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Suzuki Type Common Coupled Fixed Point for a Pair of w-Compatible Maps in Partial G-Metric Spaces

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ABSTRACT: In this paper we prove a unique common coupled fixed point theorem for a pair of w-compatible mappings satisfying Suzuki type contractive condition in partial G-metric spaces. We also give an example to illustrate our main theorem.

KEYWORDS: Partial G-metric space, w-compatible pairs, 0-P-G completeness.
MSC: 47H10, 54H25

I. INTRODUCTION

Dhage [1] introduced the concept of $D-$metric space to generalize the ordinary metric space and proved several results, for example, refer [1, 2, 3]. Unfortunately almost all results are invalid (refer [11, 12, 13, 20, 22]). To modify $D-$metric space, Mustafa and Sims [20] introduced the concept of $G-$metric spaces and obtained some results in their papers. Later several authors, for instance, [4, 6, 7, 9, 16, 17, 18, 19, 21, 23, 24, 25], proved fixed, common fixed and coupled fixed point therorems in $G-$metric spaces.

Recently Salimi and Vetro [8] introduced partial $G-$metric spaces by combining the concepts of partial metric spaces introduced by Mathews [10] and G-metric spaces as in [20].

In this paper we prove a unique common coupled fixed point theorem for a pair of w-compatible mappings satisfying Suzuki type contractive condition in partial G-metric spaces. We also give an example to illustrate our main theorem.

First we state the following known definitions, lemmas and propositions.

Definition 1.1 ([1]) *Let* X *be a nonempty set. A* $D-$*metric on* X *is a function* $D: X^3 \to [0,+\infty)$ *that satisfies the following conditions for each* $x, y, z, a \in X$,

1. $D(x, y, z) = 0$ if and only if $x = y = z$,
2. $D(x, y, z) = D(p\{x, y, z\})$ where p is a permutation function,
3. $D(x, y, z) \le D(x, y, a) + D(x, a, z) + D(a, y, z)$.

Then the pair (X, D) is called a D-metric space.

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Definition 1.2 ([20]) *Let* X *be a nonempty set and let* $G: X \times X \times X \to \mathbb{R}^{+}$ *be a function satisfying the following properties :*

(G_1): $G(x,y,z)=0$ if $x=y=z$,

(G_2): $0<G(x,x,y)$ for all $x,y \in X$ with $x \neq y$,

(G_3): $G(x,x,y) \le G(x,y,z)$ for all $x,y,z \in X$ with $y \neq z$,

(G_4): $G(x,y,z)=G(x,z,y)=G(y,z,x)=\ldots$,symmetry in all three variables,

(G_5): $G(x,y,z) \le G(x,a,a)+G(a,y,z)$ for all $x,y,z,a \in X$.

Then the function G is called a generalized metric or a G-metric on X and the pair (X,G) is called a G-metric space.

Definition 1.3 ([10]) *A partial metric on a nonempty set* X *is a function* $p: X \times X \to \mathbb{R}^{+}$ *such that for all* $x,y,z \in X$,

(p_1) $x=y \Leftrightarrow p(x,x)=p(x,y)=p(y,y)$,

(p_2) $p(x,x) \le p(x,y), p(y,y) \le p(x,y)$,

(p_3) $p(x,y)=p(y,x)$,

(p_4) $p(x,y) \le p(x,z)+p(z,y)-p(z,z)$.

The pair (X,p) is called a *partial metric space* (PMS).

Definition 1.4 ([8]) *Let* X *be a nonempty set and let* $P: X \times X \times X \to [0,+\infty)$ *is called a partial* G*-metric if the following conditions are satisfied:*

1. $x=y=z$ then $P(x,y,z)=P(x,x,x)=P(y,y,y)=P(z,z,z)$,
2. $P(x,x,x)+P(y,y,y)+P(z,z,z) \le 3P(x,y,z)$ for all $x,y,z,\in X$,
3. $\frac{1}{3}P(x,x,x)+\frac{2}{3}P(y,y,y)<P(x,y,y)$ for all $x,y,\in X$ with $x \neq y$,
4. $P(x,x,y)-\frac{1}{3}P(x,x,x) \le P(x,y,z)-\frac{1}{3}P(x,y,z)$ for all points $x,y,z \in X$ with $y \neq z$,
5. $P(x,y,z)=P(x,z,y)=P(y,z,x)=\cdots$ (symmetry in three variables),
6. $P(x,y,z) \le P(x,a,a)+P(a,y,z)-P(a,a,a)$ for any $x,y,z,a \in X$.

Then the pair (X,P) is called a partial G-metric space (in brief PGMS).

Example 1.1 ([8]) *Let* $X=[0,+\infty)$ *and define* $P(x,y,z)=\frac{1}{3}(\max\{x,y\}+\max\{y,z\}+\max\{x,z\})$, for all points $x,y,z \in X$. Then (X,P) is a PGMS.

The following proposition gives some properties of a partial G-metric.

