ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

Existence of Libration Points in the Photogravitational Elliptic Restricted Three Body Problem

K.V. Ravi Kumar[1], S.N. Hasan[2]

Assistant Professor, Department of Mathematics, Osmania University, Hyderabad, India [1]

Professor, Department of Mathematics, Maulana Azad National Urdu University, Hyderabad, India [2]

ABSTRACT: In this paper we study the effects of eccentricity, oblateness and effects due to radiation pressure on the location of Lagrangian equilibrium points of the elliptical restricted three body problem. We investigate the location of triangular equilibrium points and collinear points under the effects of photo-gravitational and the oblateness of primaries, i.e. both of the primaries are considered to be oblate spheroid and the source of radiation. We have found the solution for all five Lagrangian points. We find that the locations of the equilibrium points are affected by oblateness, radiation factor and eccentricities of both the primary and the secondary bodies.

KEYWORDS: Restricted three body problem (RTBP), Photo gravitation, Lagrangian points

I. INTRODUCTION

Danby (1964), Examined the stability of triangular points in the elliptic restricted three body problem. Sharma (1987), Studied the photo-gravitational restricted three body problem. Khasan (1996), discussed the librational solutions to the photo gravitational RTBP. Sahoo, S. K. and Ishwar. B, (2000), examined the collinear equilibrium points in the generalised photo-gravitational elliptic restricted three body problem. Om Prakash Raman, Rama Shankar Sharma (2013), studied the location of libration points in the generalised photo-gravitational elliptic restricted three body problem, In the present paper we study the location of collinear and triangular libration points in the generalised photo-gravitational elliptic restricted three body problem. We have taken both the primaries to be oblate and radiating. Effects of eccentricity, oblateness and radiation factor are studied in detail which makes our investigations different from the classical case. A comparison with the classical case is also presented in this paper.

II. EXISTANCE OF TRIANGULAR LIBRATION POINTS

The equations of motion of an infinitesimal body, position of which with respect to the centre of mass is denoted by (ξ,η,ζ), in the gravitational field of two oblate, radiating major bodies with masses are μ and $1-\mu$ are given by (for detailed derivation of basic equations of motion without oblateness refer Khasan, 1996):

$$\xi'' - 2\eta' = \frac{\partial\Omega^*}{\partial\xi}, \eta'' - 2\xi' = \frac{\partial\Omega^*}{\partial\eta} \, and \, \zeta'' = \frac{\partial\Omega^*}{\partial\zeta} \quad \text{----- (1)}$$

Where $$\Omega^*(\xi,\eta) = \frac{1}{(1-e^2)^{1/2}}\left[\frac{\xi^2+\eta^2}{2} + \frac{1}{n^2}\left\{\frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)q_1 A_1}{2r_1^3} + \frac{\mu q_2 A_2}{2r_2^3}\right\}\right] \quad \text{---- (2)}$$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

A_1, A_2 are the oblateness coefficients, q_1, q_2 radiation pressures , 'e' the eccentricity of the orbit of the secondary body with respect to the primary,'a' the semi-major axis of the ellipse and $n=\sqrt{\dfrac{(1+\frac{3A_1}{2}+\frac{3A_2}{2})(1+e^2)^{1/2}}{a(1-e^2)}}$ the mean angular velocity and r_1 and r_2 denote the distances of the bodies respectively from their common centre of mass

Where $r_1^2=(\xi+\mu)^2+\eta^2+\zeta^2$ and $r_2^2=(\xi+\mu-1)^2+\eta^2+\zeta^2$

Multiplying the above equations with $2\xi', 2\eta' \& 2\zeta'$ respectively, then adding, integration we get the Jacobian equation $\xi'^2+\eta'^2+\zeta'^2=2\Omega^*+C$. Where 'C' is the Jacobi constant.

The Integral Equation F $(\xi,\eta,\zeta)=2\Omega^*+C$. Where

$$F(\xi,\eta,\zeta)=\frac{1}{(1-e^2)^{1/2}}\left[\frac{\xi^2+\eta^2}{2}+\frac{1}{n^2}\left\{\frac{2(1-\mu)q_1}{r_1}+\frac{2\mu q_2}{r_2}+\frac{2(1-\mu)q_1A_1}{2r_1^3}+\frac{2\mu q_2A_2}{2r_2^3}\right\}\right]+c \quad \text{---- (3)}$$

For equilibrium positions, we require the following conditions to be satisfied: $\dfrac{\partial F}{\partial \xi}=\dfrac{\partial F}{\partial \eta}=\dfrac{\partial F}{\partial \zeta}=0$

This gives us

$$\frac{1}{(1-e^2)^{1/2}}\left[\xi-\frac{1}{n^2}\left\{\frac{(1-\mu)(\xi+\mu)q_1}{r_1^3}+\frac{\mu(\xi+\mu-1)q_2}{r_2^3}+\frac{3(1-\mu)(\xi+\mu)q_1A_1}{2r_1^5}+\frac{3\mu(\xi+\mu-1)q_2A_2}{2r_2^5}\right\}\right]=0 \text{--}(4)$$

$$\frac{\eta}{(1-e^2)^{1/2}}\left[1-\frac{1}{n^2}\left\{\frac{(1-\mu)q_1}{r_1^3}+\frac{\mu q_2}{r_2^3}+\frac{3(1-\mu)q_1A_1}{2r_1^5}+\frac{3\mu q_2A_2}{2r_2^5}\right\}\right]=0 \text{--}(5)$$

if

$$\zeta\left[\frac{(1-\mu)q_1}{r_1^3}+\frac{\mu q_2}{r_2^3}+\frac{3(1-\mu)q_1A_1}{2r_1^5}+\frac{3\mu q_2A_2}{2r_2^5}\right\}=0 \text{--}(6)$$

$\zeta=0$, $\eta\neq 0$, we get planar libration points.

If $\eta\neq 0$, from (5), $1-\dfrac{1}{n^2}\left\{\dfrac{(1-\mu)q_1}{r_1^3}+\dfrac{\mu q_2}{r_2^3}+\dfrac{3(1-\mu)q_1A_1}{2r_1^5}+\dfrac{3\mu q_2A_2}{2r_2^5}\right\}]=0\text{--}(7)$

Using (7) in (4) and μ. $(1-\mu)\neq 0$, we get $\dfrac{q_1}{r_1^3}+\dfrac{3A_1q_1}{2r_1^5}=n^2$ and $\dfrac{q_2}{r_2^3}+\dfrac{3A_2q_2}{2r_2^5}=n^2$

It is clear that $\xi_2-\xi_1=1$, the distance between two primaries so that $\xi_1=-\mu$, $\xi_2=1-\mu$.

