

# Regular $\alpha$ Generalized Open Sets In Intuitionistic Fuzzy Topological Spaces

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**ABSTRACT:** The purpose of this paper is to introduce and study the concept of intuitionistic fuzzy regular  $\alpha$  generalized open sets in intuitionistic fuzzy topological spaces. We investigate some of their properties and also we introduce intuitionistic fuzzy regular  $\alpha$   $T_{1/2}$  space and obtain some characterizations and several preservation theorems.

**KEYWORDS:** Intuitionistic fuzzy set, Intuitionistic fuzzy topology, Intuitionistic fuzzy topological space, Intuitionistic fuzzy regular  $\alpha$  generalized open set, Intuitionistic fuzzy regular  $\alpha$   $T_{1/2}$  space.

## I. INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh [10] in 1965. Later, Chang [2] proposed fuzzy topology in 1967. The concept of intuitionistic fuzzy sets, introduced by Atanassov [1] is a generalization of fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces in 1997. In this paper, we introduce intuitionistic fuzzy regular  $\alpha$  generalized open set. We investigate some of their properties. We also introduce intuitionistic fuzzy regular  $\alpha$   $T_{1/2}$  space and obtain some characterizations and several preservation theorems.

## II. PRELIMINARIES

*Definition 2.1:* [1] An intuitionistic fuzzy set (IFS in short)  $A$  in  $X$  is an object having the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  where the function  $\mu_A : X \rightarrow [0,1]$  and  $\nu_A : X \rightarrow [0,1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ .

*Definition 2.2:* [1] Let  $A$  and  $B$  be two IFSs of the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$ . Then

- (i)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$
- (ii)  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$
- (iii)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (iv)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (v)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation  $A = \langle x, \mu_A, \nu_A \rangle$  instead of  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ . The IFS  $0 \sim = \{ \langle x, 0, 1 \rangle / x \in X \}$  and  $1 \sim = \{ \langle x, 1, 0 \rangle / x \in X \}$  are respectively the empty set and the whole set of  $X$ .

*Definition 2.3:* [3] An intuitionistic fuzzy topology (IFT in short) on  $X$  is a family  $\tau$  of IFS in  $X$  satisfying the following axioms:

- (i)  $0 \sim, 1 \sim \in \tau$
- (ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$
- (iii)  $\cup G_i \in \tau$  for any family  $\{ G_i / i \in J \} \subseteq \tau$

In this case the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS in short) in  $X$ . The complement  $A^c$  of an IFOS  $A$  in  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS in short) in  $X$ .

*Definition 2.4:* [3] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

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(i)  $\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$

(ii)  $\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\text{cl}(A^c) = (\text{int}(A))^c$  and  $\text{int}(A^c) = (\text{cl}(A))^c$ .

**Definition 2.5:** [5] An IFS  $A$  in an IFTS  $(X, \tau)$  is said to be

(i) intuitionistic fuzzy semiopen if  $A \subseteq \text{cl}(\text{int}(A))$

(ii) intuitionistic fuzzy  $\alpha$  open if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$

(iii) intuitionistic fuzzy preopen if  $A \subseteq \text{int}(\text{cl}(A))$

(iv) intuitionistic fuzzy regular open if  $A = \text{int}(\text{cl}(A))$

**Definition 2.6:** [4] An intuitionistic fuzzy point (IFP in short), written as  $p_{(\alpha, \beta)}$ , is defined to be an IFS of  $X$  given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP  $p_{(\alpha, \beta)}$  is said to belong to a set  $A$  if  $\alpha \leq \mu_A$  and  $\beta \geq \nu_A$

**Definition 2.7:** [7] An IFTS  $(X, \tau)$  is said to be an  $\text{IFT}_{1/2}$  space if every IFGCS in  $(X, \tau)$  is an IFCS in  $(X, \tau)$ .