Proposition 1.1 ([8]) *Let* (X,P) *be a PGMS, then* $x,y,z,a \in X$, *the following properties hold:*

1. $P(x,y,z)=P(x,x,x)=P(y,y,y)=P(z,z,z)$, then $x=y=z$
2. If $P(x,y,z)=0$ then $x=y=z$;
3. If $x \neq y$, then $P(x,y,y)>0$
4. $P(x,y,z) \le P(x,x,y)+P(x,x,z)-P(x,x,x)$ for any $x,y,z,a \in X$.
5. $P(x,y,y) \le 2P(x,x,y)-P(x,x,x)$;
6. $P(x,y,z) \le P(x,a,a)+P(y,a,a)-P(z,a,a)-2P(a,a,a)$;

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7. $P(x,y,z) \le P(x,a,z) + P(a,y,z) - \frac{2}{3}P(a,a,a) - \frac{1}{3}P(z,z,z)$ with $y \ne z$;
8. $P(x,y,y) \le P(x,y,a) + P(a,y,y) - \frac{2}{3}P(a,a,a) - \frac{1}{3}P(y,y,y)$ with $x \ne y$;

Definition 1.5 ([8]) *Let (X,P) be a PGMS. Then*

1. A sequence $\{x_n\}$ is $P-G$-converges $x \in X$, if and only if
$$P(x,x,x) = \lim_{n\to+\infty} P(x,x,x_n) = \lim_{n\to+\infty} P(x,x_n,x_n).$$
2. A sequence $\{x_n\}$ is $0-P-G$-Cauchy if and only if $\lim_{m,n\to+\infty} P(x_n,x_m,x_m) = 0.$
3. A partial G-metric spaces (X,P) is said to be $0-P-G$-complete if and only if every $0-P-G$-Cauchy sequence in X $P-G$-converges to a point $x \in X$ such that $P(x,x,x) = 0.$

Example 1.2 ([8]) *Let $X = [0,1]$ and $P: X^3 \to [0,\infty)$ be defined by $P(x,y,z) = \max\{x,y\} + \max\{y,z\} + \max\{x,z\}$,for all points $x,y,z \in X$. Then (X,P) is a $0-P-G-$complete partial $G-$metric space.*

Lemma 1.1 ([8]) *Let (X,P) be a partial G-metric space and $\{x_n\}$ be a sequence in X. Assume that $\{x_n\}$ $P-G$-converges to $x \in X$ and $P(x,x,x) = 0$. Then $\lim_{n\to+\infty} P(x_n,y,y) = P(x,y,y)$ for all $y \in X$.*

Similarly we can have the following Lemma.

Lemma 1.2 *Let (X,P) be a partial G-metric space and $\{x_n\}$ be a sequence in X. Assume that $\{x_n\}$ $P-G$-converges to $x \in X$ and $P(x,x,x) = 0$. Then $\lim_{n\to+\infty} P(x_n,x_n,y) = P(x,x,y)$ for all $y \in X$.*

Bhaskar and Lakshmikantham [14] developed some coupled fixed point theorems for a mapping satisfying mixed monotone property in partially ordered metric spaces. Later Lakshmikantham and Ciric [15] extended the notion of mixed monotone property to mixed g-monotone property and generalized the results of [14]. Abbas et al. [5] introduced $w-$compatible mappings and proved some common coupled fixed point theorems in cone metric spaces.

Definition 1.6 ([14]) *An element $(x,y) \in X \times X$ is called a coupled fixed point of a mapping $F: X \times X \to X$ if $x = F(x,y)$ and $y = F(y,x)$.*

Definition 1.7 ([15]) *An element $(x,y) \in X \times X$ is called*

(i) a coupled coincident point of mappings $F: X \times X \to X$ and $g: X \to X$ if $gx = F(x,y)$ and $gy = F(y,x)$.

(ii) a common coupled fixed point of mappings $F: X \times X \to X$ and $g: X \to X$ if $x = gx = F(x,y)$ and $y = gy = F(y,x)$.

Definition 1.8 ([5]) *The mappings $F: X \times X \to X$ and $g: X \to X$ are called a w - compatible pair if $g(F(x,y)) = F(gx,gy)$ and $g(F(y,x)) = F(gy,gx)$ whenever $gx = F(x,y)$ and $gy = F(y,x)$.*

Recently Salimi and Vetro [8] proved the following Suzuki type common fixed point theorem.

Theorem 1.1 *Let (X,P) be a partial G-metric space and $g,T: X \to X$ be satisfying*

$(1.1.1)$ $T(X) \subseteq g(X)$ and $\{g,T\}$ is a weakly compatible pair

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(1.1.2) $g(X)$ is $0-P-G$-complete sub space of X

(1.1.3) assume that there exists $r\in[0,1)$ such that

$\psi(r)\ \ P(gx,Tx,Tx)\le P(gx,gy,gy)$ implies that $P(Tx,Ty,Ty)\le rP(gx,gy,gy)$

for any $x,y\in X$, where $\psi:[0,1)\to(\frac{1}{6},\frac{\sqrt{3}}{6}]$ non decreasing function given by

$$\psi(r)=\begin{cases}\frac{\sqrt{3}}{6}\ if\ 0\le r<\frac{\sqrt{3}-1}{2}\\ \frac{1}{2(1+2r)}\ if\ \frac{\sqrt{3}-1}{2}\le r<1\end{cases}$$

Then T and g have unique common fixed point in X.

Now we give our main result.

