

Screening of a Test Charge in the High Density Strongly Coupled Quantum Plasma

Rethika KT*, Anjana AV and Vishnu MB

Department of Physics, University of Calicut, Kerala, India

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*For Correspondence

Rethika KT, Department of Physics, University of Calicut, Kerala, India, Tel: 9496961065.

E-mail: rethikasurendran@gmail.com

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ABSTRACT

This paper reports an attractive force between ions that are shielded by degenerate electrons in high density strongly coupled quantum plasma. The electric potential around an isolated ion has hard core negative part that looks like the Lennard-Jones (LJ) type potential. The new attractive potential is attributed to the quantum Bohm potential within hydrodynamic approach. Here the electron exchange and electron correlations are not considered. The negative potential is responsible for the attraction of ions forming lattices and atoms/molecules also for critical point and phase transition in high density SCQ plasma at nano scales

INTRODUCTION

Shielding of electric charge generated by free charges in plasma is a primary issue in plasma physics. A test charged particle immersed in plasma will be shielded out by either ions or the electrons. The Debye potential $\phi(r)$ [1-5] arises due to the distribution of charges within the plasma. The knowledge of potential distribution predicts how a cloud of opposite polarity charges will shield a test charge particle over a certain radius. Recently Shukla and Eliasson discovered an oscillating shielded Coulomb potential also referred to as the Shukla Eliasson attractive potential around a stationary test ion in an un magnetized quantum plasma [6]. The attractive potential obtained from the continuity and momentum equations for degenerate electrons in dense quantum plasma depends on the electron number density of the plasma. These studies were based on the generalized quantum hydro dynamical (GQHD) equations [7]. The generalized electron momentum equation in the GQHD model includes the quantum statistical pressure and quantum forces due to electron tunneling through the quantum Bohm potential [8], spin magnetization of Bohr electrons as well as the electron exchange and correlation effects due to the electron spin [9]. The dynamics of the degenerate electrons in dense quantum plasmas is controlled by the electromagnetic and quantum forces. The profile of the quantum plasma potential resembles the Lennard-Jones potential in atomic gases [6,10].

STRONGLY COUPLED QUANTUM PLASMA

Consider a quantum plasma in presence of non-relativistic degenerate electron fluids and mildly coupled ions that are immobile and form the neutralizing background. Here the generalized electron momentum equation in GQHD model includes the pressure which is derived from the asymptotic formula for the ground state energy of high density quantum electro dynamic plasma. Here the electron exchange and correlation effects due to the electron spin are not considered in the momentum equation.

The thermo dynamical properties of dusty plasma are determined by the Coulomb coupling parameter. For strongly coupled plasma, the coupling parameter $\Gamma \geq 1$. For strongly coupled plasma; the results of thermal agitation between particles are comparable to their electrostatic interaction. This will give rise to both short-range and long-range order. The long range order arises from the screened Coulomb forces and the short range order is associated to the repulsion of the charged particles due to their finite size. This brings the strongly coupled plasma system in a regime which is intermediate between an ideal gas and a solid [11,12]. Strongly coupled plasma has more in common with the liquid than a weakly coupled plasma. The high and low density limit behavior of strongly coupled plasma is different from that observed in molecular systems. In high density regime, atoms and molecules becomes localized in a lattice structure whose zero point vibrations are small. For very light electrons, localization forces the particles to have a large momentum fluctuations and which correspond to a high kinetic energy. The total energy might be higher than in the electron gas state in which the division of the two forms of energy might be better balanced. A phase

transition from the low density lattice state to the high density gas state occur at $r_s=14$ ($r_s = \frac{r_0}{r_B}$, where r_0 is the mean inter particle distance or Wigner-Seitz radius, related to the density n , $\frac{4}{3}\pi r_0^3 = n^{-1}$ and r_B is the Bohr radius, $\frac{\hbar^2}{me^2}$) [13]. The limiting unperturbed state is that of the perfect Fermi gas at high densities. Electron correlation and exchange terms become important with decrease in density. The resultant expansion for ground state energy in the high density (small r_s) regime gives:

$$E_G \approx 2.21r_s^{-2} - 0.916r_s^{-1} + r_s(0.0049 \ln r_s + C) + \dots \tag{1}$$

Where, $C \sim -0.02$. This asymptotic expansion was interpolated by Isihara and Montroll through the method of Pade approximants [14].

