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SRW-Closed Sets in Soft Topological Spaces

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ABSTRACT: In this paper a new class of soft sets called Soft Regular Weakly Closed sets (briefly SRW-Closed sets) in soft topological spaces is introduced and studied. This new class is defined over an initial universe and with a fixed set of parameters. Some basic properties of this new class of soft sets are investigated. This new class of SRW-Closed sets contributes to widening the scope of Soft Topological Spaces and its applications.

KEYWORDS: Soft Sets, Soft Topological Spaces, W-closed sets, SW-closed sets, RW-closed sets, SRW-closed sets

I. INTRODUCTION

Any Research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas. Here the researchers have succeeded in their knowledge building effort by introducing a new class of soft sets called Soft Regular Weakly Closed sets (briefly SRW-Closed sets) in Soft Topological Spaces.

Molodtsov (1999) initiated the theory of soft sets as a new mathematical tool for dealing uncertainty, which is completely a new approach for modeling vagueness and uncertainties. Soft Set Theory has a rich potential for application in solving practical problems in Economics, Social Sciences, Medical Sciences etc. Applications of Soft Set Theory in other disciplines and in real life problems are now catching momentum. Molodtsov successfully applied Soft Theory into several directions, such as Smoothness of Functions, Game theory, Operations Research, Riemann Integration, Perron Integration, Theory of Probability, Theory of Measurement and so on. Maji et al. (2002) gave first practical application of Soft Sets in decision making problems. Shabir and Naz(2011) introduce the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They studied some basic concepts of soft topological spaces also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms. . In this paper a new class of sets called Soft Regular Weakly Closed sets (SRW-Closed sets) are introduced and few of their properties are investigated.

II. RELATED WORK

Some concepts in mathematics can be considered as mathematical tool for dealing with uncertainties namely theory of vague sets, theory of rough sets and etc. But all of these theories have their all difficulties. The concept of soft set now introduce by Molodtsov[10] in 1999 as a general mathematical tool for modeling uncertainty present in real life. Later on Maji et al [9] proposed several operations on soft sets and some basic properties and then Pei and Miao [12] investigated the relationships between soft sets and information systems. Shabir and Naz [12] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Latter on Benchalli and Wali [3] introduced RW-Closed sets in Topological Spaces.

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III. PRELIMINARIES

Let X be an initial universe set and E be the set of parameters. Let $P(X)$ denote the power set of X.

Definition 2.1[10]

For $A \subseteq E$, the pair (F, A) is called a **Soft Set** over X, where $F$is a mapping given by $F: A \to P(X)$.

In other words, a soft set over X is a parameterized family of subsets of the universe$X$. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε- approximate elements of the soft set (F, A).

Definition 2.2 [11]

A soft set (F, A) over X is said to be **Null Soft Set** denoted by Φ if for all $e \in A$, $F(e) = \varphi$. A soft set (F, E) over $X$is said to be an **Absolute Soft Set** denoted by $\tilde{A}$ if for all $e \in A$,$F(e) = X$.

Definition 2.3 [9]

The **Union** of two soft sets(F, A) and (G, B)over X is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e) = F(e)$, if $e \in A \setminus B$, $H(e) = G(e)$ if $e \in B \setminus A$ and$H(e) = F(e) \cup G(e)$if $e \in A \cap B$ and is denoted as

$(F, A) \cup (G, B) = (H, C)$.

Definition 2.4 [9]

The **Intersection** of two soft sets (F, A) and (G, B)over X is the soft set (H, C), where$C = A \cap B$ and

$H(e) = F(e) \cap G(e)$for all $e \in C$ and is denoted as$(F, A) \cap (G, B) = (H, C)$.

Definition 2.5 [11]

The **Relative Complement** of (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, A)$ where $F^C: A \to P(X)$ is a mapping given by $F^C(e) = X - F(e)$ for all $e \in A$.

Definition 2.6 [11]

The **Difference**(H, E) of two soft sets (F, E) and (G, E)over X, denoted by $(F, E) \setminus (G, E)$ is defined as

$H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition 2.7 [11]

Let (F, A) and (G, B) be soft sets over X, we say that (F, A) is a **Soft Subset** of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$and $G(e)$ are identical approximations. We write $(F, A) \subseteq (G, B)$.

Definition 2.8 [11]

Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a **Soft Topology** on$X$ if

i. $\Phi, \tilde{E}$ belongs to τ
ii. The union of any number of soft sets in τ belongs to τ.
iii. The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called **Soft Topological Spaces** over X.

The members of τ are called **Soft Open** sets in X and complements of them are called **Soft Closed** sets in$X$.

