The Zhang and Fu’s Similarity Measure on Intuitionistic Fuzzy Multi Sets

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ABSTRACT: In this paper we introduce a novel Similarity measure for Intuitionistic fuzzy multi sets (IFMS) based on Zhang and Fu’s measure, as Similarity Measure is an important topic in fuzzy set theory. The unique feature of this method is that it considers multi membership, non membership and hesitancy degree for the same element. Since the proposed measure of IFMS is mathematically valid and demonstrates the superiority of the existing methods, we apply this measure to medical diagnosis and, pattern recognition problems.

KEYWORDS: Intuitionistic fuzzy set, Fuzzy Multi sets, Intuitionistic Fuzzy Multi sets, Similarity measure.

I. INTRODUCTION

The Intuitionistic Fuzzy sets (IFS) proposed by Krassimir T. Atanassov [1], [2] was the generalisation of the Fuzzy set (FS) introduced by Lofti A. Zadeh [3]. The object, partially belong to a set with a membership degree ($\mu$) between 0 and 1 are represented by the FS whereas the IFS represent the uncertainty with respect to both membership ($\mu \in [0,1]$) and non membership ($\vartheta \in [0,1]$) such that $\mu + \vartheta \leq 1$. Here, the number $\pi = 1 - \mu - \vartheta$ is called the hesitation degree or intuitionistic index.

The study of distance and similarity measure of IFSs gives lots of measures, each representing specific properties and behaviour in real-life decision making and pattern recognition works. For measuring the degree of similarity between vague sets, Chen and Tan [4] proposed two similarity measures. The Hamming, Euclidean distance and similarity measures were introduced by Smidt and Kacprzyk [5], [6], [7], [8]. The Geometric distance and similarity measures were given by Xu [9]. Most of the similarity measures reflect the degree of membership and non membership; Li et al [10] made a comparative study for similarity measures between IFSs and found the inadequate conditions for similarity measures. Therefore the hesitation degree was introduced for similarity measure. Zhang and Fu [11] proposed a new similarity measure for IFSs by considering the hesitation degree also. Later some modifications were made by Binyamin et al [12] on Zhang and Fu’s method for better results.

The Multi set [13] allows the repeated occurrences of any element and hence the Fuzzy Multi set (FMS) can occur more than once with the possibly of the same or the different membership values was introduced by R. R. Yager [14]. Recently, the new concept Intuitionistic Fuzzy Multi sets (IFMS) was proposed by T.K Shinoj and Sunil Jacob John [15] which allows the repeated occurrences of different membership and non membership functions.

As various distance and similarity methods of IFS are extended for IFMS distance and similarity measures [16], [17], [18], [19] and [20]. And in this paper we extend the Zhang and Fu’s measure of IFSs to IFMSs. The numerical results of the examples show that the developed similarity measures are well suited to use any linguistic variables. The organization of this paper is as follows: In section 2, the Fuzzy Multi sets, Intuitionistic Fuzzy Multi sets and similarity measures of IFMS are presented. The section 3 deals with the proposed Zhang and Fu’s measure for the IFMS. The
significance of this new measure along with the numerical evaluation is in the section 4. The section 5 and 6 are dedicated for the application of the similarity measure in medical diagnosis and pattern recognition.

II. PRELIMINARIES

Definition: 2.1

Let X be a nonempty set. A fuzzy set A in X is given by

\[ A = \{(x, \mu_A(x)) / x \in X\} \quad (2.1) \]

where \( \mu_A : X \rightarrow [0, 1] \) is the membership function of the fuzzy set A (i.e.) \( \mu_A(x) \in [0,1] \) is the membership of \( x \in X \) in A. The generalizations of fuzzy sets are the Intuitionistic fuzzy (IFS) set proposed by Atanassov [1], [2] is with independent memberships and non memberships.

