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Restricting R&D Cooperative Networks for Maximum Outcomes

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Abstract: Firms always seek to create positional advantages in a network of research and development (R&D). In real R&D cooperation, it is impossible to describe the network as a fully connected network. However, it can exhibit characteristics of complex networks. In this paper, we extend the theoretical outcomes by Goyal and Moraga-Gonzalez through discussing the structural properties on the outcomes. Firstly, in terms of the con- flict between individual and social advantages of the R&D network, how restricting R&D interactions between firms allows convergence of these advantages? Secondly, In regard to the complex interactions between firms, what R&D systems that can preserve and improve the individual and social benefit?

Keywords
Network game, R&D cooperation, Network Connectivity, Maximum outcomes.

I. INTRODUCTION

The involvement of the network concept in R&D cooperation between firms has effective con- tributions in economics. By using the networks, we are able to expect the better structures to generate maximum outcomes. We are also able to determine changes in the outcomes generated by expanding the R&D organization [1-3]. Moreover, the advantages of the networks appear through interdependence of the outcomes when considering all agents. For example, the R&D interactions by some individuals have effects on the incentives of other existing individuals in direct or indirect ways. Also, performance evaluation of the individuals is affected by their structural locations and by the architecture of the R&D cooperation [1,3,4].

On the other hand, the involvement of the networks opens the insight for a number of important questions such as knowledge diffusion, strategies of investment and competition. Given the network framework, it is essential to expect the upcoming structure that ensures maximum individual and social outcomes. The emphasis of many papers in studying R&D interactions was on some types of R&D networks such as star and complete networks. [1]This included massive changes in the outcomes resulting in creating cooperative links [1-5]. In other papers, the study focused on the main structure of the networks that carry characteristics of the complexity. For example, a small world network has taken an attention of many authors, especially in the knowledge diffusion between firms [6-12].

In this paper, the strategic network formation of the R&D interactions is based on the network game by Goyal and Moraga-Gonzalez [1]. In the game, each two firms have a choice to form a link between each other to share their outcomes of R&D. If the two firms are not cooperating, they are not linked the network, but there is an R&D spillover between them to ensure a partial benefit from investments of non-cooperating firms in R&D.

[1] A star network is characterized by a node in the center of the network (hub) linked to all other nodes (peripheral nodes) while none of the peripheral nodes has a link with any other. A complete network is a graph such that each two nodes in that network are linked.

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The paper contributes to the theory of the R&D network formation. The objective is to extend the outcomes of Goyal and Moraga-Gonzalez in two matters [1]. In the first matter, the focus is on the conflict between the individual and social perspectives in terms of profitable network structures. The authors stated that the two perspectives are not always consistent, especially when the rate of the substitution between the products is high as the case in the homogeneous products. According to their outcomes, the conflict between the individual and social benefit is not small, particularly when the R&D spillover increases. They stated that the individual profit increases with growing their own links and this allows the complete network to be always individually profitable. Whereas, the profitable structure in the social perspective is characterized by low R&D interactions. Thus, since the possible R&D relationships, increase with growing the market size, the discrepancy between the individual and social benefits of the R&D cooperation network will be massive when considering a large market size[2].

In this paper, we reduce the conflict between the individual and social benefits by restricting the R&D relationships between firms. One way to restrict the possible relationships is to consider bipartite networks. In such networks, firms are divided into two groups such that firms belonged to one group cannot cooperate. This restriction for any number of firms generates a small number of the R&D interactions compared to the case when all relationships are considered. The outcomes show that restricting number of the R&D relationships reduces the conflict between the individual and social benefits. When comparing the outcomes with that generated by involving all possible relationships, we found that restricting the networks allows convergence of the individual and social perspectives for the R&D cooperation, regardless of the R&D spillover.

In the second matter, we focus on R&D structures that exhibit features of the complex networks. We study two types of the small world networks, connected and non-connected networks. In the connected small world networks, we assume that the cooperation forms one connected network Watts and Strogatz, whereas in the non-connected small world networks, we assume that the cooperation forms a network consists of a large group of connected firms and small groups of connected firms [13]. The outcomes suggest that in the most cases, the expenditures of firms in R&D, and the individual and social benefits are maximized when firms form a small world network with low interactions.

The theoretical outcomes in this paper consist with the empirical findings in the real R&D networks. Firstly, forming R&D cooperative links is profitable for firms, but belonging to a fully connected network with few interactions is not individually preferable. This is observed in this paper and the paper by Goyal and Moraga-Gonzalez where the profit is affected negatively by increasing the R&D agreements of other firms [1]. The empirical outcomes have shown that the overall cooperative networks are irregular (firms have different number of links), in addition to that the number of links is very smaller than the number of cooperators [14-17]. The authors concluded that developing the R&D networks are based on existing highly connected firms. These results indicate that many firms possess a small number of links.

Secondly, forming a component consisted of a large number of R&D relationships is individually preferable. Our findings show that the profits of firms become better when the resultant network contains a giant component consisted of most of the R&D relationships. In the empirical R&D literature, the network of the cooperation in R&D cannot be described as a fully connected network, but it consists of a number of components where each component is formed by multilateral agreements. In addition to this, the giant component exists and contains most of the R&D cooperation [18,19]. The authors found that growing the giant component allows the network to exhibit the characteristics of the small world network.

The paper is structured as follows. In the second section, we provide foundations in the social network and economics, and then we introduce the R&D network model by Goyal and Moraga-Gonzalez. In the third section, we present our outcomes. In the fourth section, we conclude our study.

II. BACKGROUND

[2] The market size means the number of firms.