**Definition 2.8:** [8] Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then

(i)  $\alpha\text{int}(A) = \cup \{ G / G \text{ is an IF}\alpha\text{OS in } X \text{ and } G \subseteq A \}$

(ii)  $\alpha\text{cl}(A) = \cap \{ K / K \text{ is an IF}\alpha\text{CS in } X \text{ and } A \subseteq K \}$

**Result 2.9:** [8] Let  $A$  be an IFS in  $(X, \tau)$ . Then

(i)  $\alpha\text{cl}(A) = A \cup \text{cl}(\text{int}(\text{cl}(A)))$

(ii)  $\alpha\text{int}(A) = A \cap \text{int}(\text{cl}(\text{int}(A)))$

**Definition 2.10:** [9] An IFS  $A$  in an IFTS  $(X, \tau)$  is intuitionistic fuzzy  $\alpha$  generalized closed set (IF $\alpha$ GCS in short) if  $\alpha\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $X$ .

**Definition 2.11:** [6] An IFS  $A$  of an IFTS  $(X, \tau)$  is called intuitionistic fuzzy regular  $\alpha$  generalized closed set (IFR $\alpha$ GCS in short) if  $\alpha\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFROS in  $X$ .

### III. INTUITIONISTIC FUZZY REGULAR $\alpha$ GENERALIZED OPEN SETS

In this section we introduce the notion of intuitionistic fuzzy regular  $\alpha$  generalized open sets and study some of their properties.

**Definition 3.1:** An IFS  $A$  of an IFTS  $(X, \tau)$  is called intuitionistic fuzzy regular  $\alpha$  generalized open set (IFR $\alpha$ GOS in short) if  $\alpha\text{int}(A) \supseteq U$  whenever  $A \supseteq U$  and  $U$  is an IFRCS in  $X$ .

The family of all IFR $\alpha$ GOSs of an IFTS  $(X, \tau)$  is denoted by  $\text{IFR}\alpha\text{GO}(X)$ .

Note that the complement  $A^c$  of an IFR $\alpha$ GCS  $A$  in an IFTS  $(X, \tau)$  is an IFR $\alpha$ GOS in  $X$ .

**Example 3.2:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  where  $G_1 = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$  where  $\mu_a=0.6, \mu_b=0.7, \nu_a=0.4, \nu_b=0.2$  and  $G_2 = \langle x, (0.1, 0.2), (0.7, 0.7) \rangle$  where  $\mu_a=0.1, \mu_b=0.2, \nu_a=0.7, \nu_b=0.7$  let  $A = \langle x, (0.6, 0.7), (0.3, 0.1) \rangle$  be any IFS in  $(X, \tau)$ . Then  $A \supseteq U$  where  $U = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$  is an IFRCS in  $X$ . Now  $\alpha\text{int}(A) = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle \supseteq U$ . Therefore  $A$  is an IFR $\alpha$ GOS in  $(X, \tau)$ .

**Theorem 3.3:** Every IFOS, IFROS and IF $\alpha$ OS is an IFR $\alpha$ GOS but the converses are not true in general.

**Proof:** Straight forward.

**Example 3.4:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  where  $G_1 = \langle x, (0.6, 0.5), (0.4, 0.2) \rangle$  and  $G_2 = \langle x, (0.2, 0.1), (0.8, 0.8) \rangle$ . Let  $A = \langle x, (0.6, 0.6), (0.3, 0.2) \rangle$  be any IFS in  $X$ . Then  $A \supseteq U$  where  $U = \langle x, (0.4, 0.2), (0.6, 0.5) \rangle$  is an IFRCS in  $X$ . Now  $\alpha\text{int}(A) = \langle x, (0.6, 0.5), (0.4, 0.2) \rangle \supseteq U$ .  $A$  is an IFR $\alpha$ GOS but not an IFOS in  $X$ , since  $\text{int}(A) = \langle x, (0.6, 0.5), (0.4, 0.2) \rangle \neq A$ .