Definition: 2.2

An Intuitionistic fuzzy set (IFS), A in X is given by

\[ A = \{(x, \mu_A(x), \sigma_A(x)) / x \in X\} \quad (2.2) \]

where \( \mu_A : X \rightarrow [0, 1] \) and \( \sigma_A : X \rightarrow [0,1] \) with the condition \( 0 \leq \mu_A(x) + \sigma_A(x) \leq 1 \), \( \forall x \in X \). Here \( \mu_A(x) \) and \( \sigma_A(x) \in [0,1] \) denote the membership and the non membership functions of the fuzzy set A; For each Intuitionistic fuzzy set in X, \( \pi_A(x) = 1 - \mu_A(x) - [1 - \mu_A(x)] = 0 \) for all \( x \in X \) that is \( \pi_A(x) = 1 - \mu_A(x) - \sigma_A(x) \) is the hesitancy degree of \( x \in X \) in A. Always \( 0 \leq \pi_A(x) \leq 1 \), \( \forall x \in X \).

The complementary set \( A^c \) of A is defined as \( A^c = \{(x, \sigma_A(x), \mu_A(x)) / x \in X\} \quad (2.3) \)

Definition: 2.3

Let X be a nonempty set. A Fuzzy Multi set (FMS) A in X is characterized by the count membership function Mc such that Mc : X \( \rightarrow \) Q where Q is the set of all crisp multi sets in \([0,1]\). Hence, for any \( x \in X \), Mc(x) is the crisp multi set from \([0,1]\). The membership sequence is defined as

\[ (\mu^1_A(x), \mu^2_A(x), \ldots, \mu^p_A(x)) \text{ where } \mu^1_A(x) \geq \mu^2_A(x) \geq \cdots \geq \mu^p_A(x). \]

Therefore, A FMS A is given by

\[ A = \{(x, \mu^1_A(x), \mu^2_A(x), \ldots, \mu^p_A(x)) / x \in X\} \quad (2.4) \]

Definition: 2.4

Let X be a nonempty set. A Intuitionistic Fuzzy Multi set (IFMS) A in X is characterized by two functions namely count membership function Mc and count non membership function NMc such that Mc : X \( \rightarrow \) Q and NMc : X \( \rightarrow \) Q where Q is the set of all crisp multi sets in \([0,1]\). Hence, for any \( x \in X \), Mc(x) is the crisp multi set from \([0,1]\) whose membership sequence is defined as \( (\mu^1_A(x), \mu^2_A(x), \ldots, \mu^p_A(x)) \) where \( \mu^1_A(x) \geq \mu^2_A(x) \geq \cdots \geq \mu^p_A(x) \) and the corresponding non membership sequence NMc(x) is defined as \( (\theta^1_A(x), \theta^2_A(x), \ldots, \theta^p_A(x)) \) where the non membership can be either decreasing or increasing function. such that \( 0 \leq \mu^i_A(x) + \theta^i_A(x) \leq 1 \), \( \forall x \in X \) and \( i = 1,2, \ldots, p \). Therefore,

An IFMS A is given by

\[ A = \{(x, \mu^1_A(x), \mu^2_A(x), \ldots, \mu^p_A(x)), \ (\theta^1_A(x), \theta^2_A(x), \ldots, \theta^p_A(x)) / x \in X\} \quad (2.5) \]

where \( \mu^1_A(x) \geq \mu^2_A(x) \geq \cdots \geq \mu^p_A(x) \). The complementary set \( A^c \) of A is defined as

\[ A^c = \{(x, \theta^1_A(x), \theta^2_A(x), \ldots, \theta^p_A(x)), \ (\mu^1_A(x), \mu^2_A(x), \ldots, \mu^p_A(x)) / x \in X\} \quad (2.6) \]
Definition: 2.5

The **Cardinality** of the membership function $M_c(x)$ and the non membership function $N_{M_c}(x)$ is the length of an element $x$ in an **IFMS** $A$ denoted as $\eta$, defined as $\eta = |M_c(x)| = |N_{M_c}(x)|$

If $A, B, C$ are the **IFMS** defined on $X$, then their cardinality $\eta = \text{Max}\{ \eta(A), \eta(B), \eta(C) \}$.