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Network

A network is a combination of sets of nodes $N=\{i,j,k,\dots\}$ and $E=\{ij, jk,\dots\}$ connect those nodes. $G(N, E)$ denotes a network and for simplicity, we write G. If each link between any two nodes runs in both directions, G is called an undirected network. Also, if there is no parallel links (links that have the same end nodes) nor loops (links where their start and end nodes are the same), it is called a simple network [20,21].
A set of nodes that are linked to node $i \in N$ is defined as neighbors of that node $N_i = \{j \in N: ij \in E\}$. The length of the neighbors set of node i is a degree of that node: $deg(i) = |N_i|$. If all nodes have the same degree, the network is called regular; otherwise it is called irregular. If $N' \subseteq N$ and $E' \subseteq E$, then $G'(N', E')$ is a subgraph of the network $G(N, E)$. A component of the graph G is defined as a connected subgraphs and the largest connected component is called a giant component.
If n and m are numbers of nodes and links in the network G, the average degree is $AV = 2m/n$ and the density is $D = 2m/(n(n-1))$. The clustering coefficient for each node $i \in N$ measures the proportion of neighbors of i that link to each other:

$$Ci = \frac{2|jk: j,k \in N_i, jk \in E|}{\deg(i)(\deg(i)-1)} \quad (1)$$

The average clustering coefficient for network G is the total of clustering coefficients for all nodes in the network divided by the number of nodes: $C(G) = \frac{\sum_{i=1}^{n} C_i}{n}$.

A path between any two firms is a sequence of links between those firms. Therefore, two firms are connected if there is a path between them; otherwise they are disconnected. Generally, a network is connected when there is a path between every pair of nodes. The length of the path is the number of links constructed that path. The length of the shortest path between two firms i and j is the distance between them (d_{ij}). The average distance in the network G can be calculated by the following formula:

$$AD(G) = \frac{\sum_{i \neq j} d_{ij}}{n(n-1)} \quad (2)$$

If two nodes are disconnected, the distance is set infinity and this means the average path length of the entire network is only well defined if the network is connected.

The Model

Consider the linear-quadratic function of consumers given by the following equation:

$$U = a\sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\lambda \sum_{j \neq i} q_i q_j\right) + I \quad (3)$$

Here the demand parameters $a > 0$ denotes the willingness of consumers to pay and q_i is the quantity consumed of product or good i and I measures the consumer's consumption of all other products Hackner [22]. The parameter $\lambda \in [-1, 1]$ captures the marginal rate of differentiation between products. If $\lambda < 1$, goods are differentiated (complements or substitutes) and if $\lambda = 1$ ($\lambda = 0$), goods are homogeneous (independent).

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The inverse demand function for each good i is

$$pi = a - qi - \lambda \sum_{i=1}^{n} qj,\ i = 1, 2, 3, \ldots, n \qquad (4)$$

The profit π_i for each firm i is
$$\pi i = (pi - ci)qi, \qquad (5)$$

where p_i is the price of good i produced by firm i and c_i is the production cost. Total welfare is the total surplus of consumers and producers.

$$TW = \frac{1-\lambda}{2}\sum_{i=1}^{n} q_i^2 + \frac{\lambda}{2}\left(\sum_{i=1}^{n} q_i\right)^2 \quad (6)$$

Consumer surplus industry profit

R&D Network Model

If we describe the R&D cooperation between firms as a network, then firms are represented by nodes and the R&D relationships are represented by links. Based on the R&D network model by Goyal and Moraga-Gonzalez, the formation of a link requires the cooperation of both firms where they have a choice to form a link between them or to delete it [1]. If any two firms cooperate, a link is established between them such that the cost of the link formation is assumed to be negligible. However, if firms do not cooperate, they are not linked and there is an R&D spillover $\beta \in [0, 1)$ between them. The spillover is set to ensure partial benefits between non- cooperated firms.

Stages of the model:

In Goyal and Moraga-Gonzalez, the strategic formation of the cooperative links is modeled as a three-stage game [1].

The stage of network formation: Each firm chooses its research partners where firms and the cooperative links together constitute a network of cooperation in R&D.

The stage of R&D expenditure: Given the R&D network, each firm chooses the amounts of investment in R&D simultaneously and independently in order to reduce the cost of production.

The stage of the competition: Given the R&D investments of each firm and the effective R&D investment (as determined by the R&D network), firms compete in the product market by setting quantities (Cournot competition) in order to maximize their profits.

R&D cooperation network:

1. Bipartite network: A bipartite graph is a graph whose nodes is divided into two disjoint sets $V1$ and $V2$ where nodes that belong to the same set cannot be linked. In the R&D bipartite network, there are two groups of firms such that firms in one group cannot cooperate (Fig. 1).

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Fig. 1. Examples of bipartite networks.

2. Small world network: A small world network is a graph in which most nodes are not linked to one another, but the nodes can be reached from every other node by only a small number of links Watts and Strogatz [13]. This definition is appropriate for connecting networks. Goyal et al. provided an alternative approach for non-connected networks using the giant component [23]. They defined a non-connected network as a small world network if it satisfies the following conditions: (1) The network size is large compared to the average degree i.e., n » $2m/n$, (2) The clustering coefficient is high compared to the clustering coefficient in the random network with the same number of nodes $C(G)$ » $2m/n^2$, (3) The giant component exists and covers a large fraction of the network, (4) The average distance in the giant component is small (of order $\ln(n)$).

By assuming that the R&D interactions satisfy the conditions of the small world network, if the interactions compose one connected component, then we have a connected small world network; otherwise we have a non-connected small world network (Fig. 2).

