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ANALYSIS OF ACCELERATED LIFE TESTING USING GEOMETRIC PROCESS FOR FRECHET DISTRIBUTION

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Abstract: In Accelerated life testing (ALT), a log-linear relationship between life and stress is used. However instead of original parameters, transformed parameters are used for estimation which loose originality due to various assumptions. In this paper the geometric process is used for the analysis of ALT under constant stress for the Frechet Distribution. The maximum likelihood procedure is followed to estimate the parameters of the model and also evaluate the performance of confidence intervals through simulation.

Keywords: Maximum Likelihood Estimation; Reliability Function; Fisher Information Matrix; Asymptotic Confidence Interval; Simulation.

I. INTRODUCTION

Accelerated life testing (ALT) is the most common way to assess a product's life and gives desired information on its life under normal use. It is widely used in industry for improving the performance of products. It allows the experimenter to apply more severe stresses on the parameters of lifetime distributions to obtain information quickly than would be possible under normal operating conditions. Under such test settings, products are tested at higher-than usual levels of stress to induce early failure as operating in normal conditions require large number of units and hence more time is consumed which is not feasible. The goal of accelerated life analysis is to utilize the test data to extrapolate a product's life distribution and its associated parameters at a normal stress level. Such a test saves time and expenses compared to tests at normal conditions.

The most difficult task in Accelerated life testing is to choose what stress is to be applied and in what manner. There are various types of Stress loading in ALT which includes constant, cyclic, step, progressive and random stress. Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some failure data are missing).

Many methodologies have been developed related to accelerated life testing and are widely used in industry. See Yurkowski, Schafer and Finkelstein[1], Nelson[2], Meeker[3], Viertl[4]. Lawless[5], Cox and Oakes[6], Lipson and Sheth[7], Viertl[4] have recent book chapters on ALT methodologies.

In the current study, we discuss the application of constant stress in accelerated life testing. Constant-stress ALT for two parameter Birnbaum-Saunders (BS) distribution is introduced by Owen[8]. Reliability bounds and critical time for the Birnbaum-Saunders distribution, see Chang[9], Dupuis and Mills[10], Xu and Tang[11,12]. Work based on censored data includes Rieck[13], Jeng[14], Ng et al. M[15] and Wang et al[16] . Yang[17] proposed an optimal design of 4-level constant-stress ALT plans considering different censoring times. Chen et al.[18] discuss the optimal design of multiple stresses constant accelerated life test plan on non-rectangle test region. Fan and Yu[19] discuss the reliability analysis of the constant stress accelerated life tests when a parameter in the generalized gamma lifetime distribution is linear in the stress level.

The concept of geometric process (GP) was first introduced by Lam in 1988[20, 21] when he studied the problem of repair replacement. The statistical inference for geometric process is studied in both nonparametric and parametric ways. Lam and Zhang[22] introduced the geometric process model in the analysis of a two-component series system with one repairman. Huang[23] utilizes the geometric process in the analysis of accelerated life test with complete and censored exponential samples under the constant stress. Chan, Lam, and Leung[31] estimated the parameters of the GP

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Vol. 2, Issue 9, September 2013

with Gamma distribution by parametric methods including maximum likelihood method along with some nonparametric methods previously proposed by Lam. Chen[32] provided the Bayesian approach to the estimation of parameters in a GP with several popular life distributions including the exponential and lognormal distributions. Large amount of studies in maintenance problems and system reliability have shown that a geometric process model is a good and simple model for analysis of data with a single trend or with multiple trends. So far, there is no study that utilizes the geometric process in the analysis of accelerated life test with censored data.

In this paper, the geometric process is used for the Frechet distribution for constant stress with complete data. Constant stress is most simple and is also verified for some materials and products. It is also easy to maintain constant stress.