For strongly coupled quantum plasma,

$$r_s = \left(\frac{3}{4\pi n}\right)^{\frac{1}{3}} \tag{2}$$

$$\text{Pressure, } P = \frac{-nr_s}{3} \frac{\partial E_G}{\partial r_s} \tag{3}$$

QUANTUM HYDRODYNAMICS

Quantum hydrodynamics has a wide range of applicability in the framework of semiconductor physics, molecular systems, metallic nanoparticles, thin metal films and in nuclear physics [15]. The quantum fluid model has been applied to several problems involving charged particle systems such as the description of quantum diodes in degenerate plasmas and quantum ion acoustic waves in carbon nanotubes. The model has also been applied to dense astrophysical plasmas and high grain free electron lasers [16].

The QHD equations are,

$$\frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0 \tag{4}$$

$$m \left(\frac{\partial u}{\partial t} + u \cdot \nabla u \right) = e \nabla \phi - n^{-1} \frac{\partial P}{\partial n} \nabla n + \nabla V_B \tag{5}$$

$$\nabla^2 \phi = 4\pi e (n - n_0) - 4\pi Q \delta(r) \tag{6}$$

Here the Quantum Bohm potential, $V_B = -\frac{\hbar^2}{2m} \frac{1}{\sqrt{n}} \nabla^2 \sqrt{n}$ Letting $n = n_0 + n_1$, where $n_1 \ll n_0$. Linearize the resultant equations to obtain the density perturbation n_1 that can be inserted in to eqn. (6). The Fourier transformation in space leads to the electric potential around an isolated ion. Thus we have:

$$\phi(r) = \frac{Q}{2\pi^2} \int \frac{e^{ik \cdot r}}{k^2 D} d^3k \tag{7}$$

Where D is the dielectric constant, r denotes the position relative to the instantaneous position of the test charge.

The inverse Dielectric constant can be written as:

$$\frac{1}{D} = \frac{\frac{k^2}{k_s^2} + \alpha \frac{k^4}{k_s^4}}{1 + \frac{k^2}{k_s^2} + \alpha \frac{k^4}{k_s^4}} \tag{8}$$

Where $k_s^2 = \frac{m\omega_p^2}{\left(\frac{\partial P}{\partial n}\right)}$ is the inverse Thomas Fermi screening length and $\alpha = \frac{\hbar^2 \omega_p^2}{\left(\frac{\partial P}{\partial n}\right)^2}$ measures the importance of quantum recoil effect and $\omega_p^2 = \frac{4\pi n_0 e^2}{m}$.

$\frac{\partial P}{\partial n}$, at $n=n_0$ can be calculated by using eqns. (1)-(3).

$$\frac{\partial P}{\partial n} = (1.78 \times 10^{-16}) n_0^{\frac{2}{3}} - (0.35 \times 10^{-8}) n_0^{\frac{1}{3}} - 0.0205 + (3.51 \times 10^4 \ln(n_0) - 1.73 \times 10^6) n_0^{-\frac{1}{3}} \tag{9}$$

Inserting eqn. (8) in (7),

$$\phi(r) = \frac{Q}{4\pi^2} \int \left(\frac{1+b}{k^2+k_+^2} + \frac{1-b}{k^2+k_-^2} \right) e^{ik \cdot r} d^3k \tag{10}$$

where, $b = \frac{1}{\sqrt{1-4\alpha}}$

and $k_{\pm}^2 = k_s^2 \frac{[1 \mp \sqrt{1-4\alpha}]}{2\alpha}$

RESULTS

The above integral was evaluated for different values of α . The boundary condition is $\phi \rightarrow 0$ at $r \rightarrow \infty$.

Eqn. (10) becomes:

$$\phi(r) = \frac{Q}{2r} [(1+b)e^{-k_+r} + (1-b)e^{-k_-r}] \tag{11}$$

For $\alpha \rightarrow 0$,

$$\phi(r) = \frac{Q}{r} e^{-k_s r} \tag{12}$$

For $\alpha \rightarrow \frac{1}{4}$,

$$\phi(r) = \frac{Q}{r} \left(1 + \frac{k_s}{\sqrt{2}} r\right) e^{-k_s r} \tag{13}$$

For $\alpha = 1$,

$$\phi(r) = \frac{Q}{r} \left(\cos(k_i r) + \frac{1}{\sqrt{3}} \sin(k_i r)\right) e^{-k_r r} \tag{14}$$

Where $k_r = \frac{k_s}{2} \sqrt{3}$

$$k_i = \frac{k_s}{2}$$

For $\alpha \gg 1$, We get the exponential cosine screened potential ^[17]:

$$\phi(r) = \frac{Q}{r} (\cos(k_s r_*)) e^{-k_s r_*} \tag{15}$$

Where $r_* = \frac{r}{(4\alpha)^{\frac{1}{4}}}$.