Definition: 2.6

$\text{Sim} (A, B)$ is said to be the **similarity measure** between $A$ and $B$, where $A, B \in X$ and $X$ is an **IFMS**, as $\text{Sim} (A, B)$ satisfies the following properties

1. $\text{Sim} (A, B) \in [0,1]$
2. $\text{Sim} (A, B) = 1$ if and only if $A = B$
3. $\text{Sim} (A, B) = \text{Sim} (B, A)$

**ZHANG AND FU’S SIMILARITY MEASURE OF IFSs**

The similarity measure of **IFSs** proposed by **Zhang and Fu’s** was as follows

$$\text{Sim}_{ZF} (A, B) = 1 - \frac{1}{2n} \sum_{i=1}^{n} \{ |\mu_A(x_i) - \mu_B(x_i)| + |(1 - \vartheta_A(x_i)) - (1 - \vartheta_B(x_i))| \}$$

consisting of the membership and non membership functions.

Later Zhang and Fu’s new similarity measure was

$$\text{Sim}_{ZF} (A, B) = 1 - \frac{1}{2n} \sum_{i=1}^{n} \{ |\delta_A - \delta_B| + |\alpha_A - \alpha_B| \}$$

where $\delta_A(x_i) = \mu_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \mu_A(x_i)$ and

$\alpha_A(x_i) = \vartheta_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \vartheta_A(x_i)$

And if there are three parameters like membership, non membership and hesitation function then the modified Zhang and Fu’s similarity measure of **Binyamin et al [12]** for **IFSs** becomes

$$\text{Sim}_{modified} \text{ZF} (A, B) = 1 - \frac{1}{2n} \sum_{i=1}^{n} \{ |\delta_A - \delta_B| + |\alpha_A - \alpha_B| + |\beta_A - \beta_B| \}$$

where $\delta_A(x_i) = \mu_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \mu_A(x_i)$

$\alpha_A(x_i) = \vartheta_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \vartheta_A(x_i)$ and $\beta_A(x_i) = (1 - \delta_A(x_i) - \alpha_A(x_i))$

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III. PROPOSED ZHANG AND FU’S SIMILARITY MEASURES FOR INTUITIONISTIC MULTI FUZZY SETS

In IFS, the similarity measures are considered for the membership and non-membership functions only once. But in IFMS, it should be considered more than once because of their multi-membership and non-membership functions. And, their considerations are combined together by means of Summation concept based on their cardinality.

Definition: 3.1

\[ IFMS_{ZF1}(A,B) = \frac{1}{n} \sum_{j=1}^{n} \left[ 1 - \frac{1}{2n} \sum_{i=1}^{n} \left( |\mu_{A}^{j}(x_i) - \mu_{B}^{j}(x_i)| + |(1-\theta_{A}^{j}(x_i)) - (1-\theta_{B}^{j}(x_i))| \right) \right] \]

of the membership and non-membership functions. Also the new similarity measure becomes

\[ IFMS_{ZF2}(A,B) = \frac{1}{n} \sum_{j=1}^{n} \left[ 1 - \frac{1}{2n} \sum_{i=1}^{n} \left( |\delta_{A}^{j}(x_i) - \delta_{B}^{j}(x_i)| + |\alpha_{A}^{j} - \alpha_{B}^{j}| \right) \right] \]

Where \( \delta_{A}^{j}(x_i) = \mu_{A}^{j}(x_i) + (1 - \mu_{A}^{j}(x_i) - \theta_{A}^{j}(x_i)) \mu_{B}^{j}(x_i) \) and

\( \alpha_{A}^{j}(x_i) = \theta_{A}^{j}(x_i) + (1 - \mu_{A}^{j}(x_i) - \theta_{A}^{j}(x_i)) \theta_{B}^{j}(x_i) \)