**Example 3.5:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  where  $G_1 = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$  and  $G_2 = \langle x, (0.1, 0.2), (0.8, 0.8) \rangle$ . Let  $A = \langle x, (0.6, 0.7), (0.3, 0.2) \rangle$  be any IFS in  $(X, \tau)$ . Then  $A \supseteq U$  where  $U = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$  is an IFRCS in  $X$ . Now  $\alpha\text{int}(A) = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle \supseteq U$ .  $A$  is an IFR $\alpha$ GOS but not an IFROS in  $X$ , since  $\text{int}(\text{cl}(A)) = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle \neq A$ .

**Example 3.6:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  where  $G_1 = \langle x, (0.5, 0.7), (0.4, 0.2) \rangle$  and  $G_2 = \langle x, (0.2, 0.2), (0.8, 0.8) \rangle$ . Let  $A = \langle x, (0.5, 0.7), (0.3, 0.2) \rangle$  be any IFS in  $(X, \tau)$ . Then  $A \supseteq U$  where  $U = \langle x, (0.4, 0.2), (0.5, 0.7) \rangle$  is an IFRCS in  $X$ . Now  $\alpha\text{int}(A) = \langle x, (0.5, 0.7), (0.4, 0.2) \rangle \supseteq U$ .  $A$  is an IFR $\alpha$ GOS but not an IF $\alpha$ OS in  $X$ , since  $\text{int}(\text{cl}(\text{int}(A))) = \langle x, (0.5, 0.7), (0.4, 0.2) \rangle \not\subseteq A$ .

**Remark 3.7:** Every IFR $\alpha$ GOSs and every IFPOSs are independent to each other.

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*Example 3.8:* In Example 3.6, let  $A = \langle x, (0.6,0.7), (0.2,0.2) \rangle$  be any IFS in  $X$ . Then  $A \supseteq U$  where  $U = \langle x, (0.4,0.2), (0.5,0.7) \rangle$  is an IFRCS in  $X$ . Now  $\alpha \text{int}(A) = \langle x, (0.5,0.7), (0.4,0.2) \rangle \supseteq U$ .  $A$  is an IFR $\alpha$ GOS but not an IFPOS in  $X$ , since  $\text{int}(\text{cl}(A)) = \langle x, (0.5,0.7), (0.4,0.2) \rangle \not\supseteq A$ .

*Example 3.9:* Let  $X = \{a,b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  where  $G_1 = \langle x, (0.2,0.3), (0.6,0.7) \rangle$  and  $G_2 = \langle x, (0.8,0.7), (0.1,0.1) \rangle$ . Let  $A = \langle x, (0.7,0.8), (0.1,0.2) \rangle$  be any IFS in  $(X, \tau)$ . Then  $\text{int}(\text{cl}(A)) = 1\sim \supseteq A$ . Therefore  $A$  is an IFPOS in  $X$  but not an IFR $\alpha$ GOS in  $X$ , since  $A \supseteq U$  where  $U$  is an IFRCS in  $X$  but  $\alpha \text{int}(A) = \langle x, (0.2,0.3), (0.6,0.7) \rangle \not\supseteq U$ .

*Remark 3.10:* Every IFR $\alpha$ GOSs and every IFSOSs are independent to each other.

*Example 3.11:* Let  $X = \{a,b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  where  $G_1 = \langle x, (0.6,0.7), (0.4,0.2) \rangle$  and  $G_2 = \langle x, (0.1,0.2), (0.7,0.8) \rangle$ . Let  $A = \langle x, (0.8,0.7), (0.2,0.1) \rangle$  be any IFS in  $(X, \tau)$ . Then  $A \supseteq U$  where  $U = \langle x, (0.4,0.2), (0.6,0.7) \rangle$  is an IFRCS in  $X$ . Now  $\alpha \text{int}(A) = \langle x, (0.6,0.7), (0.4,0.2) \rangle \supseteq U$ .  $A$  is an IFR $\alpha$ GOS but not an IFSOS in  $X$ , since  $\text{cl}(\text{int}(A)) = \langle x, (0.7,0.8), (0.1,0.2) \rangle \not\supseteq A$ .