II. Main Result

Theorem 2.1 *Let* (X,P) *be a partial* G*-metric space and* $F:X\times X\to X$, $g:X\to X$ be mappings satisfying

(2.1.1) $F(X\times X)\subseteq g(X)$, $g(X)$ is $0-P-G$-complete subspace of X,

(2.1.2) F and g are w-compatible,

(2.1.3) if there exits $\theta\in[0,1)$ such that

$$\eta(\theta)\min\begin{Bmatrix}P(gx,F(x,y),F(x,y)),\\ P(gy,F(y,x),F(y,x))\end{Bmatrix}\le\max\begin{Bmatrix}P(gx,gu,gu),\\ P(gy,gv,gv)\end{Bmatrix}$$

Implies

$$P(F(x,y),F(u,v),F(u,v))\le\theta\max\begin{Bmatrix}P(gx,gu,gu),P(gy,gv,gv),\\ \frac{1}{3}P(gx,F(x,y),F(x,y)),\frac{1}{3}P(gy,F(y,x),F(y,x)),\\ \frac{1}{3}P(gu,F(u,v),F(u,v)),\frac{1}{3}P(gv,F(v,u),F(v,u)),\\ \frac{1}{6}P(gx,F(u,v),F(u,v)),\frac{1}{6}P(gy,F(v,u),F(v,u)),\\ \frac{1}{3}P(gu,F(x,y),F(x,y)),\frac{1}{3}P(gv,F(y,x),F(y,x))\end{Bmatrix}$$

for all $x,y,u,v\in X$, where $\eta:[0,1)\to(\frac{1}{5},\frac{1}{3}]$ defined by $\eta(\theta)=\frac{1}{3+2\theta}$ is a strictly decreasing function.

Then F and g have a unique common coupled fixed point.

Proof. Let $(x_0,y_0)\in X$. From (2.1.1), we can construct the sequences $\{x_n\}$ and $\{y_n\}$ in X such that $gx_{n+1}=F(x_n,y_n)$ and $gy_{n+1}=F(y_n,x_n)$, $n=0,1,2,\cdots$

Case (i) : *Suppose* $gx_n\ne gx_{n+1}$ *or* $gy_n\ne gy_{n+1}$ *for all* n. (1)

Then from Proposition 1.1 (ii), we have $P(gx_n,gx_{n+1},gx_{n+1})>0$ or

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$P(gy_n, gy_{n+1}, gy_{n+1}) > 0$ for all n.

Since

$$\eta(\theta)\min\begin{Bmatrix} P(gx_n, F(x_n,y_n), F(x_n,y_n)), \\ P(gy_n, F(y_n,x_n), F(y_n,x_n)) \end{Bmatrix} \le \eta(\theta) P(gx_n, gx_{n+1}, gx_{n+1})$$
$$< P(gx_n, gx_{n+1}, gx_{n+1})$$
$$\le \max\begin{Bmatrix} P(gx_n, gx_{n+1}, gx_{n+1}), \\ P(gy_n, gy_{n+1}, gy_{n+1}) \end{Bmatrix}$$

from (2.1.3), we have

$$P(gx_{n+1}, gx_{n+2}, gx_{n+2}) \le \theta \max\begin{Bmatrix} P(gx_n, gx_{n+1}, gx_{n+1}), P(gy_n, gy_{n+1}, gy_{n+1}), \\ \frac{1}{3}P(gx_n, gx_{n+1}, gx_{n+1}), \frac{1}{3}P(gy_n, gy_{n+1}, gy_{n+1}), \\ \frac{1}{3}P(gx_{n+1}, gx_{n+2}, gx_{n+2}), \frac{1}{3}P(gy_{n+1}, gy_{n+2}, gy_{n+2}), \\ \frac{1}{6}P(gx_n, gx_{n+2}, gx_{n+2}), \frac{1}{6}P(gy_n, gy_{n+2}, gy_{n+2}), \\ \frac{1}{3}P(gx_{n+1}, gx_{n+1}, gx_{n+1}), \frac{1}{3}P(gy_{n+1}, gy_{n+1}, gy_{n+1}) \end{Bmatrix}$$

But by using (P_6), we have

$$\frac{1}{6}P(gx_n, gx_{n+2}, gx_{n+2}) \le \frac{1}{6}[P(gx_n, gx_{n+1}, gx_{n+1}) + P(gx_{n+1}, gx_{n+2}, gx_{n+2})]$$

$$\le \frac{1}{3}\max\{P(gx_n, gx_{n+1}, gx_{n+1}), P(gx_{n+1}, gx_{n+2}, gx_{n+2})\}$$

from (P_2), we have $\frac{1}{3}P(gx_{n+1}, gx_{n+1}, gx_{n+1}) \le P(gx_{n+1}, gx_{n+2}, gx_{n+2})$.

Hence

$$P(gx_{n+1}, gx_{n+2}, gx_{n+2}) \le \theta \max\begin{Bmatrix} P(gx_n, gx_{n+1}, gx_{n+1}), P(gy_n, gy_{n+1}, gy_{n+1}), \\ P(gx_{n+1}, gx_{n+2}, gx_{n+2}), P(gy_{n+1}, gy_{n+2}, gy_{n+2}) \end{Bmatrix}.$$

Similarly

$$P(gy_{n+1}, gy_{n+2}, gy_{n+2}) \le \theta \max\begin{Bmatrix} P(gx_n, gx_{n+1}, gx_{n+1}), P(gy_n, gy_{n+1}, gy_{n+1}), \\ P(gx_{n+1}, gx_{n+2}, gx_{n+2}), P(gy_{n+1}, gy_{n+2}, gy_{n+2}) \end{Bmatrix}.$$

Thus

$$\max\{P(gx_{n+1}, gx_{n+2}, gx_{n+2}), P(gy_{n+1}, gy_{n+2}, gy_{n+2})\} \le \theta \max\begin{Bmatrix} P(gx_n, gx_{n+1}, gx_{n+1}), P(gy_n, gy_{n+1}, gy_{n+1}), \\ P(gx_{n+1}, gx_{n+2}, gx_{n+2}), P(gy_{n+1}, gy_{n+2}, gy_{n+2}) \end{Bmatrix}. \quad (2)$$

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If maximum of R.H.S of (2) is $\max\{P(gx_{n+1}, gx_{n+2}, gx_{n+2}), P(gy_{n+1}, gy_{n+2}, gy_{n+2})\}$, then from (2) we get a contradiction in view of (1).

