on the performance measures are evaluated. The efficiency of the proposed sampling system is illustrated with matched SkSP-2 and single sampling plans.

KEYWORDS: Acceptable quality level, Average sample number, Single sampling plan, Skip-lot sampling system, Operating characteristic function.

I. INTRODUCTION

The acceptance sampling plans have been widely used in industries for maintaining the high quality level of the product at the minimum inspection cost. The skip-lot sampling schemes, have the provision of inspecting only a fraction of lots. Dodge [5] introduced SkSP-1 plan in chemical and physical processes to reduce inspection cost as an extension of continuous sampling plan, CSP-1 of Dodge [4]. Perry [8] proposed skip-lot sampling plan of type SkSP-2 by using single sampling plan as the reference plan and discussed the applications of the plan. Brugger [3] provided the derivation measures by employing simplified Markov-chain approach for SkSP-2 plan. Perry [9] extended the single level SkSP-2 plan to two levels of skipping inspection and derived their performance measures.

Vijayaraghavan [12] extended the SkSP-2 beyond two levels of sampling and using a general non-cost based approach to design skip-lot sampling plans and obtained tables using Poisson model. Govindaraju [7] studied the properties of SkSP-2 plan using single sampling plan with zero acceptance number as the reference plan. Balamurali et.al [2] proposed SkSP-R plan by including resampling and provided their performance measures. Aslam et.al [1] proposed SkSP-2 plan with two-stage group acceptance sampling plan as reference plan.

II. RELATED WORK

Stephens and Larson [11] indicated that an acceptance sampling system consisting of two or more sampling inspection plans and the rules for switching between them to achieve blending of the advantageous features of each of the sampling plans. This motivated the researcher to design the general skip-lot sampling system with two different reference sampling plans as a generalization of SkSP-2. In this paper a new skip-lot sampling system having two different reference sampling plans with same sample sizes and different acceptance numbers is introduced.

Performance measures are derived for the proposed system by Markov-Chain approach. Method of designing plans is indicated. Comparison with matched SkSP-2 plan and single sampling plan are presented and indicated that improved efficiency is achieved by the new system. Effect of parameters i and f on performance measures are evaluated. Tables are constructed to facilitate selection of plans. Method of construction of tables is also indicated.

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III. OPERATING PROCEDURE OF THE SYSTEM

The operating procedure of general skip-lot sampling system (GSkSS) has the following steps.

1. Start with normal inspection, using the reference sampling plan, r_N. At this stage every lot is inspected.
2. When i consecutive lots are accepted on normal inspection, switch to skipping inspection at rate f, using the reference sampling plan r_S.
3. When a lot is rejected, switch to normal inspection. Screen each rejected lot and replace all nonconforming units found.

This new system designated as GSkSS (i, f, n, c_N, c_S) – refers to a skip-lot sampling system where the normal and skipping single sampling plans have the same sample size n, but different acceptance numbers c_N and c_S with $c_N \leq c_{S.}$ where c_N is the acceptance number for normal inspection reference plan, r_N and c_S is the acceptance number for skipping inspection reference plan,r_S. If $c_N = c_S$ the system degenerates into a skip-lot sampling plan of type SkSP-2 with parameters (i,f,n,c).

IV. MEASURES OF PERFORMANCE

The performance measures are derived using Markov-Chain approach due to Robert's [10]. The states of GSkSS are defined as

NO - Lot rejected on normal inspection using reference plan r_N.
NJ - j lots are consecutively accepted during normal inspection using reference plan r_N, j = 1, 2, ……..,i
SA - lot is accepted during skipping inspection at rate f, using reference plan r_S.
SR - lot is rejected during skipping inspection at rate f, using reference plan r_S.
SN - lot is skipped during skipping inspection at rate f, using reference plan r_S.