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Definition 2.9 [11]

Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft Interior** of (F, E) denoted by $Int(F, E)$ is the union of all soft open subsets of (F, E). Clearly (F, E) is the largest soft open set over X which is contained in (F, E). The **Soft Closure** of (F, E) denoted by $Cl(F, E)$ is the intersection of closed sets containing (F, E). Clearly (F, E) is the smallest soft closed set containing (F, E).

$$Int(F, E) = \cup \{(O.E): (O, E) \text{is soft open and } (O, E) \subseteq (F, E)\}$$
$$Cl(F, E) = \cup \{(O.E): (O, E) \text{is soft closed and } (F, E) \subseteq (O, E)\}$$

Result2.10 [11]

Let (X, τ, E) be a Soft Topological Spaces over X and (F, E) and (G, E) be a soft sets over X. Then

i. (F, E) is soft closed set if and only if $(F, E) = Cl(F, E)$
ii. $Cl((F, E) \cup (G, E)) = Cl(F, E) \cup Cl(G, E)$
iii. $Cl(Cl(F, E)) = Cl(F, E)$.

Definition 2.11[11]

In a Soft Topological Spaces (X, τ, E), a soft set (F, E)over X is called

i. a **Soft Semi Open** if $(F, E) \subseteq Cl(Int(F, E))$ and **Soft Semi Closed** if $Int(Cl(F, E)) \subseteq (F, E)$.
ii. a **Soft Regular Open** if $(F, E) = Int(Cl(F, E))$ and **Soft Regular Closed** if $(F, E) = Cl(Int(F, E))$.
iii. a **Soft Weakly Closed**(briefly **SW-Closed**) if $Cl((F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is soft semi open in X.
iv. a **Soft Regular Semi Open** if there exists a soft regular open set (U, E) such that $(U, E) \subseteq (F, E) \subseteq Cl(U, E)$.

Result 2.12[11]

i. Every soft regular semi open set in (X, τ, E) is soft semi open.
ii. If (F, E) is soft regular semi open in (X, τ, E) then $(X, E) \setminus (F, E)$ is also soft regular semi open.

Definition 2.13 [3]

A subset A of a Topological Spaces (X, τ) is called

i. A **Semi Open** if $A \subseteq Cl(Int(A))$ and **Semi Closed** if $Int(Cl(A) \subseteq A$.
ii. a **Regular Open** if $A = Int(Cl(A))$ and **Regular Closed** if $A = Cl(Int(A))$.
iii. a **Regular Semi Open** if there exists a regular open set U such that $U \subseteq A \subseteq Cl(U)$.
iv. a **Weakly Closed** (briefly **W-Closed**) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
v. a **Regular Weakly Closed**(briefly **RW-Closed**) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in (X, τ).

IV. SRW-CLOSED SETS IN SOFT TOPOLOGICAL SPACES

Definition 3.1

Let (X, τ, E) be a Soft Topological Spaces. A soft set (F, E) is called **Soft Regular Weakly Closed** (briefly **SRW-Closed**) if $Cl(F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is soft regular semi open in (X, τ, E).

Example 3.2

Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{E}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$ where

$F_1(e_1) = \{x_1\}$ $\quad$ $F_1(e_2) = \{x_1\}$

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$F_2(e_1) = \{x_2\}$ $F_2(e_2) = \{x_2\}$
$F_3(e_1) = \{x_3\}$ $F_3(e_2) = \{x_3\}$
$F_4(e_1) = \{x_1, x_2\}$ $F_4(e_2) = \{x_1, x_2\}$
$F_5(e_1) = \{x_2, x_3\}$ $F_5(e_2) = \{x_2, x_3\}$
$F_6(e_1) = \{x_1, x_3\}$ $F_6(e_2) = \{x_1, x_3\}$

Then (X, τ, E) is a Soft Topological Spaces. Define soft sets (G, E) and (H, E) over X such that
$G(e_1) = \{x_1, x_3\}$,$G(e_2) = \{x_1\}$and $H(e_1) = \{x_1, x_2\}$, $H(e_2) = \{x_2\}$.
Here both (G, E) and (H, E) are SRW-Closed sets in (X, τ, E).

Theorem 3.3

Every soft closed set is a SRW-Closed set but not conversely.

Proof

Let (F, E) be a soft closed set in (X, τ, E) and (U, E) be soft regular semi open set such that $(F, E) \subseteq (U, E)$. Consider $Cl(F, E) = (F, E) \subseteq (U, E)$. Therefore $(F, E)$is SRW-Closed set.

In Example3.2, (G, E) is a SRW-Closed set but not soft closed set.

Theorem 3.4

Every SW-Closed set is a SRW-Closed set but not conversely.

Proof

The proof follows from the definitions and the fact that every soft regular semi open set is soft semi open set.
In Example3.2, (G, E) is a SRW-Closed set but not SW-Closed set.

Theorem 3.5

If (F, E) and (G, E) are SRW-Closed sets in (X, τ, E)then $(F, E) \cup (G, E)$is SRW-Closed set in (X, τ, E).