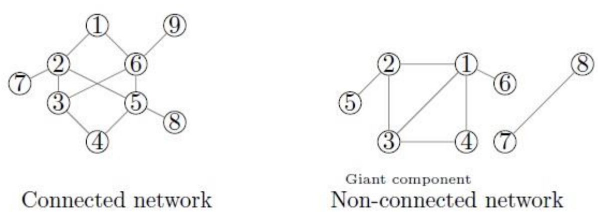

Fig. 2. An example of connected and non-connected small world networks.

Cost reduction:

In Goyal and Moraga-Gonzalez, the effective R&D investment for each firm is defined by the following equation [1]:

$$X_i = x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k,\ i = 1, \ldots\ldots, n \quad (7)$$

Where x_i denotes R&D investment of firm i, N_i is the set of firms participating in R&D with firm i and $\beta \in [0,1)$ is an exogenous parameter that captures knowledge spillovers acquired from firms not engaged in R&D with firm i. The total investment of n firms in a network is

i=1 x_i.

The effective R&D investment reduces firm i's marginal cost (c) of production −

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$$c_i = \bar{c} - x_i - \sum_{j \in N_i} x_j - \beta \sum_{k \notin N_i} x_k, \ i = 1, \ldots, n \quad (8)$$

The profit π_i of firm i is given by the following equation:

$$\pi_i = \left(a - q_i - \lambda \sum_{j \neq i}^{n} q_j - \bar{c} + x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k \right) q_i - \gamma x_i^2 \ \ i = 1, \ldots, n \quad (9)$$

where the marginal cost satisfies $a > \bar{c}$. The expression γx_i^2 is the cost of the R&D investment where $\gamma > 0$ indicates the effectiveness of R&D expenditure D'Aspremont and Jacquemin [24].

Effectiveness γ:

To have suitable values of the effectiveness, the investment and cost functions should be non-negative and the second order condition for maximizing profit function (i.e. $\frac{\partial^2 \pi}{\partial x^2} < 0$)should be satisfied. From Goyal and Moraga-Gonzalez , we have the following [1]:

1. For independent goods, $\gamma > max\{n/4, an/4\bar{c}\}$ (10)
2. For homogeneous goods, $\gamma > max\{n^2/(n+1)^2, a/4\bar{c}\}$ (11)

Stability and efficiency of R&D networks:

Based on the profit of firms, the pairwise stability is defined as follows [25].

Definition 1: *A network G is stable if for any any two firms i, j in G the following two conditions are satisfied:*

If $\boldsymbol{ij} \in G, \pi i(G) \geq \pi i(G - \boldsymbol{ij})$ *and* $\pi j(G) \geq \pi j(G - \boldsymbol{ij})$,
If $\boldsymbol{ij} \notin G$ *and if* $\pi i(G) < \pi i(G + \boldsymbol{ij})$, *then* $\pi j(G) > \pi j(G + \boldsymbol{ij})$,

$G - \boldsymbol{ij}$ is the network resulting from deleting a link $\boldsymbol{ij}$ from the network G and $G + \boldsymbol{ij}$ is the network resulting from adding a link $\boldsymbol{ij}$ to the network G.

Based on the total welfare, the efficiency of a network is defined as follows:

Definition 2: A network G is efficient if no other network Gadding or can be generated from G by deleting links, such that TW(Ǵ) > TW(G).

Nash Equilibria

Assume that the marginal cost function is constant and equal for all firms. Under Cournot competition, we identify the sub-game perfect Nash equilibrium by using backwards induction.

Consider an industry consisted of n firms, where each firm choosing an amount of output to produce. If the firm i's output

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level is denoted as q_i, then to find the equilibrium for the production quality of that firm, we solve $\frac{\partial \pi_i}{\partial q_i} = 0$ This yields the best response function of quality of good i:

$$q_i = \frac{a - c_i - \lambda \sum_{j \neq i} q_j}{2} \quad (12)$$

Substituting the best response functions (equation 12 for each i) into each other yields the symmetric equilibrium that is the Nash equilibrium for the production quantity:

$$q_i^* = \frac{(2-\lambda)a - (2+(n-2)\lambda)c_i - \lambda \sum_{j \neq i} c_j}{(2-\lambda)((n-1)\lambda + 2)} \quad (13)$$

To find the equilibrium profit, we substitute the equilibrium output (13) into the profit function which gives

$$\pi_i^* = \left[\frac{(2-\lambda)a - (2+(n-2)\lambda)c_i - \lambda \sum_{j \neq i} c_j}{(2-\lambda)((n-1)\lambda + 2)} \right]^2 - \gamma x_i^2 \quad (14)$$

Calculating the equilibrium investment x_i depends on the structure of the R&D network. By knowing the structure, we find the cost function c_i to substitute it into the profit function (14). Then, we calculate the best response function of R&D investment for each firm $i(\frac{\partial \pi_i}{\partial q_i} = 0)$. By plugging the best response functions into each other, we have the symmetric equilibrium for the R&D investment. The final list of the equilibria in the case of regular networks is given in the Appendix [1].

III. THE OUTCOMES

Bipartite Networks

The purpose of considering the bipartite networks is to limit the possible R&D interactions among firms. This contributes to examine the impact of the density of the network in reducing the conflict between the stability and efficiency of the R&D networks. We consider two cases, regular and irregularnetworks.

