OPTIMIZATION OF SURFACE AREA OF POROUS STRUCTURES

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ABSTRACT

Heat transfer from a surface depends on the surface area of the surface exposed to convection. Porous structures are one that has high surface area to volume ratio. For a given porosity the surface area can be maximized by varying the fiber width. In the present study a unit cell model like a Rubik's cube is considered. The cubes formed by changing number of single cubes in a raw are also considered. The fiber width corresponding to maximum surface area is found for porosity 0.45.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A1</td>
<td>SA of initial solid cell, cm²</td>
</tr>
<tr>
<td>A2</td>
<td>Total SA of porous cell, cm²</td>
</tr>
<tr>
<td>A2i</td>
<td>Inner SA of porous cell, cm²</td>
</tr>
<tr>
<td>A2o</td>
<td>Outer SA of porous cell, cm²</td>
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<td>SA</td>
<td>Surface area, cm²</td>
</tr>
<tr>
<td>p</td>
<td>Fiber width, cm</td>
</tr>
<tr>
<td>r</td>
<td>Porosity</td>
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<tr>
<td>R</td>
<td>Number of unit cells in a raw</td>
</tr>
<tr>
<td>V1</td>
<td>Initial volume of solid cell, cm³</td>
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<tr>
<td>V2</td>
<td>Volume of void, cm³</td>
</tr>
<tr>
<td>p</td>
<td>Fiber width, cm</td>
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</table>

1. INTRODUCTION

Removal of heat from electronic equipment is essential for enhancing its durability. High heat removal depends on the surface area projected to the surrounding medium. The size of structure used for enhancing heat transfer depends on space and weight restriction. The usage of porous body in thermal system will improve heat transfer rate according to law of convection and reduces weight and space requirement.

Several studies have been endeavored to the characteristics of heat transfer for porous media with various conditions including the Reynolds number, flow conditions, Darcy number, porosity, shape and conditions of bodies. Huang et.al. [1] analyzed the changes in the flow pattern and heat transfer characteristics due to the existence of the multiple porous block structure. He had explored the effects of several governing dimensionless parameters, such as the Darcy number, Reynolds number, Prandtl number, and the inertia parameter. Jin et.al. [2]
studied numerically flow and heat transfer characteristics of forced convection in an open channel which is partially filled with porous medium. The maximum temperature at the heat source and the associated pressure drop were presented for varying Re, Da, S and Rk. They found that as S increases or Da decreases, the fluid flow rate increases. Hartnett et.al. [3] investigated the fluid flow and heat transfer for oscillatory flow with the heat transfer in a cavity filled with porous medium. He found that the permeability of porous medium exhibits more influence in flow and thermal phenomena and it diminishes the circulation strength and heat flux as the value is decreased. Zhao et.al. [4] had investigated analytically the forced convection in a saturated porous medium subjected to heating with a permeable wall perpendicular to the flow direction. The forced convection heat transfer of a porous medium in a laminar channel flow is numerically investigated by Shung et.al. [5]. Results shows that when the mean porosity is larger than 0.5, the average Nusselt numbers are enhanced and better than the solid block case. They found that a porous medium with larger porosity and the proper bead diameter could provide more heat dissipation. A numerical study was carried out by Huang et.al [6] for enhanced heat transfer from multiple heated blocks in a channel by porous covers. The results shows that the recirculation caused by porous-covering block will significantly enhance the heat transfer rate on both top and right faces of second and subsequent blocks.

The flow resistance and heat transfer characteristics of the air flow for laminar to fully turbulent ranges of Reynolds numbers are investigated experimentally and numerically by Huang [7]. He found that the core flow enhancement is an efficacious method for enhancing heat transfer. Jian et.al [8] has studied numerically forced convective heat transfer in three-dimensional porous pin fin channels. They have explored that the overall heat transfer performances in porous pin fin channels are much better than those in traditional solid pin fin channels. They found that for the same physical parameters, the overall heat transfer efficiencies in the long elliptic porous pin fin channels are the highest while they are the lowest in the short elliptic porous pin fin channels.

Nessrine [9] had analyzed the fluid flow and heat transfer in a pipe partially filled with porous media and provided with a flat piston during an expansion stroke. Darvishi et.al [10] had introduced an analytic (series) solution to describe the thermal performance of a porous radial fin with natural convection in the fluid saturating the fin and radiation heat loss from the top and bottom surfaces of the fin.