The one-step transition matrix for the GSkSS plan is presented in Table A

TABLE-A TRANSITION PROBABILITY MATRIX OF GSkSS

		State at t^{th} trial							
		N0	**N1**	**N2**	**…**	**Ni**	**SA**	**SR**	**SN**
State at $(t-1)^{st}$ trial	**N0**	Q	P	•		•	•	•	•
	N1	Q	•	P		•	•	•	•
	•								
	•								
	•								
	•								
	Ni-1	Q	•	•		P	•	•	•
	Ni	•	•	•		•	fP_1	fQ_1	(1-f)
	SA	•	•	•		•	fP_1	fQ_1	(1-f)
	SR	Q	P	•		•	•	•	•
	SN	•	•	•		•	fP_1	fQ_1	(1-f)

In the matrix,

P - the probability of acceptance of a lot according to reference plan r_N
Q - 1 – P

P_1 - the probability of acceptance of a lot according to reference plan r_S
Q_1 - $1 – P_1$

Using this transition probability matrix, the steady state probabilities π_j satisfy the following conditions

$$\pi_j \geq 0 \text{ for all (i+4) states and } \sum_{j=1}^{i+4} \pi_j = 1$$

are derived as

$$\begin{aligned}
\pi_{N0} &= (1–P^i)\, fQQ_1 / D \\
\pi_{N1} &= fQQ_1P / D \\
\pi_{Nj} &= fQQ_1P/D, \qquad j = 2, 3, \ldots\ldots, i \qquad (1) \\
\pi_{SA} &= fQP_1P^i / D \\
\pi_{SR} &= fQQ_1P^i / D \\
\pi_{SN} &= (1–f)QP^i / D, \text{ where } D = fQ_1 + P^i(Q–fQ_1)
\end{aligned}$$

The operating characteristic function of GSkSS is

$$\begin{aligned}
P_a &= 1 – \pi_{SR} – \pi_{NO} \\
&= \{fQ_1P + P^i(Q – fQ_1)\} / D \qquad (2)
\end{aligned}$$

The average fraction of total lots inspected (AFI) comprised of lots from normal inspection and a fraction, f of the lots during skipping inspection. Therefore,

$$\begin{aligned}
F &= 1 – \pi_{SN} \\
&= \{fQ_1(1–P^i) + fQP^i\} / D \qquad (3)
\end{aligned}$$

A very important property of skip-lot sampling scheme is that of reduced inspection, it will be investigated with the help of average sample numbers ASN (GSkSS) = F•ASN(R), where ASN(R) is the average sample number of units inspected per lot under normal and skipping inspection.

ASN(R) = n

V. MATHEMATICAL PROPERTIES OF P_a

The mathematical properties and relationships between P_a and each of its parameters – i, f, P and P_1 are investigated. The results obtained will hold for all values of i and f.

i) $P_a = 1 – [Q_1(1 – P) / \{P^i\, Q / f + Q_1(1 – P^i)\}]$
implies P_a is a increasing function of f (i, P, P_1 are fixed)

ii) $P_a = 1 – [\{f\, Q_1\, Q\} / \{f\, Q_1 + P^i(Q – fQ_1)\, \}]$
implies P_a is a decreasing function of i (f, P, P_1 are fixed)

iii) $P_a = P[Q_1 + P^{i-1}(Q – fQ_1)] / [fQ_1 + P^i\,(Q – fQ_1)]$
implies $P_a \geq P$ and $P_a \geq P_1$,since $P^i < P^{i-1}$

iv) P_a is a increasing function of P as well as P_1 (i, f are fixed)

VI. DESIGNING OF SYSTEM FOR GIVEN TWO FIXED POINTS ON THE OC CURVE (P_1,1-α), (P_2,β)

Tables 1 and 2 may be used to design GSkSS with two reference single sampling plans having two different acceptance numbers but same sample size for a given p_1, p_2, α and β. Consider the designing of GSkSS for the given $p_1 = 0.006$, $p_2 = 0.04$, $\alpha = 0.05$ and $\beta = 0.10$.

Compute the operating ratio, OR = p_2/p_1 = 6.66.

From Table 1 one obtains OR=6.8855. The associated np_1 is 0.334093, therefore the sample size for normal and skipping reference plans is n = np_1 / p_1~ 56. Thus the required GSkSS (i, f, n, c_N, c_S) is (4, 1/3, 56, 0, 1).

From Table 2 one obtains OR=6.7260. The associated np_1 is 0.577383, which gives the sample size as n=96. Thus the required GSkSS (i, f, n, c_N, c_S) is (14, 2/3, 96, 1, 2).

VII. CALCULATION OF AOQL OF A GIVEN PLAN

Table 2 may be used to calculate AOQL and p_m of given system. For example one requires AOQL for a given system with n=56, f=2/3, i=14, c_N =1 and c_S =2. Table 2 gives nAOQL =0.840146 and np_m = 1.615019. Therefore AOQL= 0.015002 and p_m= 0.028839. This indicates that the worst outgoing quality of using the plan GSkSS (14,2/3,56,1,2) is 0.028839 for any incoming lot quality.