On simplification, we get $r_1^2=(aq_1)^{2/3}(1-A_2-e^2)$ and $r_2^2=(aq_2)^{2/3}(1-A_1-e^2)$. ---- (8)

Using these in $r_1^2-r_2^2=2\xi+2\mu-1$, we get

$$\xi=\frac{1}{2}-\mu+\frac{1}{2}\{(aq_1)^{2/3}(1-e^2-A_2)-(aq_2)^{2/3}(1-e^2-A_1)\}\text{--}(9)$$

also from $\eta^2=r_1^2-(\xi+\mu)^2$, we get

$$\eta=\pm\left[(aq_1)^{2/3}(1-e^2-A_1)-\frac{1}{4}\{1+2(aq_1)^{2/3}(1-e^2-A_2)-2(aq_2)^{2/3}(1-e^2-A_1)\}\right]^{1/2}\text{--}(10)$$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

Hence the coordinates of L_4 and L_5 are (ξ, $\pm\eta$) respectively.

Note: - If $A_1 = A_2 = 0$, $q_1 = q_2 = 1$, $e = 0$, the coordinates of L_4, L_5 are $(\frac{1}{2}-\mu, \pm\frac{\sqrt{3}}{2})$ which are similar to result of Mc Cuskey (1963) for the classical case.

III. EXISTANCE OF COLLINEAR POINTS

To find the location of collinear libration points on the ξ -axis we assume η=0, ζ=0 in $\dfrac{\partial F}{\partial \xi}=0$.

From eq (4), $$\xi-\frac{1}{n^2}\{\frac{(1-\mu)q_1}{(\xi+\mu)^2}+\frac{\mu q_2}{(\xi-1+\mu)^2}+\frac{3(1-\mu)q_1A_1}{2(\xi+\mu)^4}+\frac{3\mu q_2A_2}{2(\xi-1+\mu)^4})]=0--(11)$$

The equation (11) is the ninth degree equation in ξ, the collinear libration points lie on the line joining the two major bodies. To find the locations of these points we divide the ξ-axis into three regions- viz. region beyond ξ_2, between ξ_1 and ξ_2 and less than ξ_1.

Case (i):- For $\xi > \xi_2$,

Consider $\xi - \xi_2 = \rho$; so that $\xi - \xi_1 = 1 + \rho$, we get $\xi = 1 + \rho - \mu$;

Substituting these values in equation (11), we get:

$$2n^2\rho^9+2n^2(5-\mu)\rho^8+2n^2(10-4\mu)\rho^7+[2n^2(10-6\mu)-2q_1(1-\mu)-2\mu q_2]\rho^6+$$
$$[2n^2(5-4\mu)-4q_1(1-\mu)-8\mu q_2]\rho^5+[2n^2(1-\mu)-2q_1(1-\mu)-12\mu q_2-3(1-\mu)q_1A_1-3\mu q_2A_2]\rho^4-$$
$$[8\mu q_2+12\mu q_2A_2]\rho^3-[2\mu q_2+18\mu q_2A_2]\rho^2-12\mu q_2A_2\rho-3\mu q_2A_2=0----(12)$$

To find the location of collinear points, we adopt a method described in Sahoo, S.K. and Ishwar, B (2000) by assuming γ be the value of ρ in the classical restricted three body problem, i.e. when $e = 0$, $A_1 = A_2 = 0$ and $q_1 = 1$, $q_2 = 1$. In the presence of oblateness, radiation and eccentricity terms it is taken that the value of $\rho = \gamma + \delta$ *where* $\delta << 1$.
Let $q_1 = 1 - \beta_1$, $q_2 = 1 - \beta_2$, *where* $\beta_1 < 1$, $\beta_2 < 1$.
Substituting the value of ρ in the equation (12), we get the equation:

$$\delta\,(P_1 + Q_1\beta_1 + Q_2\beta_2 + R_1A_1 + R_2A_2) = (L_1 + M_1\beta_1 + M_2\beta_2 + N_1A_1 + N_2A_2), \qquad --(13)$$

Where $P_1=18n^2\gamma^8+16n^2(5-\mu)\gamma^7+14n^2(10-4\mu)\gamma^6+6\{2n^2(10-6\mu)-2\}\gamma^5+5\{2n^2(5-4\mu)-4(1+\mu)\}\gamma^4$
$+4\{2n^2(1-\mu)-2(1+5\mu)\}\gamma^3-24\mu\gamma^2-4\mu\gamma$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

$$Q_1 = 12(1-\mu)\gamma^5 + 20(1-\mu)\gamma^4 + 8(1-\mu)\gamma^3$$
$$R_1 = -12(1-\mu)\gamma^3$$
$$Q_2 = 12\mu\gamma^5 + 40\mu\gamma^4 + 48\mu\gamma^3 + 24\mu\gamma^2 + 4\mu\gamma$$
$$R_2 = -\{12\mu\gamma^3 + 36\mu\gamma^2 + 36\mu\gamma + 12\mu\}$$
$$L1 = -\{2n^2\gamma^9 + 2n^2(5-\mu)\gamma^8 + 2n^2(10-4\mu)\gamma^7 + [2n^2(10-6\mu)-2]\gamma^6 +$$
$$[2n^2(5-4\mu)-4-4\mu]\gamma^5 + [2n^2(1-\mu)-2(1-\mu)-12\mu]\gamma^4 - 8\mu\gamma^3 - 2\mu\gamma^2\}$$
$$M_1 = -\{2(1-\mu)\gamma^6 + 4(1-\mu)\gamma^5 + 2(1-\mu)\gamma^4\}$$
$$M_2 = -\{2\mu\gamma^6 + 8\mu\gamma^5 + 12\mu\gamma^4 + 8\mu\gamma^3 + 2\mu\gamma^2\}$$
$$N_1 = 3(1-\mu)\gamma^4$$
$$N_2 = 3\mu\gamma^4 + 12\mu\gamma^3 + 18\mu\gamma^2 + 12\mu\gamma + 3\mu$$

Now from the equation (13) we have

$$\delta = \frac{L_1 + M_1\beta_1 + M_2\beta_2 + N_1A_1 + N_2A_2}{P_1 + Q_1\beta_1 + Q_2\beta_2 + R_1A_1 + R_2A_2}$$

$$\delta = \frac{L_1 + M_1\beta_1 + M_2\beta_2 + N_1A_1 + N_2A_2}{P_1(1 + \frac{Q_1}{P_1}\beta_1 + \frac{Q_2}{P_1}\beta_2 + \frac{R_1}{P_1}A_1 + \frac{R_2}{P_1}A_2)}$$
$$= \frac{1}{P_1}\{L_1 + (M_1 - \frac{L_1Q_1}{P_1})\beta_1 + (M_2 - \frac{L_1Q_2}{P_1})\beta_2 + (N_1 - \frac{L_1R_1}{P_1})A_1 + (N_2 - \frac{L_1R_2}{P_1})A_2\}$$