II. THE MODEL AND TEST PROCEDURE

A. *The geometric process*

A geometric process is a simple monotone process that was first introduced in 1988. It is a generalization of renewal process. It describe a stochastic process which is a sequence of independent non-negative random variables { X_K, k=1, 2,… } such that the distribution function of X_n is $\left(\lambda^{k-1}X_k\right)$, where a is a positive constant and $\left\{\lambda^{k-1}X_K\right\}$ forms a renewal process. If λ>1, it is a decreasing geometric process and if λ<1, it is an increasing geometric process. Then we consider a replacement model, as follows: the successive survival times of the system after repair form a decreasing geometric process or a renewal process while the consecutive repair times of the system constitute an increasing geometric process.

B. *The Frechet distribution*

The probability density function (pdf) of a two parameter Frechet distribution is given by:

$$f(x/\alpha,\theta)=\alpha\theta^{\alpha}(x)^{-\alpha-1}e^{-\left(\frac{x}{\theta}\right)^{-\alpha}} \quad x>0,\alpha>0,\theta>0 \tag{2.1}$$

Where α>0 is the shape parameter and θ>0 is a scale parameter.

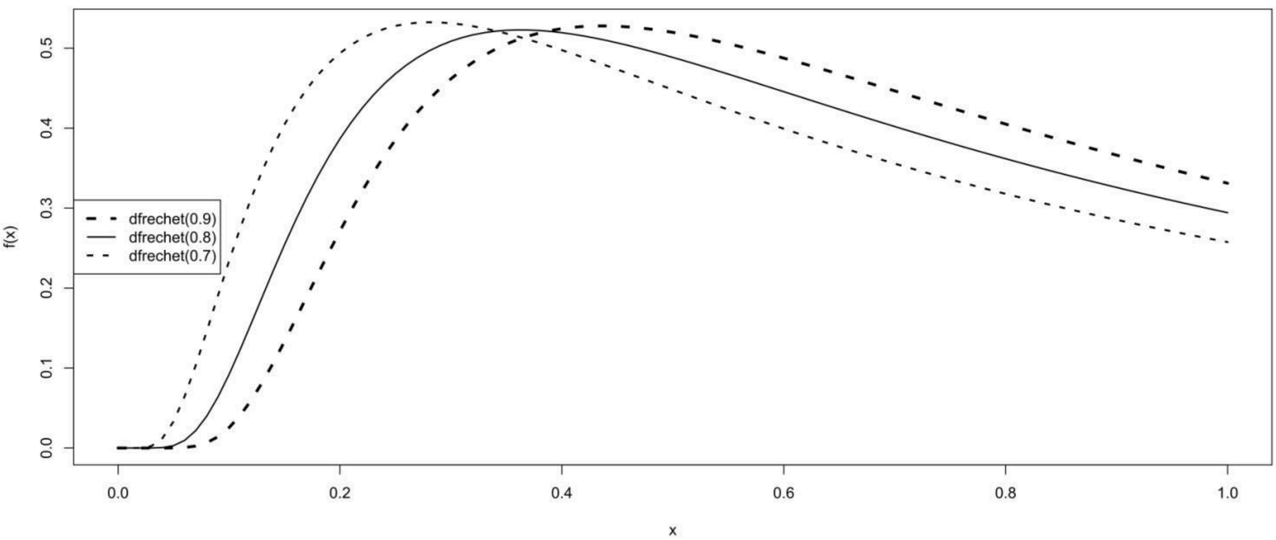

Fig. 1: Probability density function with different shape parameters

It is clear from Figure 1 that probability density function of the Frechet distribution can take different shapes.

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(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 9, September 2013

Under any constant stress the failure time of a test unit follows a Frechet distribution with distribution function is given by

$$F(x/\alpha,\theta)=e^{-\left(\frac{x}{\theta}\right)^{-\alpha}} \qquad x>0 \qquad (2.2)$$

The survival function of the Frechet distribution takes the following form:

$$S(x/\alpha,\theta)=1-e^{-\left(\frac{x}{\theta}\right)^{-\alpha}} \qquad x>0 \qquad (2.3)$$

Where α>0 is a shape parameter and θ>0 is scale parameter.