Regular bipartite networks:

For a k-regular bipartite network with $k > 0$, we have two sets V_1 and V_2 where the number of firms and the sum of degrees in the two sets should be equal [26].This indicates that the number of firms n in k-regular bipartite should be even. For regular networks, Goyal and Moraga-Gonzalez stated that the maximum total welfare with respect to the activity level depends on the market structure [1]. If firms are in a dependent product market, the total welfare is maximized when firms form a complete network (i.e., $k = n - 1$). However, in a homogeneous product market, the total welfare is maximized at an

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intermediate activity level. In addition, the authors found that in both markets, the complete network is stable. This points out that the difference between the individual and social benefits of R&D networks occurs in a homogeneous product market.

When considering bipartite networks with arbitrary n firms, the maximum value of the density is $D = 1/(n-1)$. For this density, the cooperative activity level in the network is $k = n/2$. where n is even. Fig. 3 shows the possible regular bipartite networks for six firms.

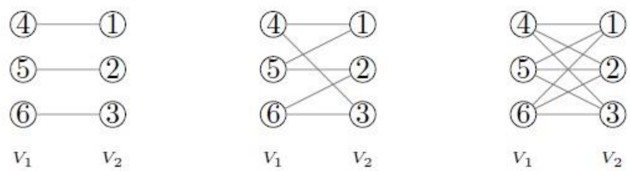

Fig. 3. The possible regular bipartite networks for six firms.

At the activity level $k^* = n/2$, the regular bipartite network is stable since the profits of firms increase with their new links. The following proposition states that for independent and homogeneous products, the bipartite network is efficient at the activity level k^*. This indicates that in a homogeneous product market, when R&D interactions form regular bipartite networks, then there is not a conflict between the individual and social benefits.

Proposition 1 *Suppose n firms form a regular bipartite R&D network with* $\beta = 0$, *the total welfare is maximized at the activity level* $k^* = n/2$.
The proof is given in the Appendix.

In addition, the maximum value of the industry profit in the regular bipartite networks is realized when the activity level between firms is $k^* = n/2$. This differs from the case when considering all possible networks generated by an arbitrary number of firms n. Goyal and Moraga-Gonzalez found that the industry profit and total welfare are maximized at different activity levels [1]. Fig. 4 shows the maximum values of the industry profit and the total welfare with respect to the activity level for different market sizes.

Irregular bipartite networks:

For irregular networks, we consider an example of four firms in a market. With this size, there are 11 possible distinct networks given in Figs. 5-7. However, when considering only the bipartite networks, the possible networks decrease to five as given in Fig. 5.

Fig. 6 shows the individual profit and the total welfare for independent and homogeneous products. It can be seen that the optimal structures of the R&D cooperation under the individual and social perspectives are consistent if the products are independent; meaning that the network G_5 is stable and efficient. This indicates that the conflict between the stability and efficiency does not appear in an independent product market. However, in a homogeneous product market, considering low density networks does not prevent the occurrence of the conflict between the stability and efficiency. This result differs from our finding in regular bipartite networks where there is not a conflict between the stability and efficiency of the R&D interactions. Moreover, the density of the efficient network decreases with increasing the R&D spillover. This indicates that

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the spillover has a role in raising the conflict between the stability and efficiency.

Proposition 2 *Suppose n firms form an irregular bipartite R&D network, the total welfare is maximized at the activity level* $k^* = n/2$ *if* $\beta \in [0, \beta_1]$.

Nevertheless, considering the bipartite networks for homogeneous products reduces the gap between the stable and efficient networks. To see this, we present the equilibrium total welfare for all distinct relationships generated with four firms. Fig. 7 shows the possible networks with four networks and Fig. 8 displays the total welfare generated from those networks for independent and homogeneous goods. Since the profit increases with the cooperative links, the complete network G_1 is stable. Whereas, the efficient network characterized by a low density that reaches to zero as the spillover approaches the highest value [1].

When comparing the stability and efficiency in the two sets of networks (given in Fig's 5 and 7), we have the following. First, the complete bipartite network G_5 is stable when we limit the interactions among firms, but this network when considering all possible relationships (i.e., G_9) is far from being stable. Moreover, when focusing on the bipartite networks, there is a small possibility that the stable network is efficient; whereas the stable network G_1 is never efficient.

Fig. 9 illustrates the density of the stable and efficient for the networks given in Fig's. 5 and 7. From the figure, we can observe that the gap between the density of the stable and efficient networks is small when considering the bipartite networks. However, when considering all different networks generated by four firms, the gap between the density is large, especially when the spillover increases.

Small World Networks

In this section, we investigate the impact of the small world properties on the equilibrium outcomes. We consider two cases of the networks. In the first case, we assume that the cooperation of firms in R&D forms a connected small world network. Then, we examine the impact of the network density on the R&D investment and profit of firms and on the social welfare. In the second case, we assume that the cooperation in R&D forms a non-connected small world network such that most firms is located in the giant component. We study the impact of increasing the R&D interactions of other components on the equilibrium outcomes.

To study the outcomes in a small world network, we need an appropriate number of firms. Therefore, we assume that there are nine firms in a market. We also assume that firms produce homogeneous products since the outcomes increases with the connectivity [1]. With this number, it is not simple to consider all possible R&D relationships[3]. However, we focus on some of those that exhibit the properties of the small world network.

Connected small world networks:

Assume nine firms cooperate in R&D such that the cooperation forms a connected small world network G_1 as shown in Fig. 10. Assume that the density of the network increases by linking some firms where the resulting network is G_2. Table 1 shows the density and the average clustering coefficient in the small world networks G_1 and G_2.
Fig. 11 shows the outcomes in the two networks G_1 and G_2. Firstly, since firms sell homo-possible networks.