*Example 3.12:* Let  $X = \{a,b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  where  $G_1 = \langle x, (0.5,0.2), (0.5,0.8) \rangle$  and  $G_2 = \langle x, (0.2,0.2), (0.8,0.8) \rangle$ . Let  $A = \langle x, (0.5,0.8), (0.5,0.2) \rangle$  be any IFS in  $(X, \tau)$ . Then  $\text{cl}(\text{int}(A)) = \langle x, (0.5,0.8), (0.5,0.2) \rangle = A$ . Therefore  $A$  is an IFSOS in  $X$  but not an IFR $\alpha$ GOS in  $X$ , since  $A = U$  where  $U = \langle x, (0.5,0.8), (0.5,0.2) \rangle$  is an IFRCS in  $X$  but  $\alpha \text{int}(A) = \langle x, (0.5,0.2), (0.5,0.8) \rangle \not\supseteq U$ .

*Remark 3.13:* The intersection of two IFR $\alpha$ GOS in an IFTS  $(X, \tau)$  need not be IFR $\alpha$ GOS in general.

*Example 3.14:* Let  $X = \{a,b\}$  and let  $\tau = \{0\sim, G_1, G_2, G_3, G_4, G_5, G_6, 1\sim\}$  is an IFT on  $(X, \tau)$ . Where  $G_1 = \langle x, (0.5,0.7), (0.3,0.2) \rangle$ ,  $G_2 = \langle x, (0.4,0.2), (0.4,0.8) \rangle$ ,  $G_3 = \langle x, (0.2,0.2), (0.4,0.8) \rangle$ ,  $G_4 = \langle x, (0.4,0.2), (0.4,0.7) \rangle$ ,  $G_5 = \langle x, (0.4,0.2), (0.5,0.8) \rangle$  and  $G_6 = \langle x, (0.2,0.2), (0.5,0.8) \rangle$ . Let  $A = \langle x, (0.4,0.2), (0.5,0.2) \rangle$  and  $B = \langle x, (0.3,0.2), (0.3,0.7) \rangle$  be any two IFS in  $(X, \tau)$ . Then  $A \supseteq U$  where  $U = \langle x, (0.3,0.2), (0.5,0.7) \rangle$  is an IFRCS in  $X$ .  $\alpha \text{int}(A) = \langle x, (0.4,0.2), (0.5,0.7) \rangle \supseteq U$  and then  $B \supseteq U$  where  $U = \langle x, (0.3,0.2), (0.5,0.7) \rangle$  is an IFRCS in  $X$ .  $\alpha \text{int}(B) = \langle x, (0.3,0.2), (0.4,0.7) \rangle \supseteq U$ . Therefore  $A$  and  $B$  are IFR $\alpha$ GOS in  $X$  but  $A \cap B = \langle x, (0.3,0.2), (0.5,0.7) \rangle$  is not an IFR $\alpha$ GOS in  $X$ , since  $A \cap B \supseteq U$  but  $\alpha \text{int}(A \cap B) = \langle x, (0.2,0.2), (0.5,0.8) \rangle \not\supseteq U$ .

*Theorem 3.15:* Let  $(X, \tau)$  be an IFTS. Then for every  $A \in \text{IFR}\alpha\text{GO}(X)$  and for every  $B \in \text{IFS}(X)$ ,  $\alpha \text{int}(A) \subseteq B \subseteq A \Rightarrow B \in \text{IFR}\alpha\text{GO}(X)$ .

*Proof:* Let  $A$  be any IFR $\alpha$ GOS of  $X$  and  $B$  be any IFS of  $X$ . Let  $\alpha \text{int}(A) \subseteq B \subseteq A$ . Then  $A^c$  is an IFR $\alpha$ GCS and  $A^c \subseteq B^c \subseteq \alpha \text{cl}(A^c)$ . Then  $B^c$  is an IFR $\alpha$ GCS[6] and Therefore  $B$  is an IFR $\alpha$ GOS in  $X$ . Hence  $B \in \text{IFR}\alpha\text{GO}(X)$ .