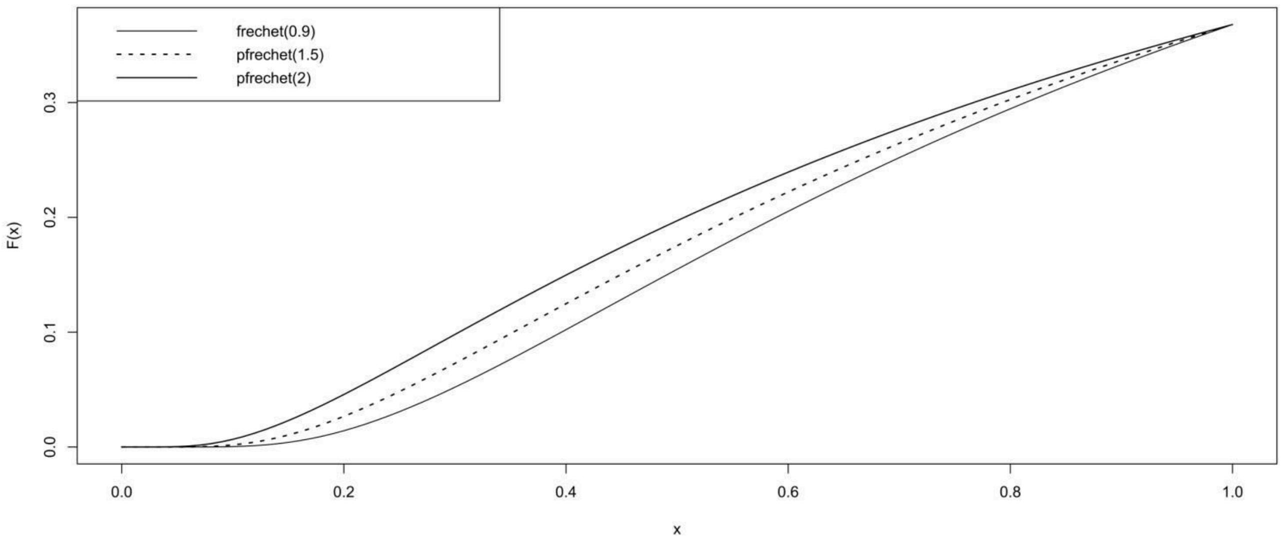

Fig. 2: Cumulative distribution function with different shape parameters

C. *Assumptions and Test procedure*

The following assumptions are made:

1. Under any stress, the lifetime of test unit follows a Frechet distribution.
2. The scale parameter is a log-linear function of stress i.e. $\theta_i = e^{a+bS_i}$ where a and b are unknown parameter.
3. Suppose that an accelerated life test under $z_k, k=1,2....s$, arithmetically increasing stress levels is performed. A random sample of $N_i, i=1,2,3...n$ identical items is placed under each stress level which start to operate at the same time. Whenever an item fails, it is removed from the test and its observed failure time x_{ki} is recorded.
4. The shape parameter α is constant.
5. Let random variables $X_0, X_1, X_2,......X_n$ denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily and sequence $\{X_k, k=0,1,.......,s\}$ forms a geometric process with ratio λ>0.

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(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 9, September 2013

The last assumption is the basic extension of this paper and the above 4 are commonly used in discussion of ALT. Based on the definition of geometric process, if density function X_0 is $f(x)$, then the probability density function of X_K will be given by $\lambda^k f(\lambda^k x)$.

Therefore, the probability density function (pdf) and cumulative distribution function of a product lifetime at the k^{th} stress level are

$$f_{x_k}(x/\alpha,\theta,\lambda) = \lambda^k \alpha\theta^\alpha (\lambda^k x)^{-\alpha-1} e^{-\left(\frac{\lambda^k x}{\theta}\right)^{-\alpha}} \qquad x,\alpha,\theta,\lambda > 0 \tag{2.4}$$

$$F(x/\alpha,\theta) = e^{-\left(\frac{\lambda^k x}{\theta}\right)^{-\alpha}} \qquad x,\alpha,\theta,\lambda > 0 \tag{2.5}$$

The above equation states that lifetime under sequence of increasing stress level form a geometric process with ratio λ.