Hence

$\max\{P(gx_{n+1}, gx_{n+2}, gx_{n+2}), P(gy_{n+1}, gy_{n+2}, gy_{n+2})\} \leq \theta \max\{P(gx_n, gx_{n+1}, gx_{n+1}), P(gy_n, gy_{n+1}, gy_{n+1})\}$.

Continuing in this way, we can show for $n = 1,2,3,\cdots$ that

$$\max\{P(gx_n, gx_{n+1}, gx_{n+1}), P(gy_n, gy_{n+1}, gy_{n+1})\} \leq \theta^n \max\{P(gx_0, gx_1, gx_1), P(gy_0, gy_1, gy_1)\}. \quad (3)$$

$$P(gx_n, gx_{n+1}, gx_{n+1}) \to 0 \ \ and \ \ P(gy_n, gy_{n+1}, gy_{n+1}) \to 0. \quad (4)$$

For $m > n$, consider

$$\begin{aligned} P(gx_n, gx_m, gx_m) &\leq P(gx_n, gx_{n+1}, gx_{n+1}) + P(gx_{n+1}, gx_{n+2}, gx_{n+2}) + \cdots + P(gx_{m-1}, gx_m, gx_m) \\ &\leq (\theta^n + \theta^{n+1} + \cdots + \theta^{m-1}) \max\{P(gx_0, gx_1, gx_1), P(gy_0, gy_1, gy_1)\} \\ &\leq \frac{\theta^n}{1-\theta} \max\{P(gx_0, gx_1, gx_1), P(gy_0, gy_1, gy_1)\} \\ &\to 0 \quad as \ \ n, m \to \infty. \end{aligned}$$

Thus

$$\lim_{n,m\to\infty} P(gx_n, gx_m, gx_m) = 0. \quad (5)$$

Similarly we have

$$\lim_{n,m\to\infty} P(gy_n, gy_m, gy_m) = 0. \quad (6)$$

From (5) and (6), it follows that $\{gx_n\}$ and $\{gy_n\}$ are $O-P-G-$ Cauchy sequences in $g(X)$.
Since $g(X)$ is $O-P-G-$complete, it follows that $\{gx_n\}$ and $\{gy_n\}$ are $O-P-G-$ converge to α and $\beta \in g(X)$ and

$$P(\alpha, \alpha, \alpha) = 0 = P(\beta, \beta, \beta). \quad (7)$$

Also there exist z_1, $z_2 \in X$ such that $\alpha = gz_1$ and $\beta = gz_2$. Since $gx_n \to \alpha$ and $gy_n \to \beta$, we may assume that $gx_n \neq \alpha$ and $gy_n \neq \beta$ for infinitely many n.

Claim :. $\max\left\{\begin{array}{l} P(gz_1, F(x,y), F(x,y)), \\ P(gz_2, F(y,x), F(y,x)) \end{array}\right\} \leq \theta \max\left\{\begin{array}{l} P(gz_1, gx, gx), P(gz_2, gy, gy), \\ \frac{1}{3} P(gx, F(x,y), F(x,y)), \\ \frac{1}{3} P(gy, F(y,x), F(y,x)) \end{array}\right\}$.

for all $x, y \in X$ with $gx \neq gz_1$ and $gy \neq gz_2$.

Let $x, y \in X$ with $gx \neq gz_1$ and $gy \neq gz_2$. Then from Proposition 1.1 (ii), we have $P(gx, gz_1, gz_1) > 0$ and $P(gy, gz_2, gz_2) > 0$.

Since $gx_n \to gz_1$ and $gy_n \to gz_2$, there exists a positive integer n_0 such that for $n \geq n_0$, we have

$$\begin{aligned} P(gz_1, gx_n, gx_n) &\leq \frac{1}{3} P(gx, gz_1, gz_1) \\ P(gz_1, gx_n, gx_{n+1}) &\leq \frac{1}{3} P(gx, gz_1, gz_1) \quad (8) \\ P(gx_n, gz_1, gz_1) &\leq \frac{1}{3} P(gx, gz_1, gz_1) \end{aligned}$$

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and

$$\begin{aligned} P(gz_2, gy_n, gy_n) &\le \frac{1}{3} P(gy, gz_2, gz_2) \\ P(gz_2, gy_n, gy_{n+1}) &\le \frac{1}{3} P(gy, gz_2, gz_2) \\ P(gy_n, gz_2, gz_2) &\le \frac{1}{3} P(gy, gz_2, gz_2) \end{aligned} \tag{9}$$