*Theorem 3.16:* If  $A$  is an IFRCS and an IFR $\alpha$ GOS in  $(X, \tau)$ . Then  $A$  is an IF $\alpha$ OS in  $(X, \tau)$ .

*Proof:* As  $A \supseteq A$ , by the hypothesis,  $\alpha \text{int}(A) \supseteq A$ . But we have  $A \supseteq \alpha \text{int}(A)$ . This implies  $\alpha \text{int}(A) = A$ . Hence  $A$  is an IFR $\alpha$ GOS.

*Theorem 3.17:* Let  $(X, \tau)$  be an IFTS. Then for every  $A \in \text{IFS}(X)$  and for every  $B \in \text{IFRC}(X)$ ,  $B \subseteq A \subseteq \text{int}(\text{cl}(B)) \Rightarrow A \in \text{IFR}\alpha\text{GO}(X)$ .

*Proof:* Let  $B$  be an IFRCS. Then  $B = \text{cl}(\text{int}(B))$ . By hypothesis,  $A \subseteq \text{int}(\text{cl}(B)) \subseteq \text{int}(\text{cl}(\text{cl}(\text{int}(B)))) \subseteq \text{int}(\text{cl}(\text{int}(B))) \subseteq \text{int}(\text{cl}(\text{int}(A)))$ . Therefore  $A$  is an IF $\alpha$ OS and by Theorem 3.3,  $A$  is an IFR $\alpha$ GOS. Hence  $A \in \text{IFR}\alpha\text{GO}(X)$ .

### IV. APPLICATIONS OF INTUITIONISTIC FUZZY REGULAR $\alpha$ GENERALIZED CLOSED SETS

In this section we provide some applications of intuitionistic fuzzy regular  $\alpha$  generalized closed sets.

*Definition 4.1:* If every IFR $\alpha$ GCS in  $(X, \tau)$  is an IF $\alpha$ CS in  $(X, \tau)$ , then the space can be called as an intuitionistic fuzzy regular  $\alpha T_{1/2}$  space ( $\text{IF}_{\tau\alpha}T_{1/2}$  in short).

*Theorem 4.2:* An IFTS  $(X, \tau)$  is an  $\text{IF}_{\tau\alpha}T_{1/2}$  space if and only if  $\text{IF}\alpha\text{O}(X) = \text{IFR}\alpha\text{GO}(X)$ .

*Proof:* Necessity: Let  $A$  be an IFR $\alpha$ GOS in  $(X, \tau)$ , then  $A^c$  is an IFR $\alpha$ GCS in  $(X, \tau)$ . By hypothesis,  $A^c$  is an IF $\alpha$ CS in  $(X, \tau)$  and therefore  $A$  is an IF $\alpha$ OS in  $(X, \tau)$ . Hence  $\text{IF}\alpha\text{O}(X) = \text{IFR}\alpha\text{GO}(X)$ .

Sufficiency: Let  $A$  be an IFR $\alpha$ GCS in  $(X, \tau)$ . Then  $A^c$  is an IFR $\alpha$ GOS in  $(X, \tau)$ . By hypothesis,  $A^c$  is an IF $\alpha$ OS in  $(X, \tau)$  and therefore  $A$  is an IF $\alpha$ CS in  $(X, \tau)$ . Hence  $(X, \tau)$  is an  $\text{IF}_{\tau\alpha}T_{1/2}$  space.

*Theorem 4.3:* Let an  $\text{IF}_{\tau\alpha}T_{1/2}$  space be an IFTS. If  $A$  is an IFS of  $X$  then the following properties are hold:

- (i)  $A \in \text{IFR}\alpha\text{GO}(X)$
- (ii)  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$
- (iii) There exists IFOS  $G$  such that  $G \subseteq A \subseteq \text{int}(\text{cl}(G))$ .

*Proof:* (i)  $\Rightarrow$  (ii): Let  $A \in \text{IFR}\alpha\text{GO}(X)$ . This implies  $A$  is an IF $\alpha$ OS in  $X$ , since  $X$  is an  $\text{IF}_{\tau\alpha}T_{1/2}$  space. Then  $A^c$  is an IF $\alpha$ CS in  $X$ . Therefore  $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq A^c$ . This implies  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ .

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(ii)  $\Rightarrow$  (iii): Let  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ . Hence  $\text{int}(A) \subseteq A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ . Then there exists IFOS  $G$  in  $X$  such that  $G \subseteq A \subseteq \text{int}(\text{cl}(G))$  where  $G = \text{int}(A)$ .

(iii)  $\Rightarrow$  (i): Suppose that there exists IFOS  $G$  such that  $G \subseteq A \subseteq \text{int}(\text{cl}(G))$ . It is clear that  $(\text{int}(\text{cl}(G)))^c \subseteq A^c$ . That is  $(\text{int}(\text{cl}(\text{int}(A))))^c \subseteq A^c$ . This implies  $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq A^c$ . That is  $A^c$  is an IF $\alpha$ CS in  $X$ . This implies  $A$  is an IF $\alpha$ OS in  $X$ . Hence  $A \in \text{IFR}\alpha\text{GO}(X)$ .

**Definition 4.4:** An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy regular  $\alpha$   $T_{1/2}^*$  space (IF $_{\alpha}T_{1/2}^*$  space in short) if every IF $\alpha$ GCS is an IFCS in  $(X, \tau)$ .

**Remark 4.5:** Every IF $_{\alpha}T_{1/2}^*$  space is an IF $_{\alpha}T_{1/2}$  space but not conversely.

**Proof:** Let  $(X, \tau)$  be an IF $_{\alpha}T_{1/2}^*$  space. Let  $A$  be an IF $\alpha$ GCS in  $(X, \tau)$ . By hypothesis,  $A$  is an IFCS. Since every IFCS is an IF $\alpha$ CS,  $A$  is an IF $\alpha$ CS in  $(X, \tau)$ . Hence  $(X, \tau)$  is an IF $_{\alpha}T_{1/2}$  space.

**Example 4.6:** Let  $X = \{a, b\}$  and let  $\tau = \{0, G_1, G_2, G_3, 1\}$  where  $G_1 = \langle x, (0.5, 0.5), (0.3, 0.1) \rangle$ ,  $G_2 = \langle x, (0.1, 0.1), (0.7, 0.7) \rangle$  and  $G_3 = \langle x, (0.5, 0.5), (0.4, 0.2) \rangle$ . Let  $A = \langle x, (0.4, 0.1), (0.5, 0.5) \rangle$  be any IFS in  $(X, \tau)$ . Then  $A \subseteq U$  where  $U = \langle x, (0.5, 0.5), (0.3, 0.1) \rangle$  is an IFROS in  $X$ . Now  $\alpha\text{cl}(A) = \langle x, (0.4, 0.1), (0.5, 0.5) \rangle \subseteq U$ . Therefore  $A$  is an IF $\alpha$ GCS in  $X$  but not an IFCS in  $X$ , since  $\text{cl}(A) = \langle x, (0.4, 0.2), (0.5, 0.5) \rangle \neq A$ .

**Theorem 4.7:** For any IFS  $A$  in  $(X, \tau)$  where  $X$  is an IF $_{\alpha}T_{1/2}^*$  space,  $A \in \text{IFR}\alpha\text{GO}(X)$  if and only if every IFP  $p_{(\alpha, \beta)} \in A$ , there exist an IF $\alpha$ GOS  $B$  in  $X$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ .

**Proof:** Necessity: If  $A \in \text{IFR}\alpha\text{GO}(X)$ , then we can take  $B = A$  so that  $p_{(\alpha, \beta)} \in B \subseteq A$ . For every IFP  $p_{(\alpha, \beta)} \in A$ .