[3] With n firms, there a possible networks

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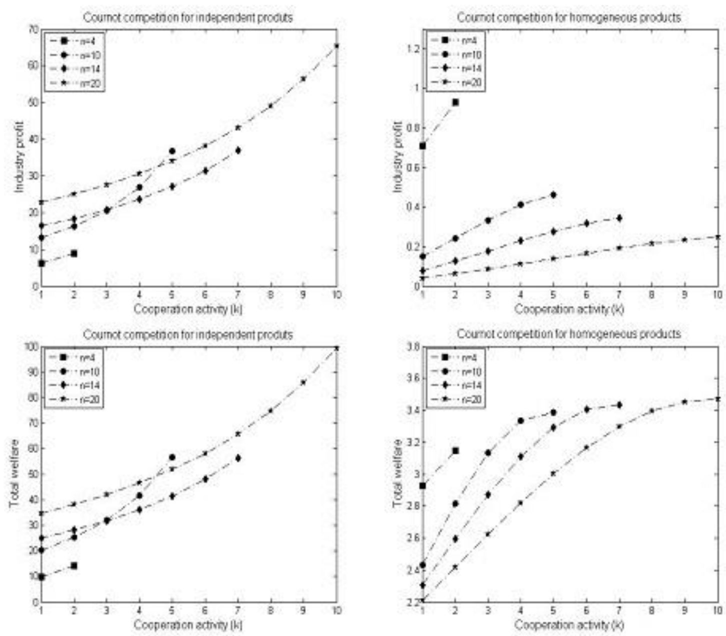

Fig. 4: The industry profit and the total welfare in the regular bipartite networks for $n = 4$, 10, 14 and 20**.**

From the graph, the industry profit and the total welfare are maximized at $k^* = n/2$. The parameters used to plot the results are $a = 12$, $\bar{c} = 10$ and $\gamma = 1$ for homogeneous products. For independent products the effectiveness $\gamma = 2, 3, 5,$ *text and* 6 with respect to the size of the market.

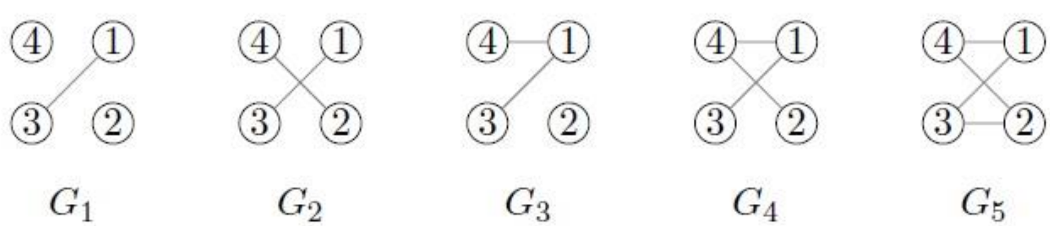

Fig. 5. The possible irregular bipartite networks with four firms.

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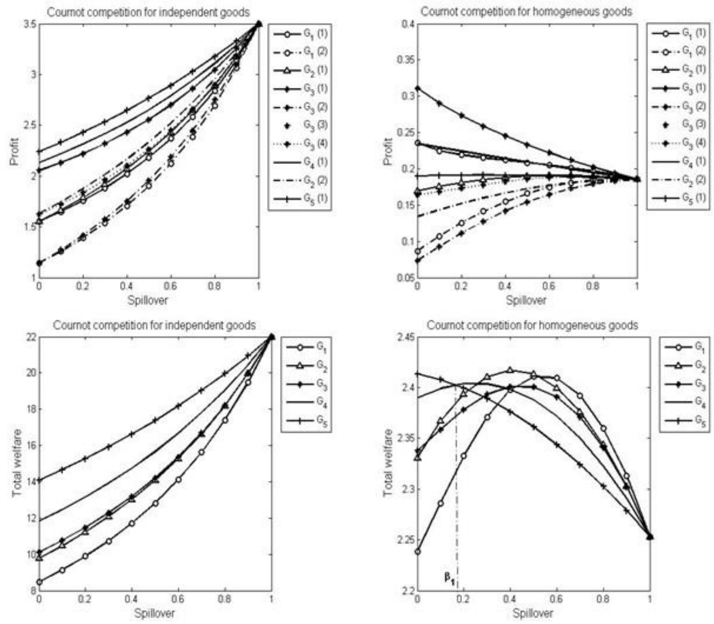

Fig. 6. The profit and the total welfare for the bipartite networks in Fig. 5. The graphs on the left side show the outcomes for independent goods and the graphs on the right side show the outcomes for homogeneous goods. The parameters used to plot the results are $a = 12$, $\bar{c} = 10$ and $\gamma = 2$.

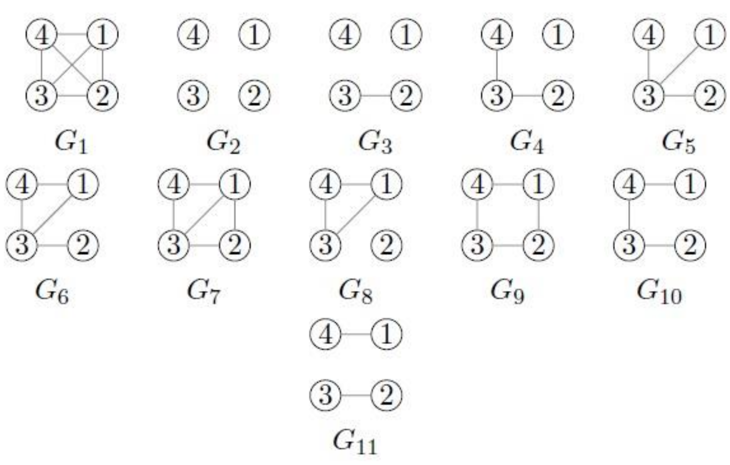

Fig. 7. The distinct networks with size four firms.