VIII. EFFECT OF PARAMETERS i AND f ON PERFORMANCE MEASURES

The effect of i and f on probability of acceptance and average fraction of lots inspected are illustrated graphically. Probability of acceptance curves for fixed i and fixed f are presented in fig.1 and the corresponding AFI curves are presented in fig.2. Figures reveal that i) For a fixed i, increase in f increases OC value when the quality is good and increases AFI irrespective of the incoming quality ii) For a fixed f, increse in i decreases OC value when the quality is poor and increases AFI.

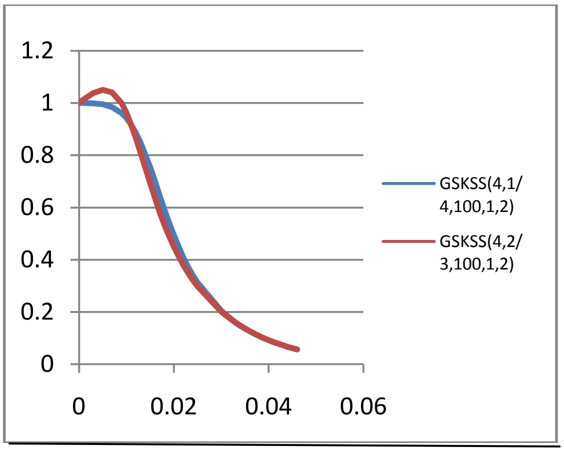
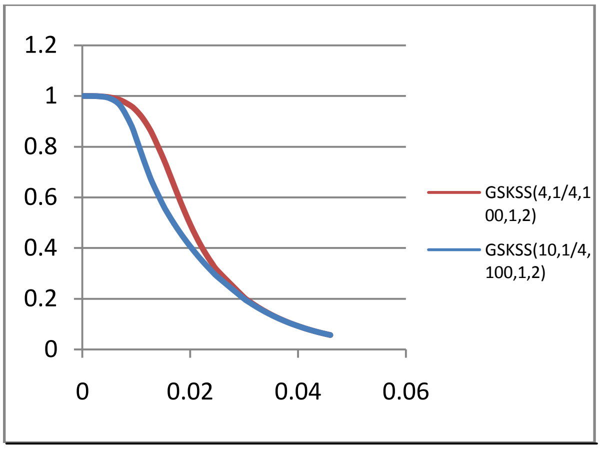


Fig-1 OC curves for same i different f



OC curves for same f different i

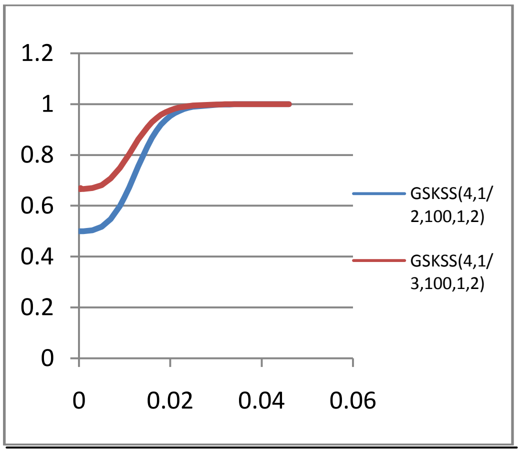
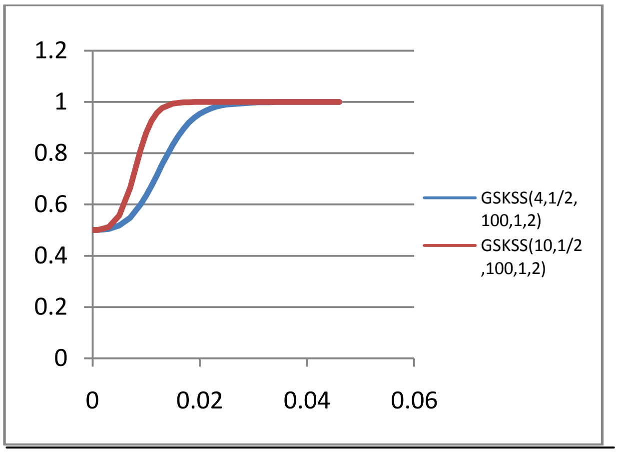


Fig-2 AFI curves for same i different f



AFI curves for same f different i

IX. COMPARISON OF GSKSS WITH SKSP-2 AND SS PLANS

Grubbs [6] suggested a measure to compare, called "Operating Ratio" which is defined as $p_{0.10}/p_{0.95}$ where $p_{0.10}$ and $p_{0.95}$ are the values of p with $P_a = 0.10$ and 0.95 respectively. This reflects the ability of a plan to discriminate between good and bad quality. The matched GSkSS, SkSP-2 and SS plans are obtained by means of nearly equal OR values.