III. MAXIMUM LIKELIHOOD ESTIMATION

Many methods for estimating parameters exist but mainly maximum likelihood is used as it is easy to handle and gives the estimates of parameter with good statistical properties.
The likelihood function for the Frechet geometric process model based on the observed data in a total stress levels in accelerated life testing, can be written as

$$L(\alpha,\theta,\lambda) = \prod_{k=1}^{s}\prod_{i=1}^{n} \lambda^k \alpha\theta^\alpha (\lambda^k x_{ki})^{-\alpha-1} e^{-\left(\frac{\lambda^k x_{ki}}{\theta}\right)^{-\alpha}} \tag{3.1}$$

Taking Log on both sides we get

$$l = \sum\sum[k\log\lambda + \log\alpha + \alpha\log\theta - (\alpha+1)\{\log\lambda^k + \log x_{ki}\} - \left(\frac{\lambda^k x_{ki}}{\theta}\right)^{-\alpha}] \tag{3.2}$$

Partial derivative w.r.t. λ, α and θ is given by

$$\frac{\partial l}{\partial\alpha} = \frac{ns}{\alpha} + ns\ln\theta - n\ln\theta - n\sum_{k=1}^{s}\log\lambda^k - \sum_{k=1}^{s}\sum_{i=1}^{n}\log x_{ki} - \sum_{k=1}^{s}\sum_{i=1}^{n}\left(\frac{\lambda^k x_{ki}}{\theta}\right)^{-\alpha}\log\left(\frac{\lambda^k x_{ki}}{\theta}\right) \tag{3.3}$$

$$\frac{\partial l}{\partial\theta} = \frac{ns\alpha}{\theta} - \alpha\theta^{\alpha-1}\sum_{k=1}^{s}\sum_{i=1}^{n}(\lambda^k x_{ki})^{-\alpha} \tag{3.4}$$

$$\frac{\partial l}{\partial\lambda} = \sum_{k=1}^{s}\frac{nk}{\lambda} - n(\alpha+1)\sum_{k=1}^{s}\frac{k}{\lambda} + \sum_{k=1}^{s}\sum_{i=1}^{n}\frac{\alpha k}{\lambda^{\alpha k+1}}\left(\frac{x_{ki}}{\theta}\right)^{-\alpha} \tag{3.5}$$

MLE's of α, θ and λ can be obtained by solving the equations $\frac{\partial l}{\partial\alpha} = 0$, $\frac{\partial l}{\partial\theta} = 0$ and $\frac{\partial l}{\partial\lambda} = 0$

Since the non -linear equations (3.3), (3.4) and (3.5) are in closed form, a numerical method, such as Newton-Rasphson method must be used to solve the equations. Thus once the values of α and θ are obtained, an estimate of λ can be obtained from equation (3.5).

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Vol. 2, Issue 9, September 2013

IV. FISHER INFORMATION MATRIX & ASYMPTOTIC CONFIDENCE INTERVAL

A. *Fisher Information Matrix*

The Fisher information matrix is obtained by taking the expected value of second and mixed partial derivatives of log(L) with respect to α, θ and λ. Unfortunately the exact mathematical expression for the expectation is very difficult to find. So, it can be approximated to the negative of second and mixed derivatives of the natural logarithm of the likelihood function evaluated at the MLE. The asymptotic Fisher Information matrix is given by:

$$F=\begin{bmatrix} -\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\theta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2 l}{\partial\alpha\partial\theta} & -\frac{\partial^2 l}{\partial\theta^2} & -\frac{\partial^2 l}{\partial\theta\partial\lambda} \\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\theta} & -\frac{\partial^2 l}{\partial\lambda^2} \end{bmatrix} \tag{4.1}$$