Since $n^2 = \frac{1}{a}(1 + \frac{3e^2}{2} + \frac{3A_1}{2} + \frac{3A_2}{2})$,
[Neglecting the higher order terms, since A_1, A_2 and e are very small]

We have $P_1 = 18n^2\gamma^8 + 16n^2(5-\mu)\gamma^7 + 14n^2(10-4\mu)\gamma^6 + 6\{2n^2(10-6\mu)-2\}\gamma^5 + 5\{2n^2(5-4\mu)-4(1+\mu)\}\gamma^4 + 4\{2n^2(1-\mu)-2(1+5\mu)\}\gamma^3 - 24\mu\gamma^2 - 4\mu\gamma$,

$P_1^{-1} = X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2$ and

$$L_1 = U_1 + V_1e^2 + V_1A_1 + V_1A_2$$
$$U_1 = -\frac{2}{a}\gamma^9 - \frac{2(5-\mu)}{a}\gamma^8 - \frac{2(10-4\mu)}{a}\gamma^7 - [\frac{2(10-6\mu)}{a} - 2]\gamma^6 -$$
$$[\frac{2(5-4\mu)}{a} - 4\mu - 4]\gamma^5 - [\frac{2(1-\mu)}{a} - 2(1-\mu) - 12\mu]\gamma^4 + 8\mu\gamma^3 + 2\mu\gamma^2$$
$$V_1 = -\frac{3}{a}[\gamma^9 + (5-\mu)\gamma^8 + (10-4\mu)\gamma^7 + (10-6\mu)\gamma^6 + (5-4\mu)\gamma^5 + (1-\mu)\gamma^4]$$

$$X_1 = \{\frac{18}{a}\gamma^8 + \frac{16(5-\mu)}{a}\gamma^7 + \frac{14(10-4\mu)}{a}\gamma^6 + [\frac{12(10-6\mu)}{a} - 12]\gamma^5 +$$
$$[\frac{10(5-4\mu)}{a} - 20\mu - 20]\gamma^4 + [\frac{8(1-\mu)}{a} - 8 - 40\mu]\gamma^3 - 24\mu\gamma^2 - 4\mu\gamma\}^{-1}$$
$$Y_1 = \{\frac{27}{a}\gamma^8 + \frac{24(5-\mu)}{a}\gamma^7 + \frac{21(10-4\mu)\gamma^6}{a} + \frac{18(10-6\mu)}{a}\gamma^5 + \frac{15(5-4\mu)}{a}\gamma^4 +$$
$$\frac{12(1-\mu)}{a}\gamma^3\}.X_1$$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

$$\delta = X_1U_1 + (X_1V_1 + Y_1U_1)(e^2 + A_1 + A_2) + (X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)\{-2(1-\mu)[(\gamma^6 + 2\gamma^5 + \gamma^4) + (6\gamma^5 + 10\gamma^4 + 4\gamma^3)X_1U_1]\beta_1 +$$
$$-2\mu[(\gamma^6 + 4\gamma^5 + 6\gamma^4 + 4\gamma^3 + \gamma^2) + (6\gamma^5 + 20\gamma^4 + 24\gamma^3 + 12\gamma^2 + 2\gamma)U_1X_1]\beta_2 +$$
$$3(1-\mu)[\gamma^4 + 4\gamma^3U_1X_1]A_1 + 3\mu[(\gamma^4 + 4\gamma^3 + 6\gamma^2 + 4\gamma + 1) + (4\gamma^3 + 12\gamma^2 + 12\gamma + 4)U_1X_1)]A_2\}$$

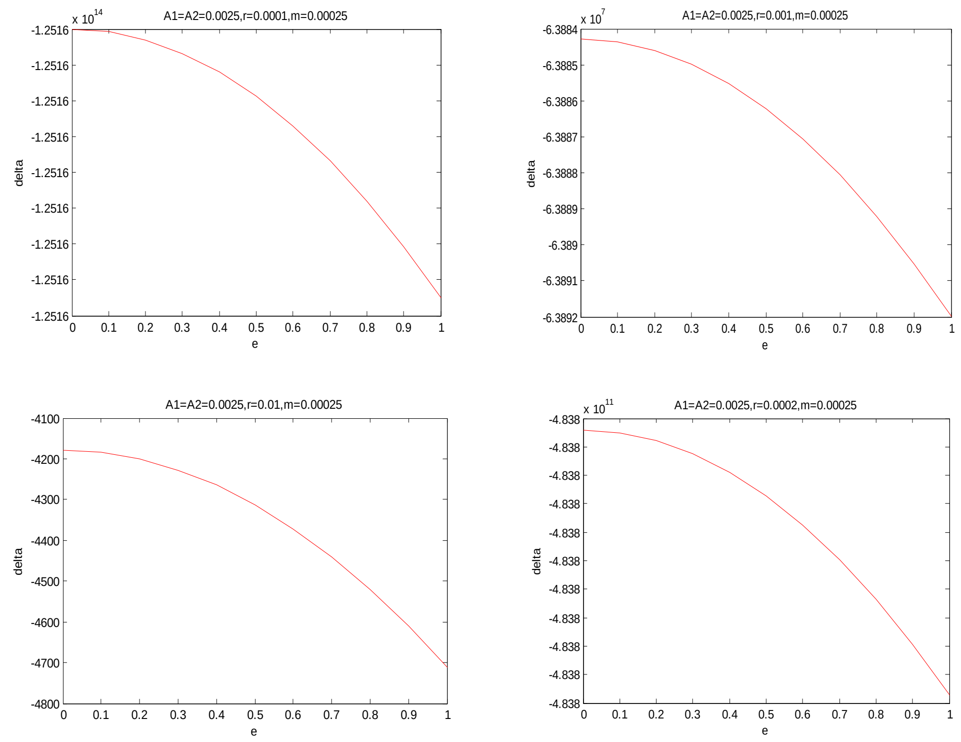
We have $\rho = \gamma + \delta$, i.e.

$$\rho = \gamma + X_1U_1 + (X_1V_1 + Y_1U_1)(e^2 + A_1 + A_2) + (X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)$$
$$\{-2(1-\mu)[(\gamma^6 + 2\gamma^5 + \gamma^4) + (6\gamma^5 + 10\gamma^4 + 4\gamma^3)X_1U_1]\beta_1 +$$
$$-2\mu[(\gamma^6 + 4\gamma^5 + 6\gamma^4 + 4\gamma^3 + \gamma^2) + (6\gamma^5 + 20\gamma^4 + 24\gamma^3 + 12\gamma^2 + 2\gamma)U_1X_1]\beta_2 +$$
$$3(1-\mu)[\gamma^4 + 4\gamma^3U_1X_1]A_1 + 3\mu[(\gamma^4 + 4\gamma^3 + 6\gamma^2 + 4\gamma + 1) + (4\gamma^3 + 12\gamma^2 + 12\gamma + 4)U_1X_1)]A_2\}$$

--- (14)

Where μ is the mass parameter, $\beta_1 = 1 - q_1$, $\beta_2 = 1 - q_2$ where q_1,q_2 *are* the radiation parameters, A_1 and A_2 are oblateness parameters of m_1 *and* m_2,e is the eccentricity and a is the semi major axis of the orbit, where γ is the value of ρ, the distance between L_1 and the smaller primary.