For $n \ge n_0$, consider

$$\begin{aligned} 3\eta(\theta)\min\begin{Bmatrix} P(gx_n, F(x_n, y_n), F(x_n, y_n)), \\ P(gy_n, F(y_n, x_n), F(y_n, x_n)) \end{Bmatrix} &\le P(gx_n, gx_{n+1}, gx_{n+1}) \\ &\le P(gx_n, gx_{n+1}, gz_1) + P(gz_1, gx_{n+1}, gx_{n+1}) \quad \text{from Prop.1.1(viii)} \\ &\le \frac{2}{3} P(gx, gz_1, gz_1) \quad \text{from (8)} \\ &= P(gx, gz_1, gz_1) - \frac{1}{3} P(gx, gz_1, gz_1) \\ &\le P(gx, gz_1, gz_1) - P(gx_n, gz_1, gz_1) \quad \text{from (8)} \\ &\le P(gx, gx_n, gx_n) \quad \text{from } (P_6) \\ &\le 2P(gx, gx, gx_n) \quad \text{from Prop. 1.1 (v)} \\ &< 3P(gx, gx, gx_n). \end{aligned}$$

Thus for all $n \ge n_0$, we have

$$\begin{aligned} \eta(\theta)\min\begin{Bmatrix} P(gx_n, F(x_n, y_n), F(x_n, y_n)), \\ P(gy_n, F(y_n, x_n), F(y_n, x_n)) \end{Bmatrix} &\le P(gx_n, gx, gx) \\ &\le \max\{P(gx_n, gx, gx), P(gy_n, gy, gy)\} \end{aligned}$$

Now from (2.1.3) we have

$$P(F(x_n, y_n), F(x, y), F(x, y)) \le \theta \max\begin{Bmatrix} P(gx_n, gx, gx), P(gy_n, gy, gy), \\ \frac{1}{3} P(gx_n, gx_{n+1}, gx_{n+1}), \frac{1}{3} P(gy_n, gy_{n+1}, gy_{n+1}), \\ \frac{1}{3} P(gx, F(x, y), F(x, y)), \frac{1}{3} P(gy, F(y, x), F(y, x)), \\ \frac{1}{6} P(gx_n, F(x, y), F(x, y)), \frac{1}{6} P(gy_n, F(y, x), F(y, x)), \\ \frac{1}{3} P(gx, F(x_n, y_n), F(x_n, y_n)), \frac{1}{3} P(gy, F(y_n, x_n), F(y_n, x_n)) \end{Bmatrix}$$

Letting $n \to \infty$ and using Lemmas 1.1,1.2, (4) and (7), we get

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$$P(gz_1,F(x,y),F(x,y))\le\theta\max\begin{Bmatrix}P(gz_1,gx,gx),P(gz_2,gy,gy),0,0\\ \frac{1}{3}P(gx,F(x,y),F(x,y)),\frac{1}{3}P(gy,F(y,x),F(y,x)),\\ \frac{1}{6}P(gz_1,F(x,y),F(x,y)),\frac{1}{6}P(gz_2,F(y,x),F(y,x)),\\ \frac{1}{3}P(gx,gz_1,gz_1),\frac{1}{3}P(gy,gz_2,gz_2)\end{Bmatrix}$$

But

$$\begin{aligned}\frac{1}{6}P(gz_1,F(x,y),F(x,y))&\le\frac{1}{6}[P(gz_1,gx,gx)+P(gx,F(x,y),F(x,y))],\ from(P_6)\\ &\le\frac{1}{3}\max\{P(gz_1,gx,gx),P(gx,F(x,y),F(x,y))\}\end{aligned}$$

Thus from Prop. 1.1 (v)

$$P(gz_1,F(x,y),F(x,y))\le\theta\max\left\{P(gz_1,gx,gx),P(gz_2,gy,gy),\frac{1}{3}P(gx,F(x,y),F(x,y)),\frac{1}{3}P(gy,F(y,x),F(y,x))\right\}$$

Similarly we can show that

$$P(gz_2,F(y,x),F(y,x))\le\theta\max\left\{P(gz_1,gx,gx),P(gz_2,gy,gy),\frac{1}{3}P(gx,F(x,y),F(x,y)),\frac{1}{3}P(gy,F(y,x),F(y,x))\right\}$$

Thus

$$\max\begin{Bmatrix}P(gz_1,F(x,y),F(x,y)),\\ P(gz_2,F(y,x),F(y,x))\end{Bmatrix}\le\theta\max\begin{Bmatrix}P(gz_1,gx,gx),P(gz_2,gy,gy),\frac{1}{3}P(gx,F(x,y),F(x,y)),\\ \frac{1}{3}P(gy,F(y,x),F(y,x))\end{Bmatrix}\qquad(10)$$

Hence the claim.

Now for $gx\ne gz_1$ and $gy\ne gz_2$ consider

$$P(gx,F(x,y),F(x,y))\le P(gx,gz_1,gz_1)+P(gz_1,F(x,y),F(x,y))\quad from(P_6)$$

$$\le P(gx,gz_1,gz_1)+\theta\max\begin{Bmatrix}P(gz_1,gx,gx),P(gz_2,gy,gy),\\ \frac{1}{3}P(gx,F(x,y),F(x,y)),\\ \frac{1}{3}P(gy,F(y,x),F(y,x))\end{Bmatrix}\quad \text{from}(10)$$

$$\le P(gx,gz_1,gz_1)+\theta\max\begin{Bmatrix}2P(gx,gz_1,gz_1),2P(gy,gz_2,gz_2),\\ \frac{1}{3}P(gx,F(x,y),F(x,y)),\\ \frac{1}{3}P(gy,F(y,x),F(y,x))\end{Bmatrix}\quad from\ \ Prop.1.1(v)$$

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Hence $P(gx, F(x,y), F(x,y)) \leq (3+2\theta)\max\left\{\begin{array}{l} P(gx, gz_1, gz_1), P(gy, gz_2, gz_2), \\ \frac{1}{6}P(gx, F(x,y), F(x,y)), \\ \frac{1}{6}P(gy, F(y,x), F(y,x)) \end{array}\right\}$