And if there are three parameters like membership, non-membership, and hesitation function then the IFMS new similarity measure becomes

\[ IFMS_{ZF}(A,B) = \frac{1}{n} \sum_{j=1}^{n} \left[ 1 - \frac{1}{2n} \sum_{i=1}^{n} \left( |\delta_{A}^{j}(x_i) - \delta_{B}^{j}(x_i)| + |\alpha_{A}^{j} - \alpha_{B}^{j}| + |\beta_{A}^{j} - \beta_{B}^{j}| \right) \right] \]

Where \( \delta_{A}^{j}(x_i) = \mu_{A}^{j}(x_i) + (1 - \mu_{A}^{j}(x_i) - \theta_{A}^{j}(x_i)) \mu_{B}^{j}(x_i) \)

\( \alpha_{A}^{j}(x_i) = \theta_{A}^{j}(x_i) + (1 - \mu_{A}^{j}(x_i) - \theta_{A}^{j}(x_i)) \theta_{B}^{j}(x_i) \)

\( \beta_{A}^{j}(x_i) = (1 - \delta_{A}^{j}(x_i) - \alpha_{B}^{j}(x_i)) \)

PROPOSITION: 3.2

The defined similarity measure \( IFMS(A,B) \) between IFMS \( A \) and \( B \) satisfies the following properties

D1. \( 0 \leq IFMS_{ZF}(A,B) \leq 1 \)

D2. \( A = B \) if and only if \( IFMS_{ZF}(A,B) = 1 \)

D3. \( IFMS_{ZF}(A,B) = IFMS_{ZF}(B,A) \)

Proof

D1. \( 0 \leq IFMS_{ZF}(A,B) \leq 1 \)
As the membership and the non membership functions of the IFMSs lies between 0 and 1, the similarity measure based on Zhang and Fu’s function also lies between 0 and 1.

D2. \( A = B \) if and only if \( IFMS_{ZF}(A, B) = 1 \)

(i) Let the two IFMS A and B be equal (i.e.) \( A = B \). This implies for any \( \mu_A^j(x_i) = \mu_B^j(x_i) \) and \( \theta_A^j(x_i) = \theta_B^j(x_i) \) which states that \( |\mu_A^j(x_i) - \mu_B^j(x_i)| \) and \( |\theta_A^j(x_i) - \theta_B^j(x_i)| = 0 \). Hence \( IFMS_{ZF}(A, B) = 1 \)

(ii) Let the \( IFMS_{ZF}(A, B) = 1 \)

The unit measure is possible only if both \( |\mu_A^j(x_i) - \mu_B^j(x_i)| \) and \( |\theta_A^j(x_i) - \theta_B^j(x_i)| = 0 \), which refers that \( \mu_A^j(x_i) = \mu_B^j(x_i) \) and \( \theta_A^j(x_i) = \theta_B^j(x_i) \) for all \( i, j \) values. Hence \( A = B \).

D3. \( IFMS_{ZF}(A, B) = IFMS_{ZF}(B, A) \)

It is obvious that \( \mu_A^j(x_i) - \mu_B^j(x_i) \neq \mu_A^j(x_i) - \mu_A^j(x_i) \) and \( \theta_A^j(x_i) - \theta_B^j(x_i) \neq \theta_A^j(x_i) - \theta_A^j(x_i) \)

But \( |\mu_A^j(x_i) - \mu_B^j(x_i)| = |\mu_A^j(x_i) - \mu_A^j(x_i)| \) and \( |\theta_A^j(x_i) - \theta_B^j(x_i)| = |\theta_A^j(x_i) - \theta_A^j(x_i)| \)

Hence \( IFMS_{ZF}(A, B) = \frac{1}{\eta} \sum_{i=1}^{n} \left[ 1 - \frac{1}{2n} \sum_{i=1}^{n} \left( |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| + |\beta_A^j - \beta_B^j| \right) \right] \)

\[ = \frac{1}{\eta} \sum_{i=1}^{n} \left[ 1 - \frac{1}{2n} \sum_{i=1}^{n} \left( |\delta_B^j - \delta_A^j| + |\alpha_B^j - \alpha_A^j| + |\beta_B^j - \beta_A^j| \right) \right] = IFMS_{ZF}(B, A) \]