In order to investigate the effects of the oblateness of the primary on L_1, As far as numerical calculation of δ is concern, we have used a = 0.0001, β_1=0.0001, β_2=0.0001and μ=0.00025, μ=0.0025 and plotted the curve between the deviation, δ, and eccentricity, e. Similarly, we have also plotted the curve between by taking into account various values of oblateness parameters and γ, which indicates that the deviation of δ decreases. We have also investigated the effect of β_1, β_2 on the position of L_1 but this effect is very insignificant and the graphs are similar to the figures even by the change of A_1, A_2, β_1, β_2.as shown in Fig.1



ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

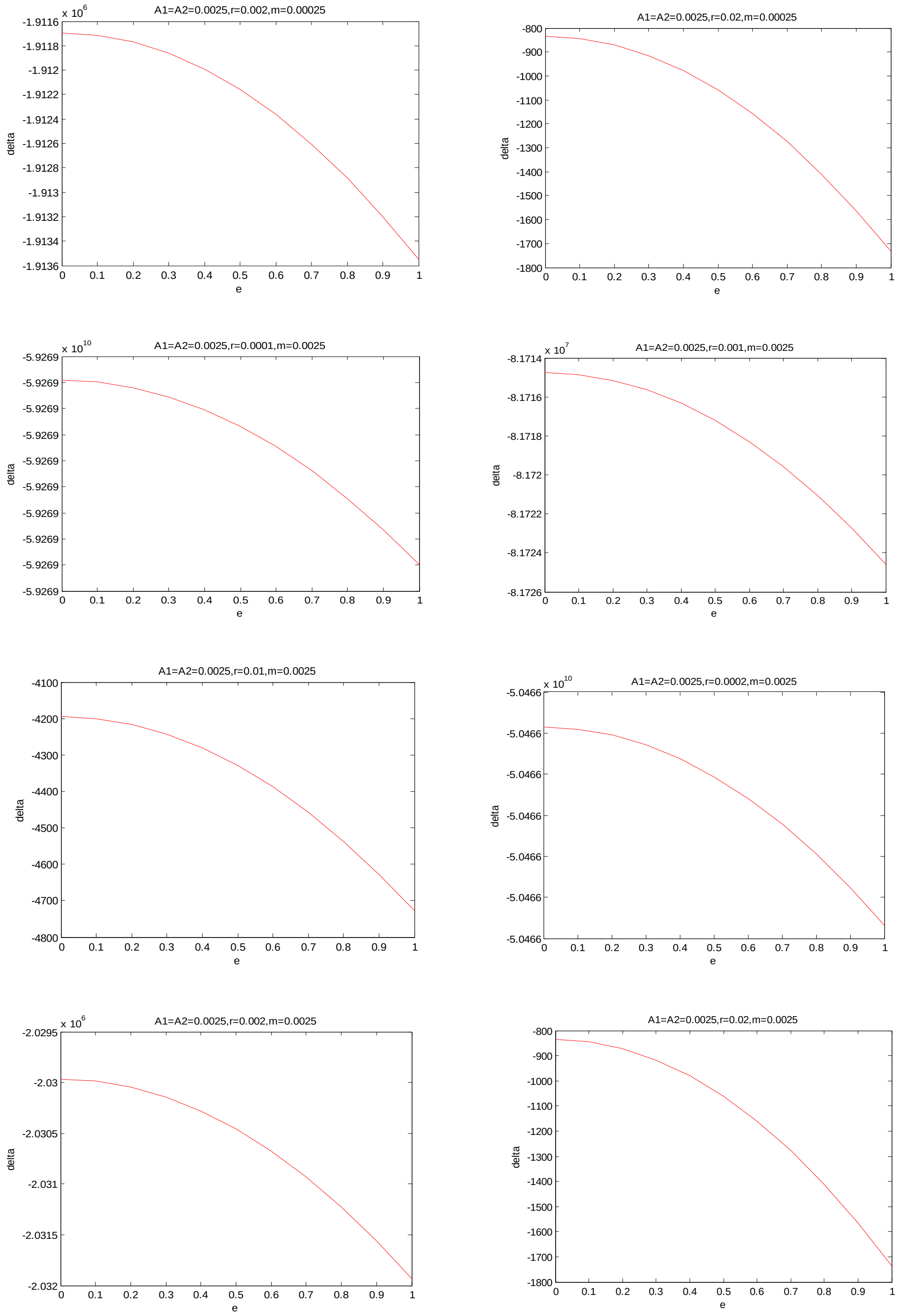


Fig.1: correlation between eccentricity,e and deviation delta, δ. we have used different values of a =0.0001, β_1=0.0001, β_2=0.0001and μ=0.00025, μ=0.0025 and plotted the curve between the deviation, δ, and eccentricity, e.γ=0.0001, γ =0.001. to see the deviation about L_1 points.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

Case(ii) :- For $\xi_1 < \xi < \xi_2$

Consider $\xi < \xi_2$, now $\xi_2 - \xi = \rho$; so that $\xi - \xi_1 = 1 - \rho$, we have
$\xi = \rho - 1 + \mu$; Substituting these values in equation (11), we get:

$$2n^2\rho^9 - 2n^2(5-\mu)\rho^8 + 2n^2(10-4\mu)\rho^7 - [2n^2(10-6\mu) + 2q_1(1-\mu) + 2\mu q_2]\rho^6 +$$
$$[2n^2(5-4\mu) + 4q_1(1-\mu) + 8\mu q_2]\rho^5 - [2n^2(1-\mu) + 2q_1(1-\mu) + 12\mu q_2 + 3(1-\mu)q_1A_1 + 3\mu q_2A_2]\rho^4 +$$
$$[8\mu q_2 + 12\mu q_2A_2]\rho^3 - [2\mu q_2 + 18\mu q_2A_2]\rho^2 + 12\mu q_2A_2\rho - 3\mu q_2A_2 = 0 \text{........(15)}$$

Assuming γ be the value of ρ in the classical restricted three body problem,

i.e. when e = 0, $A_1 = A_2 = 0$ and $q_1 = 1, q_2 = 1$.