Similarly we get

$$P(gy, F(y,x), F(y,x)) \leq (3+2\theta)\max\left\{\begin{array}{l} P(gx, gz_1, gz_1), P(gy, gz_2, gz_2), \\ \frac{1}{6}P(gx, F(x,y), F(x,y)), \\ \frac{1}{6}P(gy, F(y,x), F(y,x)) \end{array}\right\}$$

Thus

$$\max\left\{\begin{array}{l} P(gx, F(x,y), F(x,y)), \\ P(gy, F(y,x), F(y,x)) \end{array}\right\} \leq (3+2\theta)\max\left\{\begin{array}{l} P(gx, gz_1, gz_1), P(gy, gz_2, gz_2), \\ \frac{1}{6}P(gx, F(x,y), F(x,y)), \\ \frac{1}{6}P(gy, F(y,x), F(y,x)) \end{array}\right\} \qquad (11)$$

If maximum of R.H.S of (11) is $\max\left\{\begin{array}{l} \frac{1}{6}P(gx, F(x,y), F(x,y)), \\ \frac{1}{6}P(gy, F(y,x), F(y,x)) \end{array}\right\}$, then from (11) we get contradiction since

$\frac{3+2\theta}{6} = \frac{1}{2} + \frac{1}{3}\theta < 1$.

Hence from (11), we have

$$\max\left\{\begin{array}{l} P(gx, F(x,y), F(x,y)), \\ P(gy, F(y,x), F(y,x)) \end{array}\right\} \leq (3+2\theta)\max\{P(gx, gz_1, gz_1), P(gy, gz_2, gz_2)\} \qquad (12)$$

Thus from (12), we have

$$\begin{aligned} \eta(\theta)\min\left\{\begin{array}{l} P(gy, F(y,x), F(y,x)), \\ P(gx, F(x,y), F(x,y)) \end{array}\right\} &\leq \eta(\theta)\max\{P(gy, F(y,x), F(y,x)), P(gx, F(x,y), F(x,y))\} \\ &\leq \max\{P(gx, gz_1, gz_1), P(gy, gz_2, gz_2)\} \end{aligned}$$

Hence from (2.1.3), we have

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$$P(F(x,y),F(z_1,z_2),F(z_1,z_2)) \le \theta \max\left\{\begin{array}{l} P(gx,gz_1,gz_1),P(gy,gz_2,gz_2), \\ \frac{1}{3}P(gx,F(x,y),F(x,y)),\frac{1}{3}P(gy,F(y,x),F(y,x)), \\ \frac{1}{3}P(gz_1,F(z_1,z_2),F(z_1,z_2)),\frac{1}{3}P(gz_2,F(z_2,z_1),F(z_2,z_1)), \\ \frac{1}{6}P(gx,F(z_1,z_2),F(z_1,z_2)),\frac{1}{6}P(gy,F(z_2,z_1),F(z_2,z_1)), \\ \frac{1}{3}P(gz_1,F(x,y),F(x,y)),\frac{1}{3}P(gz_2,F(y,x),F(y,x)) \end{array}\right\} \quad (13)$$

Putting $x = x_n$ and $y = y_n$ in (13), we get

$$P(F(x_n,y_n),F(z_1,z_2),F(z_1,z_2)) \le \theta \max\left\{\begin{array}{l} P(gx_n,gz_1,gz_1),P(gy_n,gz_2,gz_2), \\ \frac{1}{3}P(gx_n,gx_{n+1},gx_{n+1}),\frac{1}{3}P(gy_n,gy_{n+1},gy_{n+1}), \\ \frac{1}{3}P(gz_1,F(z_1,z_2),F(z_1,z_2)),\frac{1}{3}P(gz_2,F(z_2,z_1),F(z_2,z_1)), \\ \frac{1}{6}P(gx_n,F(z_1,z_2),F(z_1,z_2)),\frac{1}{6}P(gy_n,F(z_2,z_1),F(z_2,z_1)), \\ \frac{1}{3}P(gz_1,gx_{n+1},gx_{n+1}),\frac{1}{3}P(gz_2,gy_{n+1},gy_{n+1}) \end{array}\right\}$$

Letting $n \to \infty$ and using Lemma 1.1, lemma 1.2, (4) and (7), we get

$$P(gz_1,F(z_1,z_2),F(z_1,z_2)) \le \theta \max\left\{\begin{array}{c} 0,0,0,0 \\ \frac{1}{3}P(gz_1,F(z_1,z_2),F(z_1,z_2)),\frac{1}{3}P(gz_2,F(z_2,z_1),F(z_2,z_1)), \\ \frac{1}{6}P(gz_1,F(z_1,z_2),F(z_1,z_2)),\frac{1}{6}P(gz_2,F(z_2,z_1),F(z_2,z_1)),0,0 \end{array}\right\}$$

$$\le \theta \max\{P(gz_1,F(z_1,z_2),F(z_1,z_2)),P(gz_2,F(z_2,z_1),F(z_2,z_1))\}$$

Similarly we can show that

$$P(gz_2,F(z_2,z_1),F(z_2,z_1)) \le \theta \max\{P(gz_1,F(z_1,z_2),F(z_1,z_2)),P(gz_2,F(z_2,z_1),F(z_2,z_1))\}.$$

Thus

$$\max\left\{\begin{array}{l} P(gz_1,F(z_1,z_2),F(z_1,z_2)), \\ P(gz_2,F(z_2,z_1),F(z_2,z_1)) \end{array}\right\} \le \theta \max\left\{\begin{array}{l} P(gz_1,F(z_1,z_2),F(z_1,z_2)), \\ P(gz_2,F(z_2,z_1),F(z_2,z_1)) \end{array}\right\}.$$

which in turn yields that $gz_1 = F(z_1,z_2)$ and $gz_2 = F(z_2,z_1)$.