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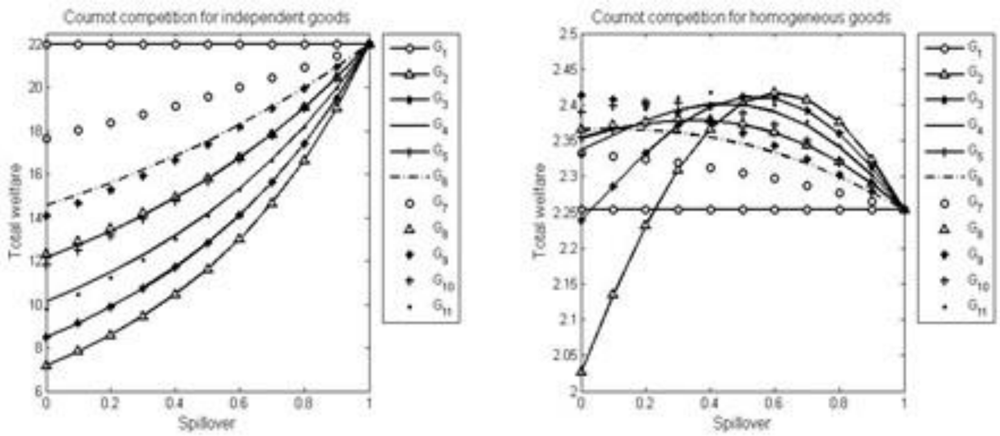

Fig. 8. The total welfare for the networks in Fig. 7. The graph on the left side is for independent goods and the graph on the right side is for homogeneous goods. The parameters used to plot the results are $a = 12$, $\bar{c} = 10$ and $\gamma = 2$.

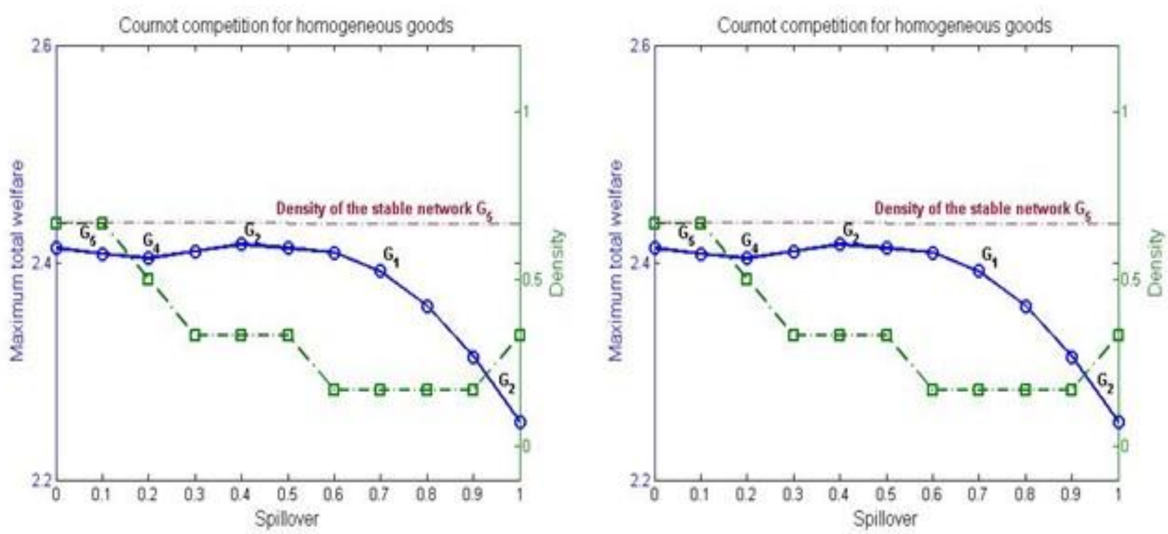

Fig. 9. The comparison of the density of the stable and efficient networks for the networks given in 5 and 7. The parameters used to plot the results are $a = 12$, $\bar{c} = 10$ and $\gamma = 2$.

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Table 1. The density and clustering coefficient of the networks G_1 and G_2.

Network	G_1	G_2
D	0.3056	0.4444
C	0.2222	0.7037

geneous goods, the impact of the cooperative links on the R&D investment is negative [1]. Meaning that, the expenditure of firms in R&D is low in a dense small world network. Secondly, by taking into account the direct and indirect impact of the link formation on the profits, the importance of the links appears for firms who create new links; whereas the others obtain lower profits. As a consequence, the profit of firms 4 and 5 in the network G_1 is high, regardless of the value of the R&D spillover. However, this statement does not continue when the density of the network increases i.e., in the network G_2. The difference in the declining rate of the profits of firms 4 and 5 in the two networks is due to that firm 4 in the network G_2 creates a new link with firm 7; whereas firm 5 remains with the same number of links. Finally, when comparing the industry profit and the total welfare in the two networks, it is found that the impact of increasing the density depends on the value of the spillover. If it is small, the increase of the density improves the social benefits and vice versa.