Elements of Fisher Information Matrix are as follows:

$$\frac{\partial^2 l}{\partial\alpha^2}=-\frac{ns}{\alpha^2}-\sum_{k=1}^{s}\sum_{i=1}^{n}\left(\frac{\lambda^k x_{ki}}{\theta}\right)^{-2\alpha}\left(\log\left(\frac{\lambda^k x_{ki}}{\theta}\right)\right)^2 \tag{4.2}$$

$$\frac{\partial^2 l}{\partial\theta^2}=-\frac{ns\alpha}{\theta^2}+\alpha(\alpha-1)\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{\theta^{\alpha-2}}{(\lambda^k x_{ki})^{\alpha}} \tag{4.3}$$

$$\frac{\partial^2 l}{\partial\lambda^2}=-\frac{n}{\lambda^2}\sum_{k=1}^{s}k+n(\alpha+1)\sum_{k=1}^{s}\frac{k}{\lambda}-\alpha\theta^{\alpha}\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{kx_{ki}^{-\alpha}(\alpha k+1)}{\lambda^{\alpha k+2}} \tag{4.4}$$

$$\frac{\partial^2 l}{\partial\theta\partial\alpha}=\frac{\partial^2 l}{\partial\alpha\partial\theta}=\frac{ns}{\theta}+\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{\alpha\theta^{\alpha-1}}{(\lambda^k x_{ki})^{\alpha}}\log\left(\frac{\lambda^k x_{ki}}{\theta}\right)-\frac{1}{\theta}\left(\frac{\lambda^k x_{ki}}{\theta}\right)^{-\alpha} \tag{4.5}$$

$$\frac{\partial^2 l}{\partial\lambda\partial\alpha}=\frac{\partial^2 l}{\partial\alpha\partial\lambda}=-\sum_{k=1}^{s}\frac{kn\lambda^{k-1}}{\lambda^k}-\sum_{k=1}^{s}\sum_{i=1}^{n}\left(\frac{x_{ki}}{\theta}\right)^{-\alpha}\frac{\alpha k}{\lambda^{\alpha k+1}}\log\left(\frac{\lambda^k x_{ki}}{\theta}\right)+\sum_{k=1}^{s}\sum_{i=1}^{n}\frac{k}{\lambda}\left(\frac{\lambda^k x_{ki}}{\theta}\right)^{-\alpha} \tag{4.6}$$

$$\frac{\partial^2 l}{\partial\theta\partial\lambda}=\frac{\partial^2 l}{\partial\lambda\partial\theta}=\frac{\alpha^2\theta^{\alpha-1}}{\lambda}\sum_{k=1}^{s}\sum_{i=1}^{n}\left(\frac{1}{x_{ki}\lambda^k}\right)^{\alpha} \tag{4.7}$$

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(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 9, September 2013

Now the variance-covariance matrix can be written as

$$F=\begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \theta} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 l}{\partial \alpha \partial \theta} & -\frac{\partial^2 l}{\partial \theta^2} & -\frac{\partial^2 l}{\partial \theta \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda \partial \theta} & -\frac{\partial^2 l}{\partial \lambda^2} \end{bmatrix}^{-1} = \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\theta}) & ACov(\hat{\alpha}\hat{\lambda}) \\ ACov(\hat{\theta}\hat{\alpha}) & AVar(\hat{\theta}) & ACov(\hat{\theta}\hat{\lambda}) \\ ACov(\hat{\lambda}\hat{\alpha}) & ACov(\hat{\lambda}\hat{\theta}) & AVar(\hat{\lambda}) \end{bmatrix} \quad (4.8)$$

B. *Confidence Interval*

One of the most confusing concepts to a novice reliability engineer is estimating the precision of an estimate. The exact distributions of MLEs of the unknown parameters $(\alpha, \theta, \lambda)$ cannot be obtained due to complexity which leads to the use of confidence intervals. The asymptotic confidence interval for α, Θ and λ are given by following equations:

$\left[\hat{\alpha} \pm Z_{1-\frac{\phi}{2}}\left(SE(\hat{\alpha})\right)\right]$, $\left[\hat{\theta} \pm Z_{1-\frac{\phi}{2}}\left(SE(\hat{\theta})\right)\right]$ and $\left[\hat{\lambda} \pm Z_{1-\frac{\phi}{2}}\left(SE(\hat{\lambda})\right)\right]$

