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Modulation and Parsvals Indentity of Two Dimensional Fractional Fourier-Mellin Transform

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ABSTRACT: In electronic world an integral transform play an important role due to its properties. Fourier-Mellin transform mainly use in the radar system, reconstruction of gray scale images, in detection of human face etc.In this paper we proved some properties like modulation and Parsvals identity of two dimensional Fourier-Mellin transform.

KEYWORDS: Two-Dimensional Fractional Fourier-Mellin Transform, Testing Function Space, Generalized function.

I.INTRODUCTION

Integral transforms find special applicability within other scientific and mathematical disciplines. Integral transforms have been in wide use during the past two centuries as a tool to solve various problems in pure and applied mathematics. Many integral transforms were originally introduced to solve specific problems, but over the course of time have been found to be of use in the solution of other problems as well.

Due to shift invariant property of Fourier transform and scaled invariant property of Mellin transform, the Fourier-Mellin transform (FMT) is a very powerful tool in image restoration, pattern recognition. G.J.Pratt used the Fourier-Mellin transformation for the comparison of plant leaves[5]. The complexity of radar target detection, identification, tracking and association, a trajectory-oriented SLAM process based on the Fourier-Mellin Transform was developed for target assumptions about their position and nature were avoided by Paul Checchin, Franck Gérossier, Christophe Blanc, Roland Chapuis and Laurent Trassoudaine [1].

Robust image registration using log-polar transform. The log-polar transformation is based on fourier-Mellin transform it is used to recover rotation and scale,it is limited in use to two-fold scale factors.The purpose of the log-polar registration module is to bring two images into alignment using only rotation, scale, and translation [2].

In the field of image registration hyperspectral image play an important role in agriculture, geology, military, atmosphere, environment and in many fields. An image registration with hyperspectral data based on Fourier-Mellin transform [3]. A further development in the use of the Fourier-Mellin transform is its application into the radar classification of ships by Zwicke etal. Fourier-Mellin transform is used to identify plant leaves at various life stages based on the leaves shape or contour. Fourier-Mellin transform is also used in estimation of optical flow [4].

Purpose of this paper is to generalized two dimensional fractional Fourier-Mellin transform in the distributional sense. Also we proposed modulation and Parsvals Identity of two dimensional Fourier-Mellin transform.

II. TWO-DIMENSIONAL FRACTIONAL FOURIER-MELLIN TRANSFORM

2.1 Definition of two-dimensional fractional Fourier-Mellin transform:

The two-dimensional fractional Fourier-Mellin transform with parameters α and θ of $f(x,y,t,q)$ denoted by 2DFRFMT$\{f(x,y,t,q)\}$ performs a linear operation, given by the integral transform. 2DFRFMT

$$\{f(x,y,t,q)\} = F_{\alpha,\theta}(\xi,\eta,\lambda,\chi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,t,q)K_{\alpha,\theta}(x,y,t,q,\xi,\eta,\lambda,\chi)\,dxdydtdq \text{----(1)}$$

where $K_{\alpha,\theta}(x,y,t,q,\xi,\eta,\lambda,\chi) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{1}{2sin\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha-2(x\xi+y\eta)}\, t^{\frac{2\pi i\lambda}{sin\theta}-1}\, q^{\frac{2\pi i\chi}{sin\theta}-1}\, e^{\frac{\pi i}{tan\theta}[\lambda^2+\chi^2+log^2t+log^2q]}$

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$$= C_{1\alpha} e^{iC_{2\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha - 2(x\xi+y\eta)} t^{C_{1\theta} i\lambda -1} q^{C_{1\theta} i\chi -1} e^{C_{1\theta} i[\lambda^2+\chi^2+log^2 t+log^2 q]}$$

where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{1}{2sin\alpha} C_{1\theta} = \frac{2\pi}{sin\theta}$, $C_{2\theta} = \frac{\pi}{tan\theta}$ $\quad 0 < \alpha < \frac{\pi}{2}$, $0 < \theta < \frac{\pi}{2}$. ---(2)