From the preceding studies it is found that the porosity of structures has great importance in improving heat transfer. Here optimization of surface area variation with respect to porosity as well as fiber width for a cubical porous block is done.

**2. MATHEMATICAL MODELLING**

The porous structure used for this study is formed by arranging a number of single porous cells one above the other. A single porous cell is obtained by taking some volume of material from a solid cell.

**Single porous cell formation and its surface area analysis**

Unit cell of dimensions 'a', 'b', 'c' (Fig 1) is made porous by taking materials away along x, y, z directions, thus we got a porous unit cell (Fig 2). Porosity is the ratio of void volume to initial volume. The variable in this study is fiber 'p' which is shown in Fig. 2.
Initial volume of solid cell is given by
\[ V_1 = abc \] (1)

Volume of void generated, \( V_2 \) is the sum of volume of material removed in 'z', 'x' 'y' directions i.e.
\[ V_2 = 16p^3 - 4p(a+b+c) + abc \] (2)

Porosity, \( r = \frac{V_2}{V_1} \) (3)

For a unit cube the all sides have unit dimensions, i.e., \( a=b=c=1 \)
Initial volume of solid cell, \( V_1 = 1 \)

Volume of void generated,
\[ V_2 = 16p^3 - 12p + 1 \] (4)

Porosity, \( r = 16p^3 - 12p + 1 \) (5)

For various porosity the fiber width can be get by solving equation (5) And are given in table 1

Surface area of initial solid cell
\[ A_1 = 2(ab+bc+ac) \] (6)

Due to the material removal from solid unit cell, 24 internal areas are formed which are shown in fig.2.

Outer surface area of porous cell
\[ A_{2o} = 2[ab-(a-2p)(b-2p)] + 2(bc-(b-2p)(c-2p))] + 2[ac-(a-2p)(c-2p)] \]
\[ A_{2o} = 8p(a+b+c) - 24p^2 \] (7)

Inner surface area of porous cell
\[ A_{2i} = 8(a-2p)p + 8(b-2p)p + 8(c-2p)p \]
\[ A_{2i} = 8p(a+b+c) - 48p^2 \] (8)

Total surface area of porous cell
\[ A_2 = A_{2o} + A_{2i} \]
\[ A_2 = 16p(a+b+c) - 72p^2 \] (9)

When \( a=b=c=1 \)

Total surface area of porous cell
\[ A_2 = 48p^2 - 72p^2 \] (10)

For various fibers width the surface area can be find out using equation (10) and are given in table1.

**Porous Rubik’s cube formation and its surface area analysis**

The Fig 3 and Fig.4 shows the porous structure formed by assembling 27 unit cells in the shape of Rubik's cube.
Volume of initial solid cell
\[ V_1 = 3a \cdot 3b \cdot 3c = 27abc \]  
Equation (11)

Volume of void generated
\[ V_2 = 27(16p^3 - 4P^2(a+b+c) + abc) \]  
Equation (12)

Porosity
\[ r = 16p^3 - 4p^2(a+b+c) + abc \]  
Equation (13)

For \( a = b = c = 1 \)
\[ r = 16p^3 - 12p^2 + 1 \]  
Equation (14)

Outer surface area of 3\(^3\) solid cells
\[ A_1 = 2[(3a \cdot 3b) + (3a \cdot 3c) + (3b \cdot 3c)] \]
\[ A_1 = 18(ab + ac + bc) \]  
Equation (15)

Outer surface area of 3\(^3\) porous cells
\[ A_{2o} = 2\{3[ab-(a-2p)(b-2p)]\} \]
\[ + 2\{3[bc-(b-2p)(c-2p)]\} \]
\[ + 2\{3[ac-(a-2p)(c-2p)]\} \]
\[ = 3[8p(a+b+c) - 24p^2] \]
\[ A_{2o} = 72p(a+b+c) - 216p^2 \]  
Equation (16)

Inner surface area of 3\(^3\) porous cells
\[ A_{2i} = 3 \cdot (inner \ surface \ area \ of \ one \ porous \ cell) \]
\[ A_{2i} = 3 \cdot (8p(a+b+c) - 48p^2) \]  
Equation (17)
\[ A_{2i} = 216p(a+b+c) - 1296p^2 \]  
Equation (18)

Total surface area of 3\(^3\) porous cells
\[ A = 288p(a+b+c) - 1344p^2 \]  
Equation (19)

For various porosities the strip width value and surface area can be calculated from above equations and are given in table 1.