Some GSkSS and reasonably well-matched SkSP-2 plans and single sampling plans are given in Table 3 Suppose that one requires a plan for $p_1 = 0.015$ $p_2 = 0.072$, $\alpha = 0.05$, and $\beta = 0.10$. The operating ratio is 4.8. From Table 3

I) The SSP with n = 91, c = 3, np_1 is 1.3663
ii) The SkSP-2 plan with i = 14, f = 1/5, n = 73, c = 2, np_1 is 1.09
iii) GSkSS with f = 1/4/, i = 6, n = 54, $c_N = 1$, $c_S = 2$, np_1 is 0.8127
iv) GSkSS with f = 1/4, i = 8, n = 54,c_N =1, c_S = 2 and np_1 is 0.7955

Further, the last two columns in table 3 when subtracted from 1.0 represent the percent reduction in the sample size of GSkSS with that of SkSP-2 and SS matched plans. This shows reduction in sample size for GSkSS over SkSP-2 plan and single sampling plan. This suggests the economy of the proposed plan.

X. CONCLUSION

A new skip-lot sampling system called GSkSS has been proposed in this paper. The two points on the OC curve approach is adopted to design parameters of the proposed system. Various measures of performance for proposed system have been derived using Markov chain model. The proposed system is found to be more efficient than the matched single and SkSP-2 plans. It is found that the proposed system requires the less number of sample units for the inspection purpose than the conventional single and SkSP-2 plans. So the proposed system is useful in reducing the cost and the time of inspection of the material or the product.

XI. CONSTRUCTION OF TABLES

The OC function of GSkSS derived in (2) is

$$P_a = \{Fq_1 P + P^i(Q - Fq_1)\} / \{Fq_1 + P^i(Q - Fq_1)\} \quad (4)$$

Under Poisson model with single sampling reference plans,

$$P = \sum_{d=0}^{C} \exp(-np)\,(np)^d / d! \quad (5)$$

$$P = \sum_{d=0}^{C+1} \exp(-np)\,(np)^d / d! \quad (6)$$

when we use the reference sampling plans with different acceptance numbers and same sample size. Now for assumed values of i, f and $P_a(p)$ (in equation 2) can be solved for np by the methods of successive approximation.

First and second columns of Tables 1 and 2 the values relating to operating ratio and np_1 corresponding to α = 0.05 and β = 0.10 are obtained and furnished.

Assuming nAOQ = $np.P_a(P)$, values of np_mthat maximizes Naoq are obtained by the method of iterations and the nAOQL values are obtained as $nAOQL_m = np_m.Pa(p_m)$. The values of np_m and nAOQL are provided in third and fourth columns of Tables 1 and 2. The tables are constructed by assuming c_N (= 0,1),c_S (= 1,2), i (= 4, 6, 8, 10, 12, 14) and f (= 2/3, 1/2, 1/3,1/4, 1/5).

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TABLE 1 PARAMETRIC VALUES OF GSkSS FOR $c_N = 0$ AND $c_S = 1$

f	i	OR	np_1	nAOQL	np_m
2/3	4	8.63486	0.266202	0.414461	0.710995
	6	9.98080	0.230203	0.372871	0.913992
	8	11.20225	0.205103	0.368453	0.987991
	10	12.33932	0.186203	0.367954	0.997991
	12	13.41262	0.171302	0.367890	0.999991
	14	14.42300	0.159302	0.367881	0.999991
1/2	4	7.84184	0.293198	0.435473	0.699995
	6	9.16466	0.250704	0.376228	0.840993
	8	10.36814	0.221603	0.368735	0.981991
	10	11.48217	0.200103	0.367990	0.996991
	12	12.52771	0.183403	0.367894	0.999991
	14	13.52315	0.169902	0.367881	0.999991
1/3	4	6.88555	0.334093	0.469534	0.708995
	6	8.16821	0.281300	0.388743	0.651995
	8	9.33217	0.246204	0.369339	0.967991
	10	10.41043	0.220703	0.368061	0.994991
	12	11.42507	0.201103	0.367904	0.998991
	14	12.39257	0.185403	0.367883	0.999991
1/4	4	6.30253	0.365188	0.496303	0.724994
	6	7.55091	0.304297	0.403669	0.619996
	8	8.68657	0.264502	0.370010	0.951991
	10	9.73964	0.235903	0.368133	0.992991
	12	10.73135	0.214103	0.367913	0.998991
	14	11.66877	0.196903	0.367884	0.999991
1/5	4	5.89579	0.390585	0.518380	0.740994