2.2 The Test Function

An infinitely differentiable complex valued smooth function $\emptyset(x, y, t, q)$ on R^n belongs to $E(R^n)$, if for each compact set $I \subset S_{a,b}$, $J \subset S_{c,d}$ where

$S_{a,b} = \{x, y: x, y \epsilon R^n, |x| \leq a, |y| \leq b, a > 0, b > 0\}$

$S_{c,d} = \{t, q: t, q \epsilon R^n, |t| \leq c, |q| \leq d, c > 0, d > 0\}$

$\gamma_{E,m,n,k,l}[\emptyset(x,y,t,q)] = \sup_{\substack{x,y\epsilon I \\ t,q\epsilon J}} \left|D^{m,n,k,l}_{x,y,t,q} \emptyset(x,y,t,q)\right| < \infty$ ---(3)

Thus $E(R^n)$ will denote the space of all $\emptyset(x, y, t, q) \epsilon E(R^n)$ with compact support contained in $S_{a,b} \cap S_{c,d}$.

Note that the space E is complete and therefore a Frechet space. Moreover, we say that $f(x, y, t, q)$ is a fractional Fourier-Mellin transformable if it is a member of E.

III. DISTRIBUTIONAL TWO DIMENSIONAL FRACTIONAL FOURIER-MELLIN TRANSFORM (2DFRFMT)

The two dimensional distributional Fractional Fourier -Mellin transform of $f(x, y, t, q) \epsilon E^*(R^n)$ can be defined by

$2DFRFMT\{f(x, y, t, q)\} = F_{\alpha,\theta}(\xi, \eta, \lambda, \chi) = \langle f(x, y, t, q), K_{\alpha,\theta}(x, y, t, q, \xi, \eta, \lambda, \chi)\rangle$---(4)

where, $K_{\alpha,\theta}(x, y, t, q, \xi, \eta, \lambda, \chi) = \sqrt{\frac{1-icot\alpha}{2\pi}} e^{\frac{1}{2sin\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha - 2(x\xi+y\eta)} t^{\frac{2\pi i\lambda}{sin\theta}-1} q^{\frac{2\pi i\chi}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[\lambda^2+\chi^2+log^2 t+log^2 q]}$

$= C_{1\alpha} e^{iC_{2\alpha}[(x^2+y^2+\xi^2+\eta^2)cos\alpha - 2(x\xi+y\eta)} t^{C_{1\theta} i\lambda -1} q^{C_{1\theta} i\chi -1} e^{C_{1\theta} i[\lambda^2+\chi^2+log^2 t+log^2 q]}$

where $C_{1\alpha} = \sqrt{\frac{1-icot\alpha}{2\pi}}$, $C_{2\alpha} = \frac{1}{2sin\alpha}$, $C_{1\theta} = \frac{2\pi}{sin\theta}$, $C_{2\theta} = \frac{\pi}{tan\theta}$ $\quad 0 < \alpha < \frac{\pi}{2}$, $0 < \theta < \frac{\pi}{2}$. ---(5)

Right hand side of equation (4) has a meaning as the application of $f(x, y, t, q) \epsilon E^*(R^n)$ to $K_{\alpha,\theta}(x, y, t, q, \xi, \eta, \lambda, \chi) \epsilon E$.
It can be extended to the complex space as an entire function given by

$2DFRFMT\{f(x, y, t, q)\} = F_{\alpha,\theta}(\xi', \eta', \lambda', \chi')$

$= \langle f(x, y, t, q), K_{\alpha,\theta}(x, y, t, q, \xi', \eta', \lambda', \chi')\rangle$---(6)

The right hand side is meaningful because for each $\xi', \eta', \lambda', \chi' \epsilon C^n$, $K_{\alpha,\theta}(x, y, t, q, \xi', \eta', \lambda', \chi') \epsilon E$ as a function of x, y, t, q.