Proof

Suppose (F, E) and (G, E) are SRW-Closed sets in (X, τ, E). Then $Cl(F, E) \subseteq (U, E)$ and $Cl(G, E) \subseteq (U, E)$ where $(F, E) \subseteq (U, E)$and $(G, E) \subseteq (U, E)$.
Hence $Cl\big((F, E) \cup (G, E)\big) = Cl(F, E) \cup Cl(G, E) \subseteq (U, E)$. That is $Cl\big((F, E) \cup (G, E)\big) \subseteq (U, E)$. Therefore $(F, E) \cup (G, E)$ is SRW-Closed set in (X, τ, E).

Remark 3.6

Intersection of two SRW Closed sets need not be a SRW-Closed set.

In Example 3.2,(H, E) and (G, E) are SRW-Closed sets in (X, τ, E). But $(H, E) \cap (G, E)$ is not SRW-Closed set in (X, τ, E).

Theorem 3.7

If a soft set $(F, E)$is SRW-Closed set in (X, τ, E) then the difference $Cl(F, E) \setminus (F, E)$ does not contain any non-empty soft regular semi open set in (X, τ, E).

Proof

We prove the result by contradiction. Let (U, E)be a non-empty soft regular semi open set such that

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$$Cl(F,E)\setminus(F,E) \supseteq (U,E) \qquad (1)$$

Therefore $(U,E) \subseteq (X,E)\setminus(F,E)$ then $(F,E) \subseteq (X,E)\setminus(U,E)$. Since (U,E) is soft regular semi open set by result 2.12(ii) $(X,E)\setminus(U,E)$ is also soft regular semi open set in (X,τ,E). Since (F,E) is SRW-Closed set in $(X,\tau,E)$and$Cl(F,E) \subseteq (X,E)\setminus(U,E)$, so $(U,E) \subseteq (X,E)\setminus Cl(F,E)$. Also by (1)$(U,E) \subseteq Cl(F,E)$. Therefore $(U,E) \subseteq Cl(F,E) \cap \left((X,E)\setminus Cl(F,E)\right) = \varphi$.This shows that (U,E) is empty, which is a contradiction.

Hence $Cl(F,E)\setminus(F,E)$ does not contain any non-empty soft regular semi open set in (X,τ,E).

Corollary 3.8

If (F,E) is SRW-Closed set in (X,τ,E) then $Cl(F,E)\setminus(F,E)$ does not contain any non-empty soft regular open set in (X,τ,E).

Proof

Follows from theorem (3.9) and the fact that every soft regular open set is soft regular semi open set.

Corollary 3.9

If (F,E) is SRW-Closed set in (X,τ,E) then $Cl(F,E)\setminus(F,E)$ does not contain any non-empty soft regular closed set in(X,τ,E).

Proof

Follows from theorem (3.9) and the fact that every soft regular open set is soft regular semi open set.

Theorem 3.10

If (F,E) is a SRW-Closed set in (X,τ,E) such that $(F,E) \subseteq (G,E) \subseteq Cl(F,E)$ then (G,E) is SRW-Closed set in (X,τ,E).

Proof

Let (F,E) be SRW-Closed set in (X,τ,E) such that$(F,E) \subseteq (G,E) \subseteq Cl(F,E)$. Let (U,E) be a soft regular semi open set of (X,τ,E) such that $(G,E) \subseteq (U,E)$. Then $(F,E) \subseteq (U,E)$.Since (F,E) is SRW-Closed set,$Cl(F,E) \subseteq (U,E)$.Now $Cl(G,E) \subseteq Cl(Cl(F,E)) = Cl(F,E) \subseteq (U,E)$. That is $Cl(G,E) \subseteq (U,E)$. Therefore (G,E) is SRW-Closed set in(X,τ,E).

Theorem 3.11

Let (F,E) is SRW-Closed set in (X,τ,E). Then (F,E) is soft closed set if and only if $Cl(F,E)\setminus(F,E)$ is soft regular semi open set in (X,τ,E).

Proof

Suppose (F,E) is soft closed set in (X,τ,E). Then $Cl(F,E) = (F,E)$and $Cl(F,E)\setminus(F,E) = \varphi$, which is a soft regular semi open set in (X,τ,E).

Conversely, suppose $Cl(F,E)\setminus(F,E)$ is soft regular semi open set in (X,τ,E). Since(F,E) is SRW-Closed set, by theorem (3.7), $Cl(F,E)\setminus(F,E)$ does not contain any nonempty soft regular semi open set in (X,τ,E). Then $Cl(F,E)\setminus(F,E) = \varphi$. Hence (F,E) is soft closed set in(X,τ,E).

V. CONCLUSION

In the present work, a new class of sets called SRW-Closed sets in Soft Topological Spaces is introduced and some of their properties are studied. This new class of sets widens the scope to do further research in the areas like Bitopological Spaces, Smooth topological Spaces and Fuzzy Soft Topological Spaces.

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