V. SIMULATION STUDY

Simulation of data is the initial task for studying different properties of parameters. A simulation is an attempt to model a hypothetical situation to study how a function works. By changing variables in the simulation, predictions may be made about the behaviour of the function. To conduct a simulation study first a sample is generated for a Uniform distribution from R function runif() and then by transformation of equation(2.5), x_{ki} k=1, 2, 3,..............s, i=1, 2, 3,...............n is obtained for n=20, 60, 80, 120 & 200. The initial values of the parameters are chosen to be α=0.2, θ=8 and λ=10, and number of stress levels are chosen as s=2 and 4. By using maxNR() function of R we can calculate functional value, ML estimates, gradient and Hessian. The functions maxNR, maxBFGSR, and maxBHHH can work with constant parameters and related changes of parameter values. Constant parameters are useful if a parameter value is converging toward the boundary of support, or for testing. One way is to put fixed to non-NULL, specifying which parameters should be treated as constants

The performance of the estimates can be evaluated through some measures of accuracy which are mean squared error (MSE) and mean absolute error (MAE). The comparison criteria we used are described as follows:

A. *Mean Square Error*

The mean square error (MSE) measures the deviation between the predicted values with the actual observations, and is defined as:

$$MSE = \frac{\sum_{i=1}^{n}(m(t_i)-m_i)^2}{n}$$

Where n is the number of observation.

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International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 2, Issue 9, September 2013

B. *Mean Absolute Error*

The mean absolute error (MAE) is similar to MSE, but the way of measuring the deviation is by the use of absolute value. It is defined as.

$$MSE = \frac{\sum_{i=1}^{n} \left| m(t_i) - m_i \right|}{n}$$

Where n is the number of observation

The lesser the values of MSE and MAE, better will be the estimated results. Further the coverage rate of 95% & 99% asymptotic confidence intervals is also calculated. The details are summarized in Table 1 and Table 2.

Table 1 Simulation results based on complete data from GP Frechet with α=0.2, θ=8, λ=10 and s=2

n	Parameters	MLE	Iteration	Functional Value	Variance	MSE	MAE	Confidence Interval Coverage (95%)	Confidence Interval Coverage (99%)
	Alpha	0.197611			5.406269e-06	0.0544950	0.2946431	(0.1930542, 0.2021688)	(0.1916126,0.2036104)
20	Theta	0.044053	9	-149741	4.396670e-07	0.2883490	0.4310914	(0.04275384,0.04535308)	(0.04234273, 0.04576419)
	Lambda	4.900806			5.583879e-04	0.1156412	0.1988324	(4.854491,4.947122)	(4.83984,4.961772)
	Alpha	0.002662			4.797012e-12	0.0000051	0.0016018	(0.002658664,0.00266725)	(0.002657306,0.002668608)
60	Theta	4.413278	3	-8968097477	5.840316e-08	0.0002055	0.0101389	(4.412805,4.413752)	(4.412655,4.413902)
	Lambda	11.663283			1.212240e-08	0.0006801	0.0184414	(11.66307,11.6635)	(11.663,11.66357)
	Alpha	0.0000735			2.375783e-15	0.0000888	0.0056173	(7.344655e-05,7.363761e-05)	(7.341633e-05, 7.366783e-05)
80	Theta	9.168872	5	-1.262195e+13	3.920588e-11	0.0044097	0.0466776	(9.16886, 9.168884)	(9.168856,9.168888)
	Lambda	11.729240			7.833306e-12	0.0116230	0.0656546	(11.72923,11.72925)	(11.72923,11.72925)
	Alpha	0.002108			1.951471e-12	0.0003081	0.0115204	(0.002104449, 0.002109925)	(0.002103583,0.002110791)
120	Theta	9.168966	4	-16858952905	2.841237e-08	0.0115606	0.0708173	(9.168635,9.169296)	(9.168531,9.1694)
	Lambda	11.738542			5.883658e-09	0.0387176	0.1296119	(11.73839,11.73869)	(11.73834,11.73874)
	Alpha	0.000026			2.756789e-15	0.0000332	0.0029106	(2.581121e-05,2.601703e-05)	(2.577866e-05,2.604958e-05)
20	Theta	9.177554	5	-8.140967e+12	5.496743e-11	0.0013684	0.0186659	(9.177539,9.177569)	(9.177535,9.177573)
	Lambda	11.704560			1.104800e-11	0.0045254	0.0339566	(11.70455,11.70457)	(11.70455,11.70457)