IV- MODULATION PROPERTY

(i) Prove that-

$2DFRFMT\{f(x, y, u, v)\cos(ax + by + cu + dv)\}(p, q, r, s)$

$= \frac{1}{2} e^{i[(ap+bq)cos\alpha - \frac{(a^2+b^2)}{2} cos\alpha asin\alpha]}$

$\{2DFRFMT f(x, y, u, v) e^{i(cu+dv)}[(p - asin\alpha), (q - bsin\alpha), r, s]\}$

$+ \frac{1}{2} e^{-i[(ap+bq)cos\alpha + \frac{(a^2+b^2)}{2} cos\alpha asin\alpha]}$

$\{2DFRFMT f(x, y, u, v) e^{-i(cu+dv)}[(p + asin\alpha), (q + bsin\alpha), r, s]\}$

Proof-

$2DFRFMT\{f(x, y, u, v)\cos(ax + by + cu + dv)\}(p, q, r, s)$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x, y, u, v) K_{\alpha,\theta}(x, y, u, v, p, q, r, s) \cos(ax + by + cu + dv)\, dxdydudv$$

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$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]} \cos(ax+by+cu+dv)\,dxdydudv$$

$$= \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]} \left[e^{i(ax+by+cu+dv)} + e^{-i(ax+by+cu+dv)}\right] dxdydudv$$

$$= \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)} e^{i(ax+by)}\,dxdydudv$$

$$+ \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]-i(cu+dv)} e^{-i(ax+by)}\,dxdydudv$$

$$= \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}x^2cot\alpha-i(pcosec\alpha-a)x}\, e^{\frac{i}{2}y^2cot\alpha-i(qcosec\alpha-b)y}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)}\,dxdydudv$$

$$+ \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}x^2cot\alpha-i(pcosec\alpha+a)x}\, e^{\frac{i}{2}y^2cot\alpha-i(qcosec\alpha+b)y}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]-i(cu+dv)}\,dxdydudv$$

$$= \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}x^2cot\alpha-icosec\alpha\,(p-asin\alpha)x}\, e^{\frac{i}{2}y^2cot\alpha-icosec\alpha\,(q-bsin\alpha)y}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)}\,dxdydudv$$

$$+ \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}x^2cot\alpha-icosec\alpha\,(p+asin\alpha)x}\, e^{\frac{i}{2}y^2cot\alpha-icosec\alpha\,(q+bsin\alpha)y}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]-i(cu+dv)}\,dxdydudv$$

$$= \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{i[(ap+bq)cos\alpha-\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}[x^2+(p-asin\alpha)^2]cot\alpha}\, e^{-icosec\alpha\,(p-asin\alpha)x}\, e^{\frac{i}{2}[y^2+(q-bsin\alpha)^2]cot\alpha}\, e^{-icosec\alpha\,(q-bsin\alpha)y}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)}\,dxdydudv$$

$$+ \frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{-i[(ap+bq)cos\alpha+\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}[x^2+(p+asin\alpha)^2]cot\alpha}\, e^{-icosec\alpha\,(p+asin\alpha)x}\, e^{\frac{i}{2}[y^2+(q+bsin\alpha)^2]cot\alpha}\, e^{-icosec\alpha\,(q+bsin\alpha)y}$$

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$$u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)}dxdydudv$$

$$=\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}e^{i[(ap+bq)cos\alpha-\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)$$

$$e^{\frac{i}{2}\{[x^2+y^2+(p-asin\alpha)^2+(q-bsin\alpha)^2]cot\alpha-2cosec\alpha[(p-asin\alpha)x+(q-bsin\alpha)y]\}}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}$$

$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}e^{i(cu+dv)}dxdydudv$$

$$+\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}e^{i[(ap+bq)cos\alpha+\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)$$

$$e^{\frac{i}{2}\{[x^2+y^2+(p+asin\alpha)^2+(q+bsin\alpha)^2]cot\alpha-2cosec\alpha[(p+asin\alpha)x+(q+bsin\alpha)y]\}}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}$$