Since the pair (F,g) is w-compatible we have

$$g\alpha = ggz_1 = g(F(z_1,z_2)) = F(gz_1,gz_2) = F(\alpha,\beta) \quad (14)$$

$$g\beta = ggz_2 = g(F(z_2,z_1)) = F(gz_2,gz_1) = F(\beta,\alpha) \quad (15).$$

Now

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$$\eta(\theta)\min\left\{\begin{array}{l}P(ga,F(a,ß),F(a,ß))\\ P\ (gß,\ F\ (ß,a),\ F\ (ß,a))\end{array}\right\} \le \eta(\theta)P(ga,ga,ga)$$

$$\le 3\eta(\theta)P\,(ga,gz_1,gz_1)$$

$$\le P(ga,gz_1,gz_1),\ \text{since}\ \ \eta(\theta)\le\frac{1}{3}$$

$$\le \max\{P(ga,gz_1,gz_1),P(gß,gz_2,gz_2)\}.$$

Hence from (2.1.3), we have

$$P(F(\alpha,\beta),F(z_1,z_2),F(z_1,z_2))\le\theta\max\left\{\begin{array}{l}P(g\alpha,gz_1,gz_1),P(g\beta,gz_2,gz_2),\frac{1}{3}P(g\alpha,g\alpha,g\alpha),\frac{1}{3}P(g\beta,g\beta,g\beta),\\ \frac{1}{3}P(gz_1,gz_1,gz_1),\frac{1}{3}P(gz_2,gz_2,gz_2),\frac{1}{6}P(g\alpha,gz_1,gz_1),\frac{1}{6}P(g\beta,gz_2,gz_2),\\ \frac{1}{3}P(gz_1,g\alpha,g\alpha),\frac{1}{3}P(gz_2,g\beta,g\beta)\end{array}\right\}$$

which in turn yields that

$$P(g\alpha,\alpha,\alpha)\le\theta\max\left\{\begin{array}{l}P(g\alpha,\alpha,\alpha),P(g\beta,\beta,\beta),\frac{1}{3}P(g\alpha,\alpha,\alpha),\frac{1}{3}P(g\beta,\beta,\beta),\\ 0,0,\frac{1}{6}P(g\alpha,\alpha,\alpha),\frac{1}{6}P(g\beta,\beta,\beta),\frac{2}{3}P(g\alpha,\alpha,\alpha),\frac{2}{3}P(g\beta,\beta,\beta)\end{array}\right\}\ from\ (7)\ and\ Prop.1.1(v)$$

$$=\theta\max\{P(g\alpha,\alpha,\alpha),P(g\beta,\beta,\beta)\}$$

Similarly we can show that

$$P(g\beta,\beta,\beta)\le\theta\max\{P(g\alpha,\alpha,\alpha),P(g\beta,\beta,\beta)\}$$

Thus

$$\max\{P(g\alpha,\alpha,\alpha),P(g\beta,\beta,\beta)\}\le\theta\max\{P(g\alpha,\alpha,\alpha),P(g\beta,\beta,\beta)\}$$

which in turn yields that $g\alpha=\alpha$ and $g\beta=\beta$ from Prop. 1.1 (ii).

Thus from (14) and (15), it follows that (α,β) is a common coupled fixed point of F and g.

Suppose $(\alpha^{'},\beta^{'})$ is another common coupled fixed point of F and g. Consider

$$\eta(\theta)\min\left\{\begin{array}{l}P(g\alpha,\ F(\alpha,ß),\ F(\alpha,ß)),\\ P(gß,\ F(ß,\alpha),\ F(ß,\alpha))\end{array}\right\}\le\eta(\theta)P(g\alpha,F(\alpha,ß),F(\alpha,ß))$$

$$=\eta(\theta)\ P(\alpha,\alpha,\alpha)$$

$$\le 3\eta(\theta)P(\alpha,\alpha^{'},\alpha^{'})\ \text{form}\,(\text{P}_2)$$

$$\le P(\text{g}\alpha,g\alpha^{'},\text{g}\alpha^{'})\quad \text{since}\ \eta(\theta)\le\frac{1}{3}$$

$$\le\max\{P(\text{g}\alpha,g\alpha^{'},\text{g}\alpha^{'}),P(\text{g}\beta,g\beta^{'},\text{g}\beta^{'})\}$$

