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Soft ĝ-Closed Sets in Soft Topological Spaces

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ABSTRACT: In this paper a new class of soft sets called Soft ĝ-Closed sets in Soft Topological Spaces is introduced and studied. This new class is defined over an initial universe and with a fixed set of parameters. Some basic properties of this new class of soft sets are investigated. This new class of Soft ĝ-Closed sets contributes to widening the scope of Soft Topological Spaces and its applications.

KEYWORDS: Soft Sets, Soft Topological Spaces, g-closed sets, ĝ-closed sets, Soft g-closed sets, Soft ĝ-closed sets

I. INTRODUCTION

Any Research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas. Here the researchers have succeeded in their knowledge building effort by introducing a new class of soft sets called Soft ĝ Closed sets in Soft Topological Spaces.

Molodtsov (1999) initiated the theory of soft sets as a new mathematical tool for dealing uncertainty, which is completely a new approach for modeling vagueness and uncertainties. Soft Set Theory has a rich potential for application in solving practical problems in Economics, Social Sciences, Medical Sciences etc. Applications of Soft Set Theory in other disciplines and in real life problems are now catching momentum. Molodtsov successfully applied Soft Theory into several directions, such as Smoothness of Functions, Game theory, Operations Research, Riemann Integration, Perron Integration, Theory of Probability, Theory of Measurement and so on. Maji et al. (2002) gave first practical application of Soft Sets in decision making problems. Shabir and Naz(2011) introduce the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They studied some basic concepts of soft topological spaces also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms. In this paper a new class of sets called Soft ĝ-Closed sets are introduced and few of their properties are investigated.

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II. RELATED WORK

Some concepts in mathematics can be considered as mathematical tool for dealing with uncertainties namely theory of vague sets, theory of rough sets and etc. But all of these theories have their all difficulties. The concept of soft set now introduce by Molodtsov[13] in 1999 as a general mathematical tool for modeling uncertainty present in real life. Later on Maji et al [11] proposed several operations on soft sets and some basic properties and then Pei and Miao investigated the relationships between soft sets and information systems. Shabir and Naz [16] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Latter on Veerakumar [18] defined ĝ-closed sets and deeply studied on ĝ-locally closed sets and GLC-functions and also study many basic properties of ĝ-closed sets together with the relationships of these sets with some other sets.

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III. PRELIMINARIES

Let X be an initial universe set and E be the set of parameters. Let $P(X)$ denote the power set of X. For a Soft set (F,E) a space (X,τ,E), $Cl(F,E)$, $Int(F,E)$, $spInt(F,E)$, $spCl(F,E)$, $ssInt(F,E)$, and $ssCl(F,E)$ denote the soft closure of (F,E), soft interior of (F,E), soft pre interior of (F,E),soft pre closure of (F,E), soft semi interior of (F,E) and soft semi closure of (F,E) respectively.

Definition 2.1[13]

For $A \subseteq E$, the pair (F,A) is called a **Soft Set** over X, where F is a mapping given by $F: A \to P(X)$.

In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε- approximate elements of the soft set (F,A).

Definition 2.2 [14]

A soft set (F,A) over X is said to be **Null Soft Set** denoted by Φ if for all $e \in A$, $F(e) = \varphi$. A soft set (F,E) over X is said to be an **Absolute Soft Set** denoted by $\widetilde{A}$ if for all $e \in A$, $F(e) = X$.

Definition 2.3 [11]

The **Union** of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C = A \cup B$, and for all $e \in C$, $H(e) = F(e)$, if $e \in A\backslash B$, $H(e) = G(e)$ if $e \in B\backslash A$ and $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$ and is denoted as $(F,A) \cup (G,B) = (H,C)$.

Definition 2.4 [11]

The **Intersection** of two soft sets (F,A) and (G,B) over X is the soft set(H,C), where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$ and is denoted as $(F,A) \cap (G,B) = (H,C)$.

Definition 2.5 [14]

The **Relative Complement** of (F,A) is denoted by $(F,A)^C$ and is defined by $(F,A)^C = (F^C,A)$ where $F^C: A \to P(X)$ is a mapping given by $F^C(e) = X - F(e)$ for all $e \in A$.