Sufficiency: Let  $A$  be an IFS in  $(X, \tau)$  and assume that there exist  $B \in \text{IFR}\alpha\text{GO}(X)$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ . Since  $X$  is an IF $_{\alpha}T_{1/2}^*$  space,  $B$  is an IFOS. Then  $A = \bigcup_{p_{(\alpha, \beta)} \in A} \{p_{(\alpha, \beta)}\} \subseteq \bigcup_{p_{(\alpha, \beta)} \in A} B \subseteq A$ . Therefore  $A = \bigcup_{p_{(\alpha, \beta)} \in A} B$ , which is an IFOS in  $X$ . Hence by Theorem 3.3,  $A$  is an IF $\alpha$ GOS in  $X$ .

**Definition 4.8:** An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy regular  $\alpha$  generalized  $T_{1/2}$  (IF $_{\alpha}T_{1/2}$  in short) space if every IF $\alpha$ GCS in  $X$  is an IF $\alpha$ GCS in  $X$ .

**Theorem 4.9:** If an IFTS  $(X, \tau)$  is an IF $_{\alpha}T_{1/2}$  space, then every IF $\alpha$ GOS is an IF $\alpha$ GOS.

**Proof:** Let  $A$  be an IF $\alpha$ GOS in  $X$ . This implies  $A^c$  is an IF $\alpha$ GCS in  $X$ . Since  $X$  is an IF $_{\alpha}T_{1/2}$  space,  $A^c$  is an IF $\alpha$ GCS in  $X$ . Hence  $A$  is an IF $\alpha$ GOS in  $X$ .

**Theorem 4.10:** Let an IF $_{\alpha}T_{1/2}$  space be an IFTS. If  $A$  is an IFS of  $X$  then the following properties are hold:

- (i)  $A \in \text{IFR}\alpha\text{GO}(X)$
- (ii)  $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $U \subseteq A$  and  $U$  is an IFCS in  $X$
- (iii) There exists IFOSs  $G$  and  $G_1$  such that  $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$ .

**Proof:** (i)  $\Rightarrow$  (ii): Let  $A \in \text{IFR}\alpha\text{GO}(X)$ . This implies  $A$  is an IF $\alpha$ GOS in  $X$ , since  $X$  is an IF $_{\alpha}T_{1/2}$  space. Then  $A^c$  is an IF $\alpha$ GCS in  $X$ . Therefore  $\alpha\text{cl}(A^c) \subseteq V$  whenever  $A^c \subseteq V$  and  $V$  is an IFOS in  $X$ . That is  $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq V$ . This implies  $V^c \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $V^c \subseteq A$  and  $V^c$  is IFCS in  $X$ . Replacing  $V^c$  by  $U$ ,  $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $U \subseteq A$  and  $U$  is an IFCS in  $X$ .

(ii)  $\Rightarrow$  (iii): Let  $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$  whenever  $U \subseteq A$  and  $U$  is an IFCS in  $X$ . Hence  $\text{int}(U) \subseteq U \subseteq \text{int}(\text{cl}(\text{int}(A)))$ . Then there exists IFOSs  $G$  and  $G_1$  in  $X$  such that  $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$  where  $G = \text{int}(A)$  and  $G_1 = \text{int}(U)$ .

(iii)  $\Rightarrow$  (i): Suppose that there exists IFOSs  $G$  and  $G_1$  such that  $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$ . It is clear that  $(\text{int}(\text{cl}(G)))^c \subseteq U^c$ . That is  $(\text{int}(\text{cl}(\text{int}(A))))^c \subseteq U^c$ . This implies  $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq U^c$ ,  $A^c \subseteq U^c$  and  $U^c$  is an IFOS in  $X$ . This implies  $\alpha\text{cl}(A^c) \subseteq U^c$ . That is  $A^c$  is an IF $\alpha$ GCS in  $X$ . This implies  $A$  is an IF $\alpha$ GOS in  $X$ . Hence  $A \in \text{IFR}\alpha\text{GO}(X)$ .

## V. CONCLUSION

Thus we have analyzed relationship between intuitionistic fuzzy regular  $\alpha$  generalized open sets and the already existing intuitionistic fuzzy open sets and obtain many interesting theorem using the new spaces introduced above.

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