In **Figure 1**, the profiles of the potential given by the eqns. (13)-(15) for different values of α are displayed. It can be seen that the new short range attractive electric potential that resembles the LJ type potential for $\alpha=10$ while for the smaller values of α the attractive potential vanishes. In high density strongly coupled plasma, with Bohm potential, the attractive potential exists only for $\alpha \geq 1$. From **Figure 2**, it is clear that the attractive potential increases with increase in α values till $\alpha=10$. After that the attractive potential decreases and reaches to 0 as α increases.

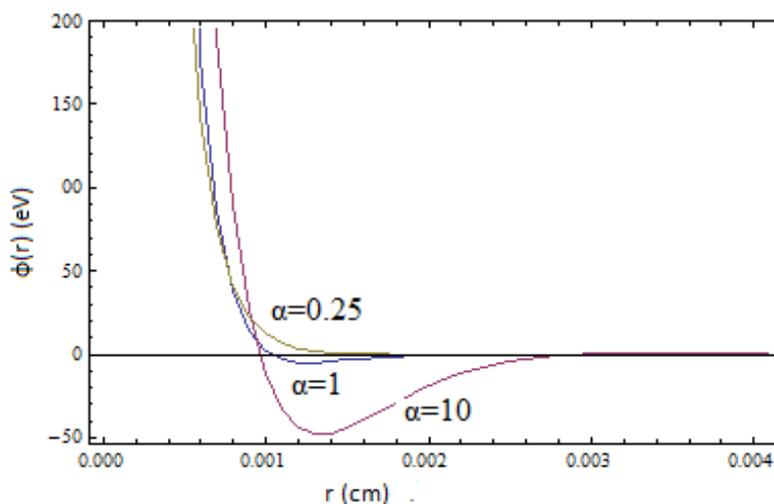


Figure 1. The electric potential ϕ as a function of r for different values of α .

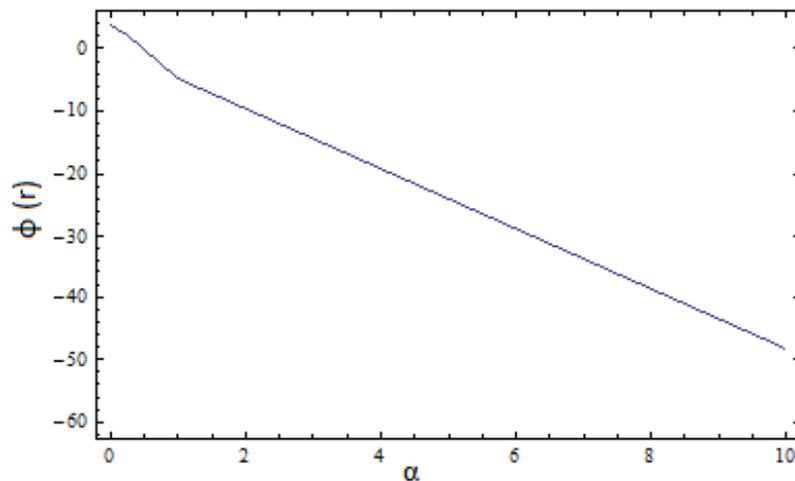


Figure 2. variation of the electric potential with α .

CONCLUSION

The electron exchange and correlation effects are not considered in this work. Pressure is derived from the asymptotic formula for the ground state energy of high density quantum plasma. These are the differences from the work of Shukla and Eliasson. The appearance of the new attractive force may be due to the density perturbations associated with the statistical pressure and due to the tunneling of degenerate electrons through the Bohm potential by overlapping electron wave functions^[6,10]. The linear response theory for quasi-stationary electrostatic perturbations involving immobile ions reveals that the electric potential around an isolated ion in a quantum plasma resembles the Lennard Jones type potential^[6,10]. Thus the newly found electric potential have a short range negative hard core electric potential.

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