By the process similar to case (i), eqn (15) becomes:

$$\delta = X_1U_1 + (X_1V_1 + Y_1U_1)(e^2 + A_1 + A_2) + (X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)$$
$$\{-2(1-\mu)[(\gamma^6 - 2\gamma^5 + \gamma^4) + (6\gamma^5 - 10\gamma^4 + 4\gamma^3)X_1U_1]\beta_1 +$$
$$-2\mu[(\gamma^6 - 4\gamma^5 - 6\gamma^4 - 4\gamma^3 + \gamma^2) + (6\gamma^5 - 20\gamma^4 + 24\gamma^3 - 12\gamma^2 + 2\gamma)U_1X_1]\beta_2 +$$
$$3(1-\mu)[\gamma^4 + 4\gamma^3U_1X_1]A_1 - 3\mu[(\gamma^4 + 4\gamma^3 - 6\gamma^2 + 4\gamma - 1) + (-4\gamma^3 + 12\gamma^2 - 12\gamma + 4)U_1X_1)]A_2\}$$

We have $\rho = \gamma + \delta$, i.e.

$$\rho = \gamma + X_1U_1 + (X_1V_1 + Y_1U_1)(e^2 + A_1 + A_2) + (X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)$$
$$\{-2(1-\mu)[(\gamma^6 - 2\gamma^5 + \gamma^4) + (6\gamma^5 - 10\gamma^4 + 4\gamma^3)X_1U_1]\beta_1 +$$
$$-2\mu[(\gamma^6 - 4\gamma^5 - 6\gamma^4 - 4\gamma^3 + \gamma^2) + (6\gamma^5 - 20\gamma^4 + 24\gamma^3 - 12\gamma^2 + 2\gamma)U_1X_1]\beta_2 +$$
$$3(1-\mu)[\gamma^4 + 4\gamma^3U_1X_1]A_1 - 3\mu[(\gamma^4 + 4\gamma^3 - 6\gamma^2 + 4\gamma - 1) + (-4\gamma^3 + 12\gamma^2 - 12\gamma + 4)U_1X_1)]A_2\}$$

Where

$$U_1 = -\frac{2}{a}\gamma^9 + \frac{2(5-\mu)}{a}\gamma^8 - \frac{2(10-4\mu)}{a}\gamma^7 + [\frac{2(10-6\mu)}{a} + 2]\gamma^6 -$$
$$[\frac{2(5-4\mu)}{a} + 4\mu + 4]\gamma^5 + [\frac{2(1-\mu)}{a} + 2(1-\mu) - 12\mu]\gamma^4 - 8\mu\gamma^3 + 2\mu\gamma^2$$
$$V_1 = -\frac{3}{a}[\gamma^9 - (5-\mu)\gamma^8 + (10-4\mu)\gamma^7 - (10-6\mu)\gamma^6 + (5-4\mu)\gamma^5 - (1-\mu)\gamma^4]$$

$$X_1 = \{\frac{18}{a}\gamma^8 - \frac{16(5-\mu)}{a}\gamma^7 + \frac{14(10-4\mu)}{a}\gamma^6 - [\frac{12(10-6\mu)}{a} + 12]\gamma^5 +$$
$$[\frac{10(5-4\mu)}{a} + 20\mu + 20]\gamma^4 - [\frac{8(1-\mu)}{a} + 8 + 40\mu]\gamma^3 + 24\mu\gamma^2 - 4\mu\gamma\}^{-1}$$
$$Y_1 = \{\frac{27}{a}\gamma^8 - \frac{24(5-\mu)}{a}\gamma^7 + \frac{21(10-4\mu)\gamma^6}{a} - \frac{18(10-6\mu)}{a}\gamma^5 + \frac{15(5-4\mu)}{a}\gamma^4 -$$
$$\frac{12(1-\mu)}{a}\gamma^3\}.\{a.[\frac{18}{a}\gamma^8 - \frac{16(5-\mu)}{a}\gamma^7 + \frac{14(10-4\mu)}{a}\gamma^6 - [\frac{12(10-6\mu)}{a} + 12]\gamma^5 +$$
$$[\frac{10(5-4\mu)}{a} + 20\mu + 20]\gamma^4 - [\frac{8(1-\mu)}{a} + 8 + 40\mu]\gamma^3 + 24\mu\gamma^2 - 4\mu\gamma]\}^{-1}$$

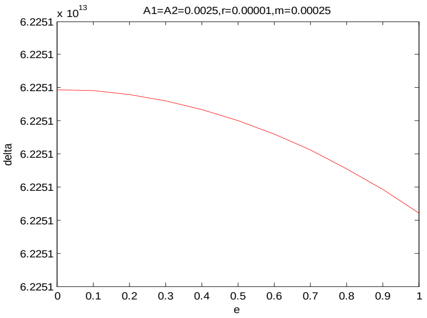
ρ, the distance between L_2 and the smaller primary.
Similar to case (i), we investigate the effects of the oblateness of the primary on L_2, which is indicated that the deviation δ is decreasing. We investigated the effect of β_1, β_2 on the position of L_2 but this effect is very insignificant, and the graphs are similar to the figure even if the values of A_1, A_2, β_1, β_2 are changed as shown in Fig.2

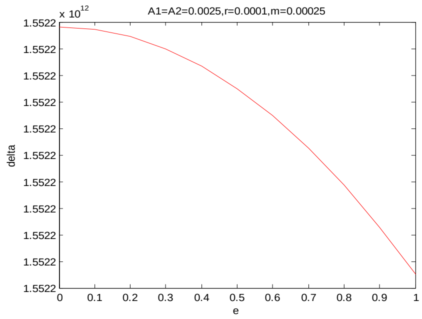
ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

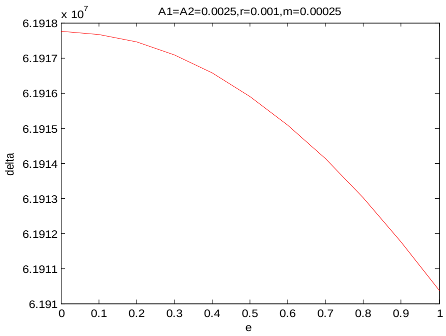
International Journal of Innovative Research in Science, Engineering and Technology