Table 2 Simulation results based on complete data from GP Frechet with α=0.2, θ=8, λ=10 and s=4

n	Parameters	MLE	Iteration	Functional Value	Variance	MSE	MAE	Confidence Interval Coverage (95%)	Confidence Interval Coverage (99%)
	Alpha	0.0072285			9.868677e-13	0.0005880	0.0146844	(0.007226545, 0.007230439)	(0.007225929, 0.007231055)
20	Theta	4.1066068	9	-7163313829	1.597182e-09	0.3094604	0.3438449	(4.106528, 4.106685)	(4.106504, 4.10671)
	Lambda	10.827578			1.519776e-09	0.0204950	0.1258608	(10.8275, 10.82765)	(10.82748, 10.82768)
	Alpha	0.0214327			3.310005e-13	0.0004715	0.0186248	(0.0214315, 0.02143376)	(0.02143115, 0.02143411)
60	Theta	3.5242816	7	-78150089474	8.680391e-11	0.2080680	0.4016367	(3.524263, 3.5243)	(3.524258, 3.524306)
	Lambda	9.8777734			1.246096e-10	0.9168920	0.8681222	(9.877751, 9.877795)	(9.877745, 9.877802)
	Alpha	0.0000729			1.547742e-18	0.0006080	0.0122025	(7.288885e-05, 7.289373e-05)	(7.288808e-05, 7.28945e-05)
80	Theta	8.5618070	7	-3.09418e+13	3.508349e-12	0.0050186	0.0351097	(8.561803, 8.561811)	(8.561802, 8.561812)
	Lambda	10.099640			3.310717e-13	0.0180159	0.0667284	(10.9964, 10.9964)	(10.9964, 10.9964)
	Alpha	0.0000581			2.947480e-19	0.0006516	0.0167205	(5.806878e-05, 5.80709e-05)	(5.806844e-05, 5.807124e-05)
120	Theta	8.5612880	7	-5.470777e+13	2.076298e-12	0.0053273	0.0479503	(8.561285, 8.561291)	(8.561284, 8.561292)
	Lambda	11.000410			1.972230e-13	0.0195275	0.0921622	(11.00041, 11.00041)	(11.00041, 11.00041)
	Alpha	0.0197678			4.308281e-13	0.0258063	0.0752017	(0.01976655, 0.01976913)	(0.01976615, 0.01976953)
200	Theta	2.4200395	12	-44158638955	3.478106e-11	0.2378319	0.3913841	(2.420028, 2.420051)	(2.420024, 2.420055)
	Lambda	7.2776506			1.154871e-10	0.8534065	0.2353123	(7.27763, 7.277672)	(7.277623, 7.277678)

VI. CONCLUSION

In this study, we introduced the geometric model for the analysis of accelerated life testing under constant stress when the life data are from a Frechet distribution .Geometric process model is a better choice as a reason of its simple nature, since it doesn't require a log- linear function of life and stress to reparameterize the original parameter.The MLEs, MSEs and MAEs of the model parameters were obtained. Based on the asymptotic normality, the 95% and 99% confidence intervals of the model parameters were also obtained.

Results in Table 1 and 2 indicate that the estimation of α and λ are more appropriate than θ. The proposed estimator $\hat{\theta}$ is consistently smaller than the true value of θ which means we tend to overestimate the mean lifetime of product at normal stress level. The result shows that when sample size increases, the variability of MLE becomes smaller and the confidence interval becomes narrower. For few cases confidence interval is précised to single value i.e. we get the exact value of estimators.

The results in Table 2 indicate that most of the estimates from the geometric process model match the values obtained by the log linear model. It is seen that when the sample size and number of stress levels are relatively small, some of the estimates obtained by geometric process model have a smaller mean square error and absolute square error than that obtained by the log linear model.From these results it may be concluded that the present model works well under complete data.

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