	6	7.11600	0.322894	0.417046	0.613996
	8	8.22928	0.279200	0.370775	0.929992
	10	9.26070	0.248104	0.368206	0.990991
	12	10.23421	0.224503	0.367922	0.998991
	14	11.15331	0.206003	0.367885	0.999991

TABLE 2 PARAMETRIC VALUES OF GSkSS FOR $c_N = 1$ AND $c_S = 2$

f	i	OR	np_1	nAOQL	np_m
2/3	4	5.166006	0.751913	0.969879	1.392009
	6	5.563704	0.698004	0.888365	1.343007
	8	5.905572	0.657597	0.851786	1.484013
	10	6.207701	0.625591	0.842714	1.581018
	12	6.479093	0.599387	0.840660	1.606019
	14	6.726006	0.577383	0.840146	1.615019
1/2	4	4.798046	0.809722	1.013632	1.409010
	6	5.214752	0.744711	0.913816	1.306005
	8	5.570888	0.697103	0.860310	1.383009
	10	5.884093	0.659997	0.844253	1.559017
	12	6.164338	0.629992	0.841015	1.601019
	14	6.420172	0.604888	0.840236	1.613019
1/3	4	4.340807	0.895336	1.080664	1.450012
	6	4.778365	0.812723	0.957070	1.294004
	8	5.149740	0.754113	0.883461	1.253002
	10	5.473510	0.709505	0.848043	1.492014
	12	5.763562	0.673800	0.841766	1.588018
	14	6.026563	0.644395	0.840420	1.610019
1/4	4	4.055592	0.958647	1.131462	1.487013
	6	4.503069	0.862431	0.991603	1.305005
	8	4.881696	0.795520	0.906686	1.222001
	10	5.211253	0.745211	0.854321	1.318006
	12	5.505329	0.705405	0.842585	1.574017
	14	5.772130	0.672799	0.840609	1.606019
1/5	4	3.853514	1.009255	1.172497	1.520015
	6	4.306781	0.901737	1.020102	1.318006
	8	4.689491	0.828125	0.927198	1.216001
	10	5.022510	0.773216	0.865997	1.198000
	12	5.319050	0.730109	0.843493	1.556017
	14	5.587724	0.695003	0.840803	1.602019

TABLE 3 COMPARISON OF SAMPLE SIZES - GSkSS WITH SkSP-2 AND SSP

SS PLAN			SKIP-LOT PLAN SkSP-2			SKIP-LOT SYSTEM GSkSS			RATIO OF SAMPLE SIZES	
C	OR	$np_{.95}$	(C, i, f)	OR	$np_{.95}$	(C_N,C_S,i, f)	OR	$np_{.95}$	A	B
2	6.50897	0.81769	(1, 8, 1/3)	6.505	0.598	(0,1, 4, 1/3)	6.8855	0.3340	0.4083	0.5585
						(0,1, 4, 1/4)	6.3025	0.3651	0.4465	0.6105
3	4.88962	1.36632	(2, 14, 1/5)	4.883	1.09	(1,2, 8, 1/4)	4.8816	0.7955	0.5822	0.7298
						(1,2, 6, 1/4)	4.7783	0.8127	0.5948	0.7455
						(1,2,8,1/5)	4.6894	0.8281	0.6060	0.7597
4	4.05735	1.97015	(3, 4, 1/2)	4.063	1.645	(1,2, 4, 1/4)	4.0555	0.9586	0.4865	0.5827
						(1,2, 4, 1/3)	4.3408	0.8953	0.4544	0.5442
						(1,2,, 6, 1/5)	4.3067	0.9017	0.4576	0.5481

A = RATIO OF SAMPLE SIZES OF GSkSS WITH SS PLAN

B = RATIO OF SAMPLE SIZES OF GSkSS WITH SkSP-2 PLAN