Definition 2.6 [14]

The **Difference** (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E)\backslash(G,E)$ is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.

Definition 2.7 [14]

Let (F,A) and (G,B) be soft sets over X, we say that (F,A) is a **Soft Subset** of (G,B) if $A \subseteq B$ and for all $e \in A$, $F(e)$and $G(e)$ are identical approximations. We write $(F,A) \subseteq (G,B)$.

Definition 2.8[14]

Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a **Soft Topology** on X if

i. $\Phi, \widetilde{E}$ belongs to τ
ii. The union of any number of soft sets in τ belongs toτ.
iii. The intersection of any two soft sets in τ belongs toτ.

The triplet (X,τ,E) is called **Soft Topological Spaces** over X.

The members of τ are called **Soft Open** sets in X and complements of them are called **Soft Closed** sets in X.

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Vol. 3, Issue 7, July 2014

Definition 2.9 [8]

Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft Interior** of (F, E) denoted by $Int(F, E)$ is the union of all soft open subsets of (F, E). Clearly (F, E) is the largest soft open set over X which is contained in (F, E). The **Soft Closure** of (F, E) denoted by $Cl(F, E)$ is the intersection of closed sets containing(F, E). Clearly (F, E) is the smallest soft closed set containing(F, E).

$$Int(F, E) = \cup\{(O.E) : (O, E) \text{ is soft open and } (O, E) \subseteq (F, E)\}$$
$$Cl(F, E) = \cup\{(O.E) : (O, E) \text{ is soft closed and } (F, E) \subseteq (O, E)\}$$

Result 2.10 [14]

Let (X, τ, E) be a Soft Topological Spaces over X and (F, E) and (G, E) be a soft sets over X. Then

i. (F, E) is soft closed set if and only if $(F, E) = Cl(F, E)$
ii. $Cl((F, E) \cup (G, E)) = Cl(F, E) \cup Cl(G, E)$
iii. $Cl(Cl(F, E)) = Cl(F, E)$**.**
iv. $ssCl(F, E) = (F, E) \cup Int(Cl(F, E))$
v. $ssInt(F, E) = (F, E) \cap Cl(Int(F, E))$
vi. $spCl(F, E) = (F, E) \cup Cl(Int(F, E))$
vii. $spInt(F, E) = (F, E) \cap Int(Cl(F, E))$

Definition 2.11 [18]

A subset A of a topological space (X, τ) is called

i. a **Semi Open** set if $A \subseteq Cl(Int(A))$ and a **Semi Closed** set if $Int(Cl(A)) \subseteq A$.
ii. a **Pre Open** set if $A \subseteq Int(Cl(A))$ and a **Pre Closed** set if $Cl(Int(A)) \subseteq A$.
iii. an **α-open** set if $A \subseteq Int(Cl(Int(A)))$ and an **α -closed** set if $Cl(Int(Cl(A))) \subseteq A$.
iv. a **Regular Open** set if $A = Int(Cl(A))$ and a **Regular Closed** set if $A = Cl(Int(A))$.
v. a **Generalized Closed** set(briefly **g-closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is open in (X, τ).The complement of a g-closed set is called a **g-Open** set.
vi. a **Semi Generalized Closed** set(briefly **sg-closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi open in (X, τ).
vii. a **Generalized α -Closed** set(briefly **αg-Closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is α-open in (X, τ).
viii. a **Regular Generalized Closed** set(briefly **rg-Closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is regular open in (X, τ).
ix. a **ĝ-Closed** set if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi open in (X, τ).The complement of a ĝ-closed set is called a **ĝ-Open** set.