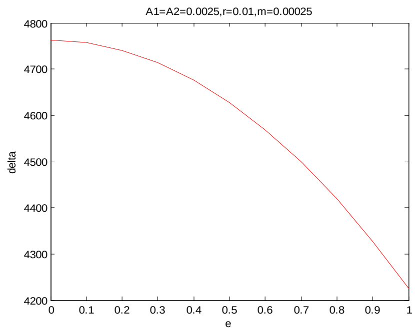
(An ISO 3297: 2007 Certified Organization)

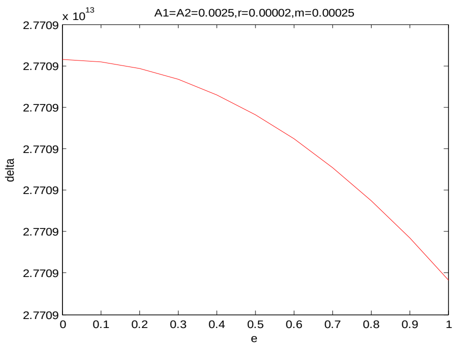
Vol. 4, Issue 6, June 2015

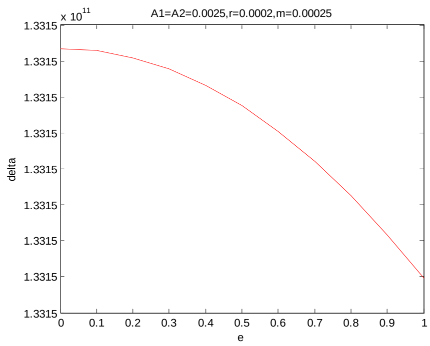


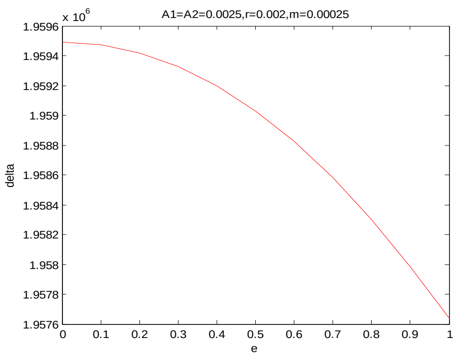


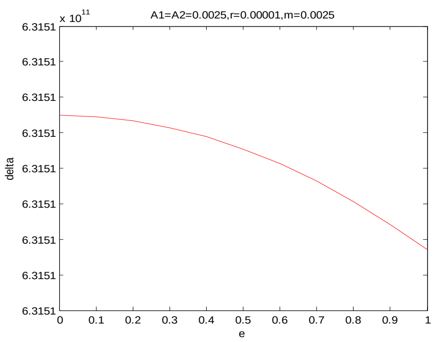










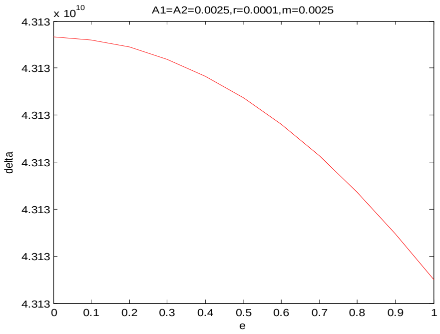


ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015



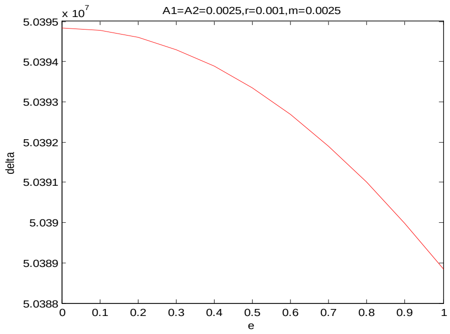


Fig.2: correlation between eccentricity and deviation delta, δ. we have used different values of a = 0.0001, β_1=0.0001, β_2=0.0001and μ=0.00025, μ=0.0025 and plotted the curve between the deviation, δ, and eccentricity, e.γ=0.0001, γ =0.001. to see the deviation about L_2 points.

Case (iii):- Assuming $\xi < \xi_1$
Consider $\xi_1 - \xi = \rho$; so that $\xi_2 - \xi = 1+\rho$, since we get $\xi = \rho + \mu$;
Substituting these values in equation (11) we get

$$2n^2\rho^9 + 2n^2(4+\mu)\rho^8 + 2n^2(6+4\mu)\rho^7 + [2n^2(4+6\mu) - 2q_1(1-\mu) - 2\mu q_2]\rho^6 +$$
$$[2n^2(1+4\mu) + 8q_1(1-\mu) - 4\mu q_2]\rho^5 + [2n^2\mu - 3q_1(1-\mu)(4+A_1) - \mu q_2(2+3A_2)]\rho^4 -$$
$$[(1-\mu)q_1(8+12A_1)]\rho^3 - [2(1-\mu)q_1(1+9A_1)]\rho^2 - 12(1-\mu)q_1A_1\rho - 3(1-\mu)q_1A_1 = 0 \quad \text{--(16)}$$

Assuming γ be the value of ρ in the classical restricted three body problem,
i.e. when $e = 0$, $A_1 = A_2 = 0$ and $q_1=1, q_2=1$.
By the process similar to case (i), Substituting the value of ρ in the equation (16), we get:

$$\delta = X_1U_1 + (X_1V_1 + Y_1U_1)(e^2 + A_1 + A_2) + (X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)$$
$$\{-2(1-\mu)[(\gamma^6 - 4\gamma^5 + 6\gamma^4 + 4\gamma^3 + \gamma^2) + (6\gamma^5 - 20\gamma^4 + 12\gamma^3 + 12\gamma^2 + 2\gamma)X_1U_1]\beta_1 +$$
$$-2\mu[(\gamma^6 + 2\gamma^5 + \gamma^4) + (6\gamma^5 + 10\gamma^4 + 4\gamma^3)U_1X_1]\beta_2 +$$
$$3(1-\mu)[(\gamma^4 + 4\gamma^3 + 6\gamma^2 + 4\gamma + 1) + (4\gamma^3 + 12\gamma^2 + 12\gamma + 4)U_1X_1]A_1 + 3\mu[(\gamma^4 + 4\gamma^3U_1X_1)]A_2\}$$