$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}e^{-i(cu+dv)}dxdydudv$$

$$=\frac{1}{2}e^{i[(ap+bq)cos\alpha-\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}$$

$$\{2DFRFMTf(x,y,u,v)e^{i(cu+dv)}[(p-asin\alpha),(q-bsin\alpha),r,s]\}$$

$$+\frac{1}{2}e^{-i[(ap+bq)cos\alpha+\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}$$

$$\{2DFRFMTf(x,y,u,v)e^{-i(cu+dv)}[(p+asin\alpha),(q+bsin\alpha),r,s]\}$$

(ii) Prove that-

$$2DFRFMT\{f(x,y,u,v)\sin(ax+by+cu+dv)\}(p,q,r,s)$$

$$=\frac{1}{2}e^{i[(ap+bq)cos\alpha-\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}$$

$$\{2DFRFMTf(x,y,u,v)e^{i(cu+dv)}[(p-asin\alpha),(q-bsin\alpha),r,s]\}$$

$$-\frac{1}{2}e^{-i[(ap+bq)cos\alpha+\frac{(a^2+b^2)}{2}cos\alpha asin\alpha]}$$

$$\{2DFRFMTf(x,y,u,v)e^{-i(cu+dv)}[(p+asin\alpha),(q+bsin\alpha),r,s]\}$$

Proof-

$$2DFRFMT\{f(x,y,u,v)\cos(ax+by+cu+dv)\}(p,q,r,s)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)K_{\alpha,\theta}(x,y,u,v,p,q,r,s)\sin(ax+by+cu+dv)dxdydudv$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}$$

$$u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}\sin(ax+by+cu+dv)dxdydudv$$

$$=\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}$$

$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}\left[e^{i(ax+by+cu+dv)}-e^{-i(ax+by+cu+dv)}\right]dxdydudv$$

$$=\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}$$

$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)}e^{i(ax+by)}dxdydudv$$

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$$-\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}\, u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1}$$
$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]-i(cu+dv)}e^{-i(ax+by)}dxdydudv$$

$$=\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)e^{\frac{i}{2}x^2cot\alpha-i(pcosec\alpha-a)x}e^{\frac{i}{2}y^2cot\alpha-i(qcosec\alpha-b)y}$$
$$u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)}dxdydudv$$

$$-\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}(p^2+q^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)e^{\frac{i}{2}x^2cot\alpha-i(pcosec\alpha+a)x}e^{\frac{i}{2}y^2cot\alpha-i(qcosec\alpha+b)y}$$
$$u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]-i(cu+dv)}dxdydudv$$

$$=\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)e^{\frac{i}{2}[(p-asin\alpha)^2cot\alpha}e^{iapcos\alpha}e^{\frac{-ia^2}{2}cos\alpha sin\alpha}e^{\frac{i}{2}x^2cot\alpha-icosec\alpha(p-asin\alpha)x}$$
$$e^{\frac{i}{2}[(q-bsin\alpha)^2cot\alpha}e^{ibqcos\alpha}e^{\frac{-ib^2}{2}cos\alpha sin\alpha}e^{\frac{i}{2}y^2cot\alpha-icosec\alpha(q-bsin\alpha)y}$$
$$u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]+i(cu+dv)}dxdydudv$$

$$-\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)e^{\frac{i}{2}[(p+asin\alpha)^2cot\alpha}e^{-iapcos\alpha}e^{\frac{-ia^2}{2}cos\alpha sin\alpha}e^{\frac{i}{2}x^2cot\alpha-icosec\alpha(p+asin\alpha)x}$$
$$e^{\frac{i}{2}[(q+bsin\alpha)^2cot\alpha}e^{-ibqcos\alpha}e^{\frac{-ib^2}{2}cos\alpha sin\alpha}e^{\frac{i}{2}y^2cot\alpha-icosec\alpha(q+bsin\alpha)y}$$
$$u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]-i(cu+dv)}dxdydudv$$