IV. SIGNIFICANCE OF THE PROPOSED SIMILARITY MEASURE

EXAMPLE : 4.1

Let \( X = \{ A_1, A_2, A_3, A_4, \ldots \} \) with \( Y = \{ A_1, A_{10} \} \) and \( Z = \{ A_1, A_{10} \} \) are the IFMS defined as

\( Y = \{ (A_1 : (0.1,0.2)), (A_{10} : (0.2,0.3)) \} \) and the Pattern \( Z = \{ (A_1 : (0.1,0.2)), (A_{10} : (0.2,0.3)) \} \)

Here, the cardinality \( \eta = 2 \) as \( |Mc(Y)| = |NmMc(Y)| = 2 \) and \( |Mc(Z)| = |NmMc(Z)| = 2 \), and the new modified similarity measure between the patterns \( (Y, Z) \)

\[ = \frac{1}{2} \sum_{i=1}^{2} \left[ 1 - \frac{1}{2(2)} \sum_{i=1}^{2} \left( |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| + |\beta_A^j - \beta_B^j| \right) \right] = 1 \]

Thus the similarity measure of any two IFMSs equals to one if and only if the two IFMSs are the same.

EXAMPLE : 4.2

Let \( X = \{ A_1, A_2, A_3, A_4, \ldots \} \) with \( A = \{ A_1, A_2 \} \), \( B = \{ A_1, A_{10} \} \) and \( C = \{ A_1, A_4 \} \) are the IFMS defined as

\( A = \{ (A_1 : (0.1,0.2)), (A_2 : (0.3,0.3)) \} \), \( B = \{ (A_9 : (0.1,0.2)), (A_{10} : (0.2,0.3)) \} \) and \( C = \{ (A_9 : (0.1,0.2)), (A_{10} : (0.2,0.2)) \} \)

Here, the cardinality \( \eta = 2 \) as \( |Mc(A)| = |NmMc(A)| = 2 \) and \( |Mc(B)| = |NmMc(B)| = 2 \). Also \( |Mc(C)| = |NmMc(C)| = 2 \), then
As Medical diagnosis contains lots of uncertainties, they are the most interesting and fruitful areas of application for Intuitionistic fuzzy set theory. Due to the increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different membership and non membership functions. The proposed similarity measure among the Patients Vs Symptoms and Symptoms Vs diseases gives the proper medical diagnosis. The unique feature of this proposed method is that it considers multi membership and non membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis.
Let \( P = \{ P_1, P_2, P_3, P_4 \} \) be a set of Patients. \( D = \{ \text{Fever, Tuberculosis, Typhoid, Throat disease} \} \) be the set of diseases and \( S = \{ \text{Temperature, Cough, Throat pain, Headache, Body pain} \} \) be the set of symptoms.

Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different membership and non membership function for each patient.

### TABLE : 5.1 – IFMs Q : The Relation between Patient and Symptoms

<table>
<thead>
<tr>
<th>( P )</th>
<th>Temperature</th>
<th>Cough</th>
<th>Throat Pain</th>
<th>Head Ache</th>
<th>Body Pain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>(0.6, 0.2)</td>
<td>(0.4, 0.3)</td>
<td>(0.1, 0.7)</td>
<td>(0.5, 0.4)</td>
<td>(0.2, 0.6)</td>
</tr>
<tr>
<td></td>
<td>(0.7, 0.1)</td>
<td>(0.3, 0.6)</td>
<td>(0.2, 0.7)</td>
<td>(0.6, 0.3)</td>
<td>(0.3, 0.4)</td>
</tr>
<tr>
<td></td>
<td>(0.5, 0.4)</td>
<td>(0.4, 0.4)</td>
<td>(0.8)</td>
<td>(0.7, 0.2)</td>
<td>(0.4, 0.4)</td>
</tr>
</tbody>
</table>