R\(^3\) porous cells are assembled to form porous cube and its surface area analysis

R is the number of unit cells in each raw and column of final porous cube

When R\(^3\) porous single cells are arranged then the porosity can be calculated as follows.
Volume of initial solid cell,
\[ V_1 = R_a R_b R_c. \]  
\[ V_1 = R_{abc} \]  \hspace{1cm} \text{(20)}

Volume of void generated
\[ V_2 = R_3 (\text{volume of void of single cell}) \]  
\[ V_2 = R_3 (16p_3 - 4p_2(a+b+c) + abc) \]  \hspace{1cm} \text{(21)}

Porosity
\[ r = 16p_3 - 4p_2(a+b+c) + abc \]  \hspace{1cm} \text{(22)}

For \( a=b=c=1 \)

Porosity
\[ r = 16p_3-12p_2+1 \]  \hspace{1cm} \text{(23)}

Outer surface area of 3 solid cells
\[ A_1 = 2[(R_a R_b) + (R_a R_c) + (R_b R_c)] \]  
\[ A_1 = 2R_2[ab+ac+bc] \]  \hspace{1cm} \text{(24)}

Outer surface area of 3 porous cells
\[ A_{2o} = \{R_2[ab-(a-2p)(b-2p)]\} \]  
\[ + \{R_2[bc-(b-2p)(c-2p)]\} \]  
\[ + \{R_2[ac-(a-2p)(c-2p)]\} \]  
\[ = R_2[8p(a+b+c) - 24p_2] \]

Inner surface area of 3 porous cells
\[ A_{2i} = R_3 \text{ (inner surface area of one porous cell) } \]  
\[ A_{2i} = R_3 (8p(a+b+c) - 48p_2) \]  \hspace{1cm} \text{(25)}

Total surface area for \( a=b=c=1 \)
\[ A_2 = R_2[24p - 24p_2] + R_3(24p - 48p_2) \]  \hspace{1cm} \text{(26)}

For various \( R \) value the strip width and surface areas for various porosity is tabulated in Table 1.

3. RESULTS AND DISCUSSIONS

For various porosity values the corresponding strip width and surface areas of porous structure are calculated and are tabulated in table 1. The yellow shaded are maximum areas obtained. For single porous cell the maximum surface area obtained is 7.999 cm² and is at 25% porosity. Corresponding strip width is 3.37mm. For 23 number of cells are arranged like cube the maximum surface area obtained is 43.2 cm² at 35% porosity. Corresponding strip width is 3.01 mm. For 33 and 43 cells the maximum areas obtained are 123.42 cm² and 266.6 cm² respectively at 40% porosity. The corresponding strip width is 2.84 mm. Further if we increase number of cells up to 103 cells the maximum surface area obtained at 45% porosity. The corresponding strip width is 2.67 mm.

If we use such porous cubes for transferring heat from a surface the heat transfer will be a maximum for the porous structure having largest area. The number of porous cells can be selected according to the space available. The porosity can be selected from the Table 1.
TABLE 1. FOR VARIOUS POROSITY, FIBER WIDTH AND SURFACE AREAS (SA IN CM$^2$) FOR VARIOUS 'R'

<table>
<thead>
<tr>
<th>Sl no</th>
<th>Porosity</th>
<th>Strip width, p(cm)</th>
<th>SA for R=1</th>
<th>SA for R=2</th>
<th>SA for R=3</th>
<th>SA for R=4</th>
<th>SA for R=5</th>
<th>SA for R=6</th>
<th>SA for R=7</th>
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<th>SA for R=10</th>
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</table>

Fig 5. SA (IN CM$^2$) AGAINST POROSITY FOR VARIOUS 'R'

A graph of surface area against porosity for various R values is given in Fig 5. The maximum surface area is obtained for R=10cm. The peak points for R=5 to R=10 is porosity = 45%.

4. CONCLUSION

The optimization of surface area of unit cell model is conducted. Generalization of equation for surface area is made. The changes in surface area according to the changes in porosity and corresponding fiber width are tabulated. The maximum surface area is obtained for 45% porosity and the optimal fiber width is 0.267 cm. Further increase in porosity will lead to decrease surface area. These surface area models can use for further heat transfer study.
REFERENCES