We have $\rho = \gamma + \delta$,

$$\rho = \gamma + X_1U_1 + (X_1V_1 + Y_1U_1)(e^2 + A_1 + A_2) + (X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)$$
$$\{-2(1-\mu)[(\gamma^6 - 4\gamma^5 + 6\gamma^4 + 4\gamma^3 + \gamma^2) + (6\gamma^5 - 20\gamma^4 + 12\gamma^3 + 12\gamma^2 + 2\gamma)X_1U_1]\beta_1 +$$
$$-2\mu[(\gamma^6 + 2\gamma^5 + \gamma^4) + (6\gamma^5 + 10\gamma^4 + 4\gamma^3)U_1X_1]\beta_2 +$$
$$3(1-\mu)[(\gamma^4 + 4\gamma^3 + 6\gamma^2 + 4\gamma + 1) + (4\gamma^3 + 12\gamma^2 + 12\gamma + 4)U_1X_1]A_1 + 3\mu[(\gamma^4 + 4\gamma^3U_1X_1)]A_2\}$$

Where

$$X_1 = \{\frac{18}{a}\gamma^8 \frac{16(4+\mu)}{a}\gamma^7 + \frac{14(6+4\mu)}{a}\gamma^6 + [\frac{12(4+6\mu)}{a} - 12]\gamma^5 +$$
$$[\frac{10(1+4\mu)}{a} - 60\mu + 40]\gamma^4 + [\frac{8\mu}{a} - 48(1-\mu) - 8\mu]\gamma^3 - 24(1-\mu)\gamma^2 - 4(1-\mu)\gamma\}$$

$$Y_1 = \{\frac{27}{a}\gamma^8 + \frac{24(4+\mu)}{a}\gamma^7 + \frac{21(6+4\mu)\gamma^6}{a} + \frac{18(4+6\mu)}{a}\gamma^5 + \frac{15(1+4\mu)}{a}\gamma^4 +$$
$$\frac{12\mu}{a}\gamma^3\}.\{\frac{18}{a}\gamma^8 \frac{16(4+\mu)}{a}\gamma^7 + \frac{14(6+4\mu)}{a}\gamma^6 + [\frac{12(4+6\mu)}{a} - 12]\gamma^5 +$$
$$[\frac{10(1+4\mu)}{a} - 60\mu + 40]\gamma^4 + [\frac{8\mu}{a} - 48(1-\mu) - 8\mu]\gamma^3 - 24(1-\mu)\gamma^2 - 4(1-\mu)\gamma\}^{-1}$$

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

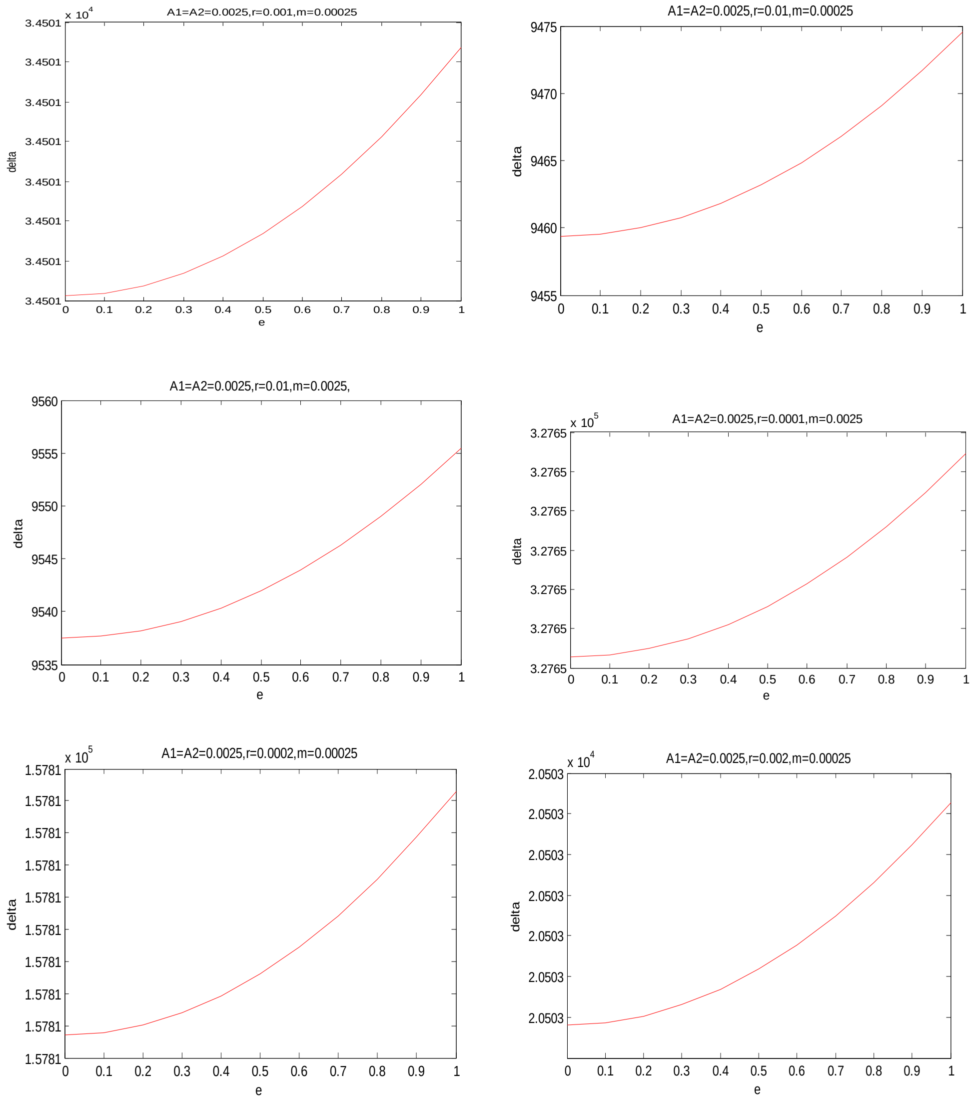
(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

$$U_1 = -\frac{2}{a}\gamma^9 - \frac{2(4+\mu)}{a}\gamma^8 - \frac{2(6+4\mu)}{a}\gamma^7 - [\frac{2(4+6\mu)}{a} - 2]\gamma^6 -$$
$$[\frac{2(1+4\mu)}{a} - 12\mu + 8]\gamma^5 - [\frac{2\mu}{a} - 12(1-\mu) - 2\mu]\gamma^4 + 8(1-\mu)\gamma^3 + 2(1-\mu)\gamma^2$$
$$V_1 = -\frac{3}{a}[\gamma^9 + (4+\mu)\gamma^8 + (6+4\mu)\gamma^7 + (4+6\mu)\gamma^6 + (1+4\mu)\gamma^5 + \mu\gamma^4]$$

ρ, the distance between L_3 and the bigger primary.

In order to investigate the effects of the oblateness of the primary on L_3, As far as numerical calculation of δ is concern, we have used a = 0.0001, β_1=0.0001, β_2=0.0001 and plotted the curve between the deviation and eccentricity. Similarly, we have also plotted the curve between by taking into account of various values of oblateness parameters, which is indicated that the deviation δ is increasing .We have also investigated the effect of β_1, β_2 on the position of L_3 but this effect is very insignificant, and the graphs are similar to the figure even if the values of A_1, A_2, β_1, β_2 are changed as shown in Fig.3.