| \( P_2 \) | (0.4, 0.5) | (0.7, 0.2) | (0.6, 0.3) | (0.3, 0.7) | (0.8, 0.1) |
| | (0.3, 0.4) | (0.6, 0.2) | (0.5, 0.3) | (0.6, 0.3) | (0.7, 0.2) |
| | (0.5, 0.4) | (0.8, 0.1) | (0.4, 0.4) | (0.2, 0.7) | (0.5, 0.3) |

| \( P_3 \) | (0.1, 0.7) | (0.3, 0.6) | (0.8) | (0.3, 0.6) | (0.4, 0.4) |
| | (0.2, 0.6) | (0.2, 0) | (0.7, 0.1) | (0.2, 0.7) | (0.3, 0.7) |
| | (0.1, 0.9) | (0.1, 0.7) | (0.8, 0.1) | (0.2, 0.6) | (0.2, 0.7) |

Let the samples be taken at three different timings in a day (morning, noon and night)

### TABLE : 5.2 – IFMs R : The Relation among Symptoms and Diseases

<table>
<thead>
<tr>
<th>( R )</th>
<th>Viral Fever</th>
<th>Tuberculosis</th>
<th>Typhoid</th>
<th>Throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>(0.8, 0.1)</td>
<td>(0.2, 0.7)</td>
<td>(0.5, 0.3)</td>
<td>(0.1, 0.7)</td>
</tr>
<tr>
<td>Cough</td>
<td>(0.2, 0.7)</td>
<td>(0.9, 0)</td>
<td>(0.3, 0.5)</td>
<td>(0.3, 0.6)</td>
</tr>
<tr>
<td>Throat Pain</td>
<td>(0.3, 0.5)</td>
<td>(0.7, 0.2)</td>
<td>(0.2, 0.7)</td>
<td>(0.8, 0.1)</td>
</tr>
<tr>
<td>Head ache</td>
<td>(0.5, 0.3)</td>
<td>(0.6, 0.3)</td>
<td>(0.2, 0.6)</td>
<td>(0.1, 0.8)</td>
</tr>
<tr>
<td>Body ache</td>
<td>(0.5, 0.4)</td>
<td>(0.7, 0.2)</td>
<td>(0.4, 0.4)</td>
<td>(0.1, 0.8)</td>
</tr>
</tbody>
</table>

### TABLE : 5.3 – The Similarity Measure between IFMs Q and R :

<table>
<thead>
<tr>
<th>Zhang And Fu’s Similarity measure</th>
<th>Viral Fever</th>
<th>Tuberculosis</th>
<th>Typhoid</th>
<th>Throat disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.7473</td>
<td>0.5650</td>
<td><strong>0.8157</strong></td>
<td>0.5287</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>0.6580</td>
<td><strong>0.7977</strong></td>
<td>0.7167</td>
<td>0.5880</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>0.5827</td>
<td>0.6413</td>
<td>0.6553</td>
<td><strong>0.8407</strong></td>
</tr>
</tbody>
</table>

The highest similarity measure from the table 4.3 gives the proper medical diagnosis.

The diagnosis refers that Patient \( P_1 \) suffers from Typhoid, Patient \( P_2 \) suffers from Tuberculosis and Patient \( P_3 \) suffers from Throat disease.
VI. PATTERN RECOGNITION OF THE PROPOSED SIMILARITY MEASURE

PATTERN RECOGNITION: 6.1

Let $X = \{A_1, A_2, A_3, \ldots, A_n\}$ with $A = \{A_1, A_2, A_3, A_4, A_5\}$ and $B = \{A_2, A_3, A_6, A_9\}$ are the IFMS defined as

Pattern I = \{ \begin{align*}
& (A_1 : (0.6,0.4), (0.5, 0.5)) , (A_2 : (0.5, 0.3), (0.4, 0.5)) , (A_3 : (0.5, 0.2), (0.4, 0.4)) , \\
& (A_4 : (0.3, 0.2), (0.3, 0.2)) , (A_5 : (0.2, 0.1), (0.2, 0.2)) \}
\end{align*} \}