Definition 2.12[3][8][9][14]

In a Soft Topological Spaces (X, τ, E), a soft set (F, E) over X is called

i. a **Soft Semi Open** if $(F, E) \subseteq Cl(Int(F, E))$ and **Soft Semi Closed** if $Int(Cl(F, E)) \subseteq (F, E)$.
ii. a **Soft Pre Open** set if $(F, E) \subseteq Int(Cl(F, E))$ and **Soft Pre Closed** set if $Cl(Int(F, E)) \subseteq (F, E)$.
iii. a **Soft α-Open** set if $(F, E) \subseteq Int(Cl(Int(F, E)))$ and **Soft α-Closed** set if $Cl(Int(Cl(F, E))) \subseteq (F, E)$.
iv. a **Soft Regular Open** if $(F, E) = Int(Cl(F, E))$ and **Soft Regular Closed** if $(F, E) = Cl(Int(F, E))$.

v. a **Soft Generalized Closed** set(briefly **Soft g-closed**) if $Cl(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and

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Vol. 3, Issue 7, July 2014

(G, E) is soft open in (X, τ, E).The complement of a soft g-closed set is called a **Soft g- Open** set.

vi. a **Soft Semi Generalized Closed** set(briefly **Soft sg Closed**) if $Cl(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is soft semi open in (X, τ, E).

vii. a **Soft generalized α -Closed** set(briefly **Soft αg-Closed**) if $Cl(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is soft α-open in (X, τ, E).

viii. a **Soft Regular Generalized Closed** set(briefly **Soft rg-Closed**) if $Cl(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is soft regular open in (X, τ, E).

IV. SOFT ĝ-CLOSED SETS IN SOFT TOPOLOGICAL SPACES

Definition 3.1

Let (X,τ, E) be a soft topological space. A soft set (F, E) is called a **Soft ĝ-Closed** set if $Cl(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is soft semi open in (X,τ, E).

Example 3.2

Let $X = \{a, b, c\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{E}, (G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E), (G_6, E), (G_7, E)\}$ where

$G_1(e_1) = \{a, b\}$ $\quad$ $G(e_2) = \{a, b\}$

$G(e_1) = \{b\}$ $\quad$ $G_2(e_2) = \{a, c\}$

$G_3(e_1) = \{b, c\}$ $\quad$ $G(e_2) = \{a\}$

$G_4(e_1) = \{b\}$ $\quad$ $G_4(e_2) = \{a\}$

$G_5(e_1) = \{a, b\}$ $\quad$ $G_5(e_2) = X$

$G_6(e_1) = X$ $\quad$ $G_6(e_2) = \{a, b\}$

$G_7(e_1) = \{b, c\}$ $\quad$ $G_7(e_2) = \{a, c\}$

Consider the soft set (G, E) over X such that

$F(e_1) = \{a\}$ $\quad$ $F(e_2) = \{c\}$

Clearly (G, E) is soft ĝ-Closed set in (X,τ, E).

Theorem 3.3

Every soft closed set is a soft ĝ-closed set.

Proof

Let (F, E) be soft closed set in (X,τ, E) and (G, E) be soft semi open set such that $(F, E) \subseteq (G, E)$. Consider $Cl(F, E) = (F, E) \subseteq (G, E)$. Therefore (F, E) is soft ĝ-closed set in (X,τ, E).

Theorem 3.4

Every soft ĝ-closed set is a soft g-closed set.

Proof

Let (F, E) be soft ĝ-closed set in (X,τ, E). Let (G, E) be any soft open in X such that $(F, E) \subseteq (G, E)$. Since every soft open set is soft semi open set. Since (F, E) is soft ĝ-Closed set then $Cl(F, E) \subseteq (G, E)$. Therefore (F, E) is soft g-closed set.

Therefore the class of ĝ-closed sets is properly contained in the class of g-closed sets and properly contains the class of closed sets.

Theorem 3.5

Every soft ĝ-closed set is soft rg-closed set.

Proof

Let (F, E) be soft ĝ-closed set in (X,τ, E) and (H, E) be soft regular open set such that $(F, E) \subseteq (H, E)$. Then

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Vol. 3, Issue 7, July 2014

$(F,E) \subseteq Int(Cl((H,E))$. Since $Int(Cl((H,E))$ is soft semi-open set containing the soft ĝ-closed set, then $Cl(F,E) \subseteq Int(Cl((H,E))$. Therefore (F,E) is soft rg-closed set in (X,τ,E).

Theorem 3.6

Union of any two soft ĝ-closed sets is soft ĝ-closed set.