Hence from (2.1.3), we have

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$$P(\alpha,\alpha^{'},\alpha^{'}) = P(F(\alpha,\beta),F(\alpha^{'},\beta^{'}),F(\alpha^{'},\beta^{'}))$$
$$\leq \theta\max\left\{\begin{matrix}P(\alpha,\alpha^{'},\alpha^{'}),P(\beta,\beta^{'},\beta^{'}),\frac{1}{3}P(\alpha,\alpha,\alpha),\frac{1}{3}P(\beta,\beta,\beta),\frac{1}{3}P(\alpha^{'},\alpha^{'},\alpha^{'}),\frac{1}{3}P(\beta^{'},\beta^{'},\beta^{'}),\\ \frac{1}{6}P(\alpha,\alpha^{'},\alpha^{'}),\frac{1}{6}P(\beta,\beta^{'},\beta^{'}),\frac{1}{3}P(\alpha^{'},\alpha,\alpha),\frac{1}{3}P(\beta^{'},\beta,\beta)\end{matrix}\right\}$$
$$\leq \theta\max\left\{\begin{matrix}P(\alpha,\alpha^{'},\alpha^{'}),P(\beta,\beta^{'},\beta^{'}),0,0,P(\alpha,\alpha^{'},\alpha^{'}),P(\alpha,\alpha^{'},\alpha^{'}),\\ \frac{1}{6}P(\alpha,\alpha^{'},\alpha^{'}),\frac{1}{6}P(\beta,\beta^{'},\beta^{'}),\frac{2}{3}P(\alpha,\alpha^{'},\alpha^{'}),\frac{2}{3}P(\alpha,\alpha^{'},\alpha^{'})\end{matrix}\right\} \text{from}(7)\text{ and Prop.}1.1(v)$$
$$= \theta\max\{P(\alpha,\alpha^{'},\alpha^{'}),P(\beta,\beta^{'},\beta^{'})\}$$

Similarly we can show that

$$P(\beta,\beta^{'},\beta^{'}) \leq \theta\max\{P(\alpha,\alpha^{'},\alpha^{'}),P(\beta,\beta^{'},\beta^{'})\}$$

Thus

$$\max\{P(\alpha,\alpha^{'},\alpha^{'}),P(\beta,\beta^{'},\beta^{'})\} \leq \theta\max\{P(\alpha,\alpha^{'},\alpha^{'}),P(\beta,\beta^{'},\beta^{'})\}$$

which in turn yields that $\alpha = \alpha^{'}$ and $\beta = \beta^{'}$.

Thus (α,β) is unique common coupled fixed point of F and g.

Case (ii): Suppose $gx_n = gx_{n+1}$ and $gy_n = gy_{n+1}$ for some n.

Then $gx_n = F(x_n, y_n)$ and $gy_n = F(y_n, x_n)$ so that (x_n, y_n) is a coupled coincidence point of F and g.

Now proceeding as in Case (i) from equation (14) on words with $gx_n = \alpha$ and $gy_n = \beta$, we can show that (α,β) is unique common coupled fixed point of F and g.

Now we give an example to illustrate our main Theorem 2.1

Example 2.1 *Let* (X, P) *be a partial* G*-metric space, where* $X = [0,1]$ *and* $P: X\times X\times X \to [0,\infty)$ be defined by $P(x, y, z) = max\{x, y\} + max\{y, z\} + max\{x, z\}$. Let $F: X\times X \to X$ and $g: X \to X$ be defined by

$$F(x, y) = \frac{x^2 + y^2}{16}, \quad gx = \frac{x}{2}, \ \forall\ x, y \in X .$$

One can easily verify the conditions $(2.1.1)$ and $(2.1.2)$.

For all $x, y \in X$, consider

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$$
\begin{aligned}
P\big(F(x,y),F(u,v),F(u,v)\big) &= 2\max\{\frac{x^2+y^2}{16},\frac{u^2+v^2}{16}\}+\frac{u^2+v^2}{16}\\
&=\frac{2}{8}\max\{\frac{x}{2}+\frac{y}{2},\frac{u}{2}+\frac{v}{2}\}+\frac{1}{8}[\frac{u}{2}+\frac{v}{2}]\\
&=\frac{2}{8}[\max\{\frac{x}{2},\frac{u}{2}\}+\{\frac{y}{2},\frac{v}{2}\}]+\frac{1}{8}[\frac{u}{2}+\frac{v}{2}]\\
&=\frac{1}{8}[2\max\{\frac{x}{2},\frac{u}{2}\}+\frac{u}{2}]+\frac{1}{8}[2\max\{\frac{y}{2},\frac{v}{2}\}+\frac{v}{2}]\\
&=\frac{1}{8}[P(gx,gu,gu)+P(gy,gv,gv)]\\
&\le\frac{1}{4}\max\{P(gx,gu,gu),P(gy,gv,gv)\}\\
&\le\frac{1}{4}\max\left\{\begin{array}{l}P(gx,gu,gu),P(gy,gv,gv),\frac{1}{3}P(gx,F(x,y),F(x,y)),\frac{1}{3}P(gy,F(y,x),F(y,x)),\\ \frac{1}{3}P(gu,F(u,v),F(u,v)),\frac{1}{3}P(gv,F(v,u),F(v,u)),\frac{1}{6}P(gx,F(u,v),F(u,v)),\\ \frac{1}{6}P(gy,F(v,u),F(v,u)),\frac{1}{3}P(gu,F(x,y),F(x,y)),\frac{1}{3}P(gv,F(y,x),F(y,x))\end{array}\right\}
\end{aligned}
$$

Hence (2.1.3) is satisfied with $\theta=\frac{1}{4}$ and $\eta(\theta)=\frac{1}{3+2\theta}=\frac{2}{7}\in(\frac{1}{5},\frac{1}{3}]$. Clearly (0,0) is the unique common coupled fixed point of F and g.

III. CONCLUSION

we presented a unique common coupled fixed point theorem for a pair of w -compatible mappings satisfying Suzuki type contractive condition in partial G -metric spaces. Our theorem generalizes many comparable results in coupled fixed points. We also give an example to illustrate our main theorem.

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