Pattern II = \{ \begin{align*}
& (A_2 : (0.5, 0.3), (0.4, 0.5)) , (A_5 : (0.2, 0.1), (0.2, 0.2)) , (A_7 : (0.7, 0.3), (0.4, 0.2)) , \\
& (A_8 : (0.4, 0.5), (0.3, 0.3)) , (A_9 : (0.2, 0.7), (0.1, 0.8)) \}
\end{align*} \}

Then the testing IFMS Pattern III be \(A_6, A_7, A_8, A_9, A_{10}\) such that \(\{ (A_6 : (0.8, 0.1), (0.4, 0.6)) , \\
(A_7 : (0.7, 0.3), (0.4, 0.2)) , (A_8 : (0.4, 0.5), (0.3, 0.3)) , (A_9 : (0.2, 0.7), (0.1, 0.8)) , (A_{10} : (0.2, 0.6), (0.0, 0.6)) \}\)

Here, the cardinality $\eta = 5$ as \(|Mc(A)| = |NM_{mc}(A)| = 5\) and \(|Mc(B)| = |NM_{mc}(B)| = 5\), then the Similarity measure between Pattern (I, III) is 0.681, Pattern (II, III) is 0.770.

The testing Pattern III belongs to Pattern II type

PATTERN RECOGNITION: 6.2

Let $X = \{A_1, A_2, A_3, \ldots, A_n\}$ with $X1 = \{A_1, A_2\}$; $X2 = \{A_3, A_4\}$; $X3 = \{A_1, A_4\}$ are the IFMS defined as

$A = \{ \begin{align*}
& (A_1 : (0.4, 0.2), (0.3, 0.1), (0.2, 0.1), (0.1, 0.4)) , \\
& (A_2 : (0.6, 0.3), (0.4, 0.5), (0.4, 0.3), (0.2, 0.6)) \}
\end{align*} \}

$B = \{ \begin{align*}
& (A_3 : (0.5, 0.2), (0.4, 0.2), (0.4, 0.1), (0.1, 0.1)) , \\
& (A_4 : (0.4, 0.6), (0.4, 0.5), (0.3, 0.4), (0.2, 0.4)) \}
\end{align*} \}

$C = \{ \begin{align*}
& (A_4 : (0.4, 0.2), (0.3, 0.1), (0.2, 0.1), (0.1, 0.4)) , \\
& (A_5 : (0.4, 0.6), (0.4, 0.5), (0.3, 0.4), (0.2, 0.4)) \}
\end{align*} \}

then the Pattern D of IFMS referred as \(\{ \begin{align*}
& (A_2 : (0.4, 0.6), (0.4, 0.5), (0.3, 0.4), (0.2, 0.4)) , \\
& (A_6 : (0.4, 0.2), (0.5, 0.5), (0.2, 0.4), (0.2, 0.5)) \}
\end{align*} \}

The cardinality $\eta = 2$ as \(|Mc(A)| = |NM_{mc}(A)| = |Hc(A)| = 2\) and \(|Mc(B)| = |NM_{mc}(B)| = |Hc(B)| = 2\), then the Proposed Similarity measure between the Pattern (A, D) is 0.7857; the pattern (B, D) is 0.7425 and the Pattern (C, D) is 0.775.

Hence, the testing Pattern D belongs to Pattern A type

VII. CONCLUSION

A new similarity measure of IFMS from IFS theory is derived. The prominent characteristic of this method is that it considers multi membership, non membership, hesitation functions and this similarity measure guarantee that the similarity measure of any two IFMS equals to one if and only if the two IFMSs are the same referred in example 4.1. Also, the example 4.2 shows that the new modified measure perform well in the case of three representatives of IFMS - membership, non membership and hesitation functions than the proposed measure with two representatives of IFMS – membership and non membership functions. Finally, this novel similarity method is applied to medical diagnosis and pattern recognitions problems.

REFERENCES