Proof

Suppose (F,E) and (H,E) are soft ĝ-closed sets in (X,τ,E). Then $Cl(F,E) \subseteq (G,E)$ and $Cl(H,E) \subseteq (G,E)$ where $(F,E) \subseteq (G,E)$ and $(H,E) \subseteq (G,E)$.
Hence $Cl\big((F,E) \cup (H,E)\big) = Cl(F,E) \cup Cl(H,E) \subseteq (G,E)$. That is $Cl\big((F,E) \cup (H,E)\big) \subseteq (G,E)$. Therefore $(F,E) \cup (H,E)$ is a soft ĝ-closed set in (X,τ,E).

In a similar way one can prove.

Intersection of any two soft ĝ-closed sets is a soft ĝ-closed set.

Theorem 3.8

Let (F,E) be a soft ĝ-closed set in (X,τ,E). Then $Cl(F,E)\setminus(F,E)$ does not contain any non-empty soft semi closed set.

Proof

Let (A,E) be a soft semi closed set in (X,τ,E) and $(A,E) \subseteq Cl(F,E)\setminus(F,E)$. Then $(A,E) \subseteq Cl(F,E)$ and $(A,E) \subseteq (F,E)^C$. This implies $(F,E) \subseteq (A,E)^C$. Then $Cl(F,E) \subseteq (A,E)^C$. This implies $(A,E) \subseteq (Cl(F,E))^C$. Therefore $(A,E) \subseteq Cl(F,E) \cap (Cl(F,E))^C$. Hence (A,E) does not contain any non-empty soft semi closed set in (X,τ,E).

Theorem 3.9

If (F,E) is a soft ĝ-closed set in (X,τ,E) and $(F,E) \subseteq (H,E) \subseteq Cl(F,E)$ then (H,E) is also a soft ĝ-closed set.

Proof

Let (F,E) be soft ĝ-Closed set in (X,τ,E) such that $(F,E) \subseteq (H,E) \subseteq Cl(F,E)$. Let (G,E) be a soft semi open set in (X,τ,E) such that $(H,E) \subseteq (G,E)$. Then $(F,E) \subseteq (G,E)$. Since (F,E) is soft ĝ-closed set, $Cl(F,E) \subseteq (G,E)$. Now $Cl(H,E) \subseteq Cl(Cl(F,E)) = Cl(F,E) \subseteq (G,E)$. That is $Cl(H,E) \subseteq (G,E)$. Therefore (H,E) is soft ĝ -Closed set in (X,τ,E).

Theorem 3.10

Let (F,E) be a soft ĝ-closed set in a soft topological space (X,τ,E). Then

(i) ssInt(F,E) is soft ĝ-closed.

(ii) spCl(F,E) is soft ĝ-closed.

(iii) If (F,E) is soft regular open then spInt(F,E) and ssCl(F,E) are also soft ĝ-closed sets.

Proof

First we note that for a (F,E) of (X,τ,E), $ssCl(F,E) = (F,E) \cup Int(Cl(F,E))$ and $spCl(F,E) = (F,E) \cup Cl(Int(F,E))$. Moreover $ssInt(F,E) = (F,E) \cap Cl(Int(F,E))$ and $spInt(F,E) = (F,E) \cap Int(Cl(F,E))$.

(i) Since $Cl(Int(F,E))$ is a soft closed set, then (F,E) and $Cl(Int(F,E))$ are soft ĝ-closed sets. By the Theorem 3.7, $(F,E) \cap Cl(Int(F,E))$ is also a soft ĝ-closed set.

(ii) spCl(F,E) is the union of two soft ĝ-closed sets (F,E) and $Cl(Int(F,E))$. Again by the Theorem 3.7, spCl(F,E) is a soft ĝ-closed.

(iii) Since (F,E) is soft regular open, then $(F,E) = Int(Cl(F,E)$. Then $ssInt(F,E) = (F,E) \cup Int\big(Cl(F,E)\big) = (F,E)$. Thus $ssCl(F,E)$ is soft ĝ-closed. Similarly $spInt(F,E)$ is also a soft ĝ-closed set.

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V. CONCLUSION

In the present work, a new class of sets called Soft ĝ-Closed sets in Soft Topological Spaces is introduced and some of their properties are studied. This new class of sets widens the scope to do further research in the areas like Bitopological Spaces, Smooth topological Spaces and Fuzzy Soft Topological Spaces.

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