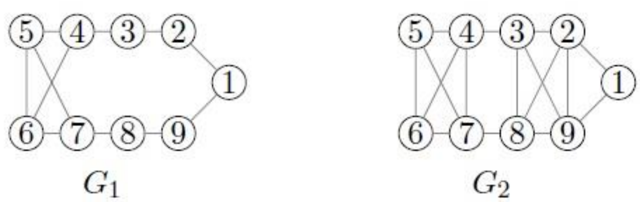

Fig. 10. Connected small world networks G_1 and G_2. In terms of the outcome, the networks have five different groups of firms: the first group contains firm 1, the second group contains firms 2 and 9, the third group contains firms 3 and 8, the fourth group contains firms 4 and 7, and the fifth group contains firms 5 and 6.

Non-connected small world networks:

In this part, we assume nine firms cooperate in R&D and the interactions form a non-connected network G_3 given in Fig. 12. In this network, six firms establish a giant component and the other three firms stay isolated. The network G_3 satisfies the conditions of the small world network given in Goyal et al. [23]. Assume the density of the network increases by linking some isolated firms and the linkage process generates networks G_4 and G_5. Table 2 shows the density, the average clustering coefficient and the average distance in the small world networks G_3, G_4 and G_5.

Fig. 13 compares the equilibrium outcomes in the three networks. Firstly, when comparing the R&D investments, firms in the low dense network G_3 spend high amounts on R&D. Secondly, firms pursue to build many R&D relationships to obtain higher profits and this may drive them to be part of the giant component. In the network G_3, the profit of firms in the giant component is high and this is because there is no external influence on the profits. Meaning that when firms outside the giant component cooperate, the new links reduce the profits of firms inside it, but this is not realized in the network G_3. Finally, the industry profit in the non-connected small world network with a low density is high, regardless of the value of the R&D spillover. This is not true for the total welfare, where the comparison is affected by the spillover. If it is small or large, the low dense small world network G_3 generates the maximum total welfare.

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Table 2. The density and clustering coefficient of the networks G_3, G_4 and G_5.

Network	G_3	G_4	G_5
D	0.25	0.2778	0.3333
C	0.2222	0.2222	0.5556
AD	1.4	1.4	1.4

Connected and non-connected small world networks:

In this part, we compare between the outcomes of the connected and non-connected small world networks given in Figs. 10 and 12. To simplify the discussion, we compare between them in terms of the total outcomes, total R&D investments, industry profit and total welfare.

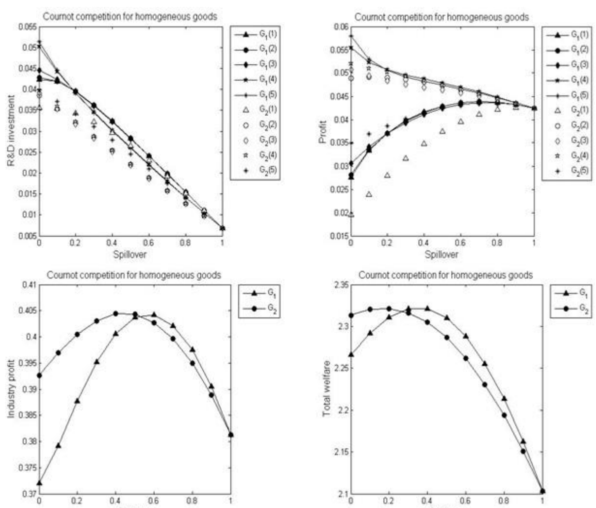

Fig. 11. The equilibrium outcomes in the small world networks G_1 and G_2. The figure shows the R&D investment, profit, the industry profit and the total welfare in the networks G_1 and G_2. The parameters used to plot the figures are $a = 12$, $\bar{c} = 10$ and $\gamma = 3$.

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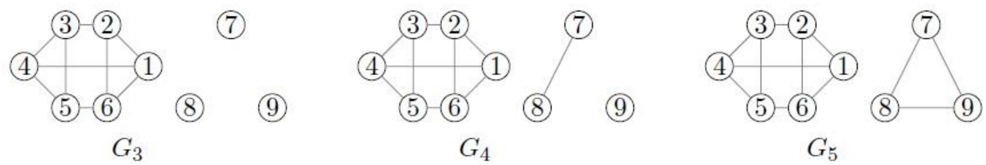

Fig. 12. Non-connected small world networks G_3, G_4 and G_5. Regarding to the outcomes, firms belong to different groups. In the networks G_3 and G_5, there are two different groups of firms: the first group contains firms from 1 to 6 and the second group contains firms 7, 8 and 9. In the network G_4, there are three groups of firms: the first group contains firms from 1 to 6, the second group contains firms 7 and 8, and the third group contains firm 9.

As shown in Fig. 14, the total R&D investments of firms in the network G_3 is the highest if the spillover is not small. This indicates that the expenditure in R&D by firms is maximized when they form a small world network with low density. When comparing the industryprofit and the total welfare, it is found that the R&D spillover plays a role in determining the profitable organization. If the spillover is moderate, the industry profit and the total welfare are maximized when firms form a dense small world network. However, if the spillover is small or high, they are maximized when firms form a low dense network G_3. These results in the total outcomes indicate that a small world network that has low interactions between firms encourages the investment in R&D and improves the social benefits. This result is consistent with the outcomes of Goyal and Moraga-Gonzalez when considering the regular networks, the industry profit and total welfare are maximized at intermediate levels of the cooperation in R&D [1].

Our results in this paper are in line with the findings in the empirical studies [13-18]. The authors have shown that the R&D cooperation network exhibits the properties of the small world networks. Also, they found that developing this type of networks depends on existing highly connected cores. This indicates that a large number of cooperating firms possessed one link which in turn had a role in declining the density of the network.