$$=\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}e^{i[(ap+bq)cos\alpha-\frac{(a^2+b^2)}{2}cos\alpha sin\alpha]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)$$
$$e^{\frac{i}{2}\{[x^2+y^2+(p-asin\alpha)^2+(q-bsin\alpha)^2]cot\alpha-2cosec\alpha[(p-asin\alpha)x+(q-bsin\alpha)y]\}}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}$$
$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}e^{i(cu+dv)}dxdydudv$$

$$-\frac{1}{2}\sqrt{\frac{1-icot\alpha}{2\pi}}e^{i[(ap+bq)cos\alpha+\frac{(a^2+b^2)}{2}cos\alpha sin\alpha]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)$$
$$e^{\frac{i}{2}\{[x^2+y^2+(p+asin\alpha)^2+(q+bsin\alpha)^2]cot\alpha-2cosec\alpha[(p+asin\alpha)x+(q+bsin\alpha)y]\}}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}$$
$$e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}e^{-i(cu+dv)}dxdydudv$$

$$=\frac{1}{2}e^{i[(ap+bq)cos\alpha-\frac{(a^2+b^2)}{2}cos\alpha sin\alpha]}$$
$$\{2DFRFMTf(x,y,u,v)e^{i(cu+dv)}[(p-asin\alpha),(q-bsin\alpha),r,s]\}$$
$$-\frac{1}{2}e^{-i[(ap+bq)cos\alpha+\frac{(a^2+b^2)}{2}cos\alpha sin\alpha]}$$
$$\{2DFRFMTf(x,y,u,v)e^{-i(cu+dv)}[(p+asin\alpha),(q+bsin\alpha),r,s]\}$$

V- PARSVALS IDENTITY

$2DFRFMT\{f(x,y,u,v)\}=F_{\alpha,\theta}(p,q,r,s)$ & $2DFRFMT\{g(x,y,u,v)\}=G_{\alpha,\theta}(p,q,r,s)$

(i) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\,\overline{g(x,y,u,v)}dxdydudv=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} F_{\alpha,\theta}(p,q,r,s)\,\overline{G_{\alpha,\theta}(p,q,r,s)}\,dpdqdrds$

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(ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}|f(x,y,u,v)|^2\,dxdydudv=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\left|F_{\alpha,\theta}(p,q,r,s)\right|^2\ dpdqdrds$

Proof-

$2DFRFMT\{g(x,y,u,v)\}(p,q,r,s)=G_{\alpha,\theta}(p,q,r,s)$

$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}g(x,y,u,v)$

$\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}u^{\frac{2\pi ir}{sin\theta}-1}v^{\frac{2\pi is}{sin\theta}-1}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}\,dxdydudv$

By using inversion formula for two dimensional fractional Fourier Transform

$g(x,y,u,v)$

$$=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\frac{1}{sin^2\alpha sin^2\theta}G_{\alpha,\theta}(p,q,r,s)\left[\frac{1-icot\alpha}{2\pi}\right]^{-1/2}$$

$$e^{\frac{-i}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)]}u^{\frac{-2\pi ir}{sin\theta}}v^{\frac{-2\pi is}{sin\theta}}e^{\frac{-\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}dpdqdrds$$

$$=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\frac{1}{sin^2\alpha sin^2\theta}G_{\alpha,\theta}(p,q,r,s)\left[\frac{1-icot\alpha}{2\pi}\right]^{-1/2}$$

$$e^{\frac{-i}{2}(x^2+y^2)cot\alpha}\,e^{\frac{-i}{2}(p^2+q^2)cot\alpha}\,e^{i(xp+yq)]cosec\alpha}\,u^{\frac{-2\pi ir}{sin\theta}}v^{\frac{-2\pi is}{sin\theta}}e^{\frac{-\pi i}{tan\theta}[r^2+s^2]}e^{\frac{-\pi i}{tan\theta}[log^2u+log^2v]}dpdqdrds$$

$$=\frac{1}{(2\pi)^{3/2}sin^2\alpha sin^2\theta\sqrt{1-icot\alpha}}e^{\frac{-i}{2}(x^2+y^2)cot\alpha\frac{-\pi i}{tan\theta}[log^2u+log^2v]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}G_{\alpha,\theta}(p,q,r,s)$$

$$e^{\frac{-i}{2}(p^2+q^2)cot\alpha}\,e^{i(xp+yq)]cosec\alpha}\,u^{\frac{-2\pi ir}{sin\theta}}v^{\frac{-2\pi is}{sin\theta}}e^{\frac{-\pi i}{tan\theta}[r^2+s^2]}dpdqdrds$$

Taking complex conjugate of above term

$$\overline{g(x,y,u,v)}=\frac{1}{(2\pi)^{3/2}sin^2\alpha sin^2\theta\sqrt{1+icot\alpha}}e^{\frac{i}{2}(x^2+y^2)cot\alpha\frac{\pi i}{tan\theta}[log^2u+log^2v]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\overline{G_{\alpha,\theta}(p,q,r,s)}$$

$$e^{\frac{i}{2}(p^2+q^2)cot\alpha}\,e^{-i(xp+yq)]cosec\alpha}\,u^{\frac{2\pi ir}{sin\theta}}v^{\frac{2\pi is}{sin\theta}}e^{\frac{\pi i}{tan\theta}[r^2+s^2]}dpdqdrds$$

Now consider,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)\,\overline{g(x,y,u,v)}dxdydudv$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)\,dxdydudv$$

$$\{\frac{1}{(2\pi)^{3/2}sin^2\alpha sin^2\theta\sqrt{1+icot\alpha}}e^{\frac{i}{2}(x^2+y^2)cot\alpha\frac{\pi i}{tan\theta}[log^2u+log^2v]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\overline{G_{\alpha,\theta}(p,q,r,s)}$$

$$e^{\frac{i}{2}(p^2+q^2)cot\alpha}\,e^{-i(xp+yq)]cosec\alpha}\,u^{\frac{2\pi ir}{sin\theta}}v^{\frac{2\pi is}{sin\theta}}e^{\frac{\pi i}{tan\theta}[r^2+s^2]}dpdqdrds\}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\frac{1}{(2\pi)^{3/2}sin^2\alpha sin^2\theta\sqrt{1+icot\alpha}}\overline{G_{\alpha,\theta}(p,q,r,s)}\,dpdqdrds$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)\,e^{\frac{i}{2}[(x^2+y^2+p^2+q^2)cot\alpha-2(xp+yq)cosec\alpha]}$$

$$u^{\frac{2\pi ir}{sin\theta}}v^{\frac{2\pi is}{sin\theta}}e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2u+log^2v]}\}\ dxdydudv$$

$$=\frac{1}{(2\pi)^{3/2}sin^2\alpha sin^2\theta\sqrt{1+icot\alpha}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\overline{G_{\alpha,\theta}(p,q,r,s)}\,dpdqdrds\qquad\sqrt{\frac{2\pi}{1-icot\alpha}}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}f(x,y,u,v)\,e^{\frac{i}{2sin\alpha}[(x^2+y^2+p^2+q^2)-2(xp+yq)]}(uv)$$

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$u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}[r^2+s^2+log^2 u+log^2 v]}\} \, dxdydudv$

$= \frac{(2\pi)^{1/2}}{(2\pi)^{3/2} sin^2\alpha sin^2\theta\sqrt{1-i^2cot^2\alpha}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} \overline{G_{\alpha,\theta}(p,q,r,s)}$

$2DFRFMT[(uv)f(x,y,u,v)](p,q,r,s)dpdqdrds$

$= \frac{