 DOI:10.15680/IJIRSET.2015.0406003

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

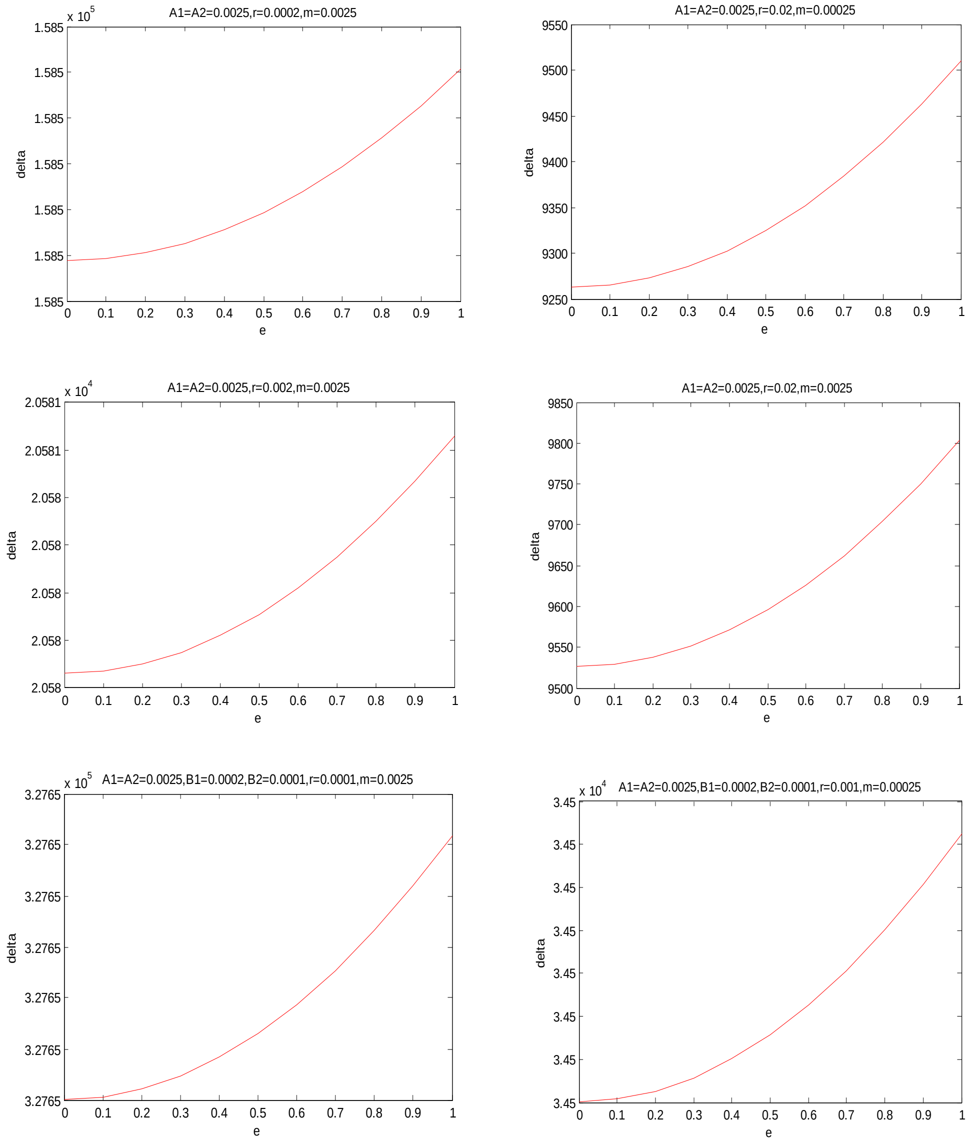


Fig.3: correlation between eccentricity and deviation delta, δ. we have used different values of a = 0.0001, β_1=0.0001, β_2=0.0001and μ=0.00025, μ=0.0025 and plotted the curve between the deviation, δ, and eccentricity, e.γ=0.0001, γ =0.001. to see the deviation about L_3 points.

IV. CONCLUSION

In this paper we study the elliptical restricted three body problem which has not been studied much in comparison to the circular RTBP further we have both major bodies radiating and oblate. We obtain the equations of motion of the infinitesimal body moving under the influence of two major bodies which are both oblate and radiating and the major bodies move in elliptical orbits around their common centre of mass, the equations (1) & (2) represent the equations of motion of the infinitesimal body. The locations of triangular librational points L_4 & L_5 are obtained and equations (9) and (10) denote their positions. We note that the position of L_4 & L_5 depend on the parameters for oblateness, radiation coefficients, eccentricity and the semi-major axis. While investigating the collinear librational points L_1, L_2 and L_3 we do not exactly find the location but rather investigate the deviations, δ of their locations from those in the classical case

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 6, June 2015

and study the dependence of the deviation on variations in eccentricity, oblateness parameters and radiation coefficients. Numerically we have obtained the positions of L_1, L_2, L_3 and using MATLAB we plot the curves between eccentricity and the deviation δ for different values of oblateness parameters A_1, A_2, parameters β_1, β_2 which dependent on the radiation parameters and the semi-major axis a. The variation in the deviation of the location from the classical case for L_1, L_2 and L_3 are given in Fig.1, Fig. 2 and Fig. 3 respectively. We note from Fig. 1 that the Lagrangian points L_1 comes closer to the secondary mass, from Fig.2, we note that L_2 moves away from primary mass and comes closer to the secondary mass and from Fig.3, we note that L_3 points recede away from primary mass as the eccentricity, e increases. We have also shown that when $e = 0$, $A_1 = A_2 = 0$ and $q_1 = 1$, $q_2 = 1$ we obtain the same results as in classical case.

REFERENCES

[1] Mc Cuskey, S.W. Introduction to Celestial Mechanics, Wesley Publishing company, New York., 1963.
[2] Danby, J.M.A. Astronomical journal 69,165-172 , 1964.
[3] Szebehely V. Theory of orbits, Academic press, Inc., 1967.
[4] Sharma, R.K. Astrophysics and space science, 135,pp 271-281, 1987.
[5] Khasan, S.N. Cosmic Research, vol 34. No.2, pp146-151, 1996.
[6] Sahoo, S.K. and Ishwar, B. Bull. Astr. Soc. India, No28,pp 579-586, 2000.
[7] Om Prakash Raman.et.al, International journal of Innovative Research in Science, Engineering and Technology,vol 2, No.10,pp5682-5686, 2013.