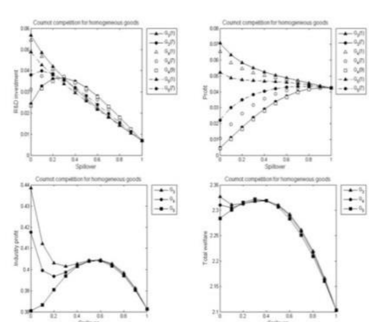

Fig. 13. The equilibrium outcomes in the non-connected small world networks G_3, G_4 and G_5. The figure shows the R&D investment, profit, the industry profit and the total welfare in the networks G_3, G_4 and G_5. The parameters used to plot the figures are $a = 12$, $\bar{c} = 10$ and $\gamma = 3$.

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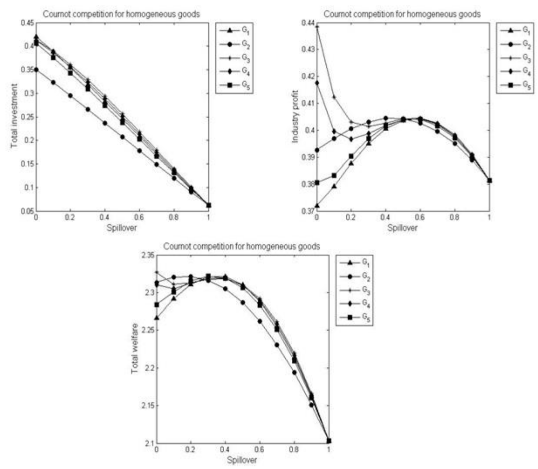

Fig. 14. Total investment, industry profit and total welfare in the five networks given in Figs. 10 and 12. The parameters used to plot the figures are $a = 12$, $\bar{c} = 10$ and $\gamma = 3$.

IV. CONCLUSION

The paper discussed two important issues related to the structure of the R&D networks. In the first issue, the attempt was to manage the R&D network formation to reduce the conflict between the individual and social desires of R&D cooperation between firms. The results suggest that while developing the strategic R&D interactions between individuals improves their profits, restricting the R&D system contributes to reconciling between the individual and social in determining the optimal structure. In the second issue, the attempt was to examine the role of the small world framework on the equilibrium outcomes. The results indicate that the density of the small world network has powerful effects on the outcomes. When forming this model of networks, restricting the R&D interactions is individually and socially profitable.

While the network game by Goyal and Moraga-Gonzalez presents important values, the R&D network formation may require enhancements [1]. One of those requirements has been discussed in this paper, that is restricting the R&D partnerships, especially when firms are in a homogeneous product market. Therefore, for future works, the free choice in forming the cooperative links by firms may need to reconsider in order to reach profitable R&D structures in both the individual and social perspectives.

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Appendix

The equilibria for homogeneous and independent products

(A) For independent goods:

R&D effort:

$$x^* = \frac{(a-c)}{4\gamma - k - 1}.$$

Quantity:

$$q^* = \frac{2\gamma(a-c)}{4\gamma - k - 1}.$$

Profit:

$$\pi^* = \frac{\gamma(4\gamma - 1)(a = c)^2}{(4\gamma - k - 1)^2}.$$

Total welfare:

$$TW^* = \frac{n\gamma(6\gamma - 1)(a = c)^2}{(4\gamma - k - 1)^2}.$$

(B) For homogeneous goods:

R&D effort:

$$x^* = \frac{(n-k)(a-c)}{\gamma(n+1)^2 - (n-k)(k+1)}.$$

Quantity:

$$q^* = \frac{\gamma(n+1)(a-c)}{\gamma(n+1)^2 - (n-k)(k+1)}.$$

Profit:

$$\pi^* = \frac{(\gamma^2(n+1)^2 - (n-k)^2)(a-c)^2}{(\gamma(n+1)^2 - (n-k)(k+1))^2}.$$

Total welfare:

$$TW^* = \frac{n(\gamma^2(n+2)(n+1)^2 - 2(n-k)^2)(a-c)^2}{2(\gamma(n+1)^2 - (n-k)(k+1))^2}.$$

Proof of Proposition 1

1. In an independent product market, the total welfare increases with the activity level k[1]. Since $k^* = n/2$ is the highest activity level in a regular bipartite network with n firms, then the total welfare is maximized at k^*.

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According to Goyal and Moraga-Gonzalez, the total welfare in a homogeneous product market is maximized at an intermediate level k_{TW} [1]. Therefore, to proof that the total welfare in k- regular bipartite networks with n firms is maximized at $k^* = n/2$, we need to show that $TW_{n/2}$ is higher than $TW_{n/2-1}$.
At the activity levels $k = n/2$ and $k = n/2 - 1$, the total welfares are

$$TW^*_{n/2} = \frac{n\gamma\left[\gamma(n+1)^2(n+2) - n^2/2\right](a-\bar{c})^2}{[\gamma(n+1)^2 - n(n+2)/4]^2}$$

$$TW^*_{n/2-1} = \frac{n\gamma\left[\gamma(n+1)^2(n+2) - (n+1)^2/2\right](a-\bar{c})^2}{[\gamma(n+1)^2 - n(n+2)/4]^2}$$

To prove the proposition, we calculate $TW^*_{n/2} - TW^*_{n/2-1}$.
Thus Since $2n\gamma(n+1) > 0$, then $TW^*_{n/2} > TW^*_{n/2-1}$

$$TW^*_{n/2} - TW^*_{n/2-1} = \frac{2n\gamma(n+1)(a-\bar{c})^2}{[\gamma(n+1)^2 - n(n+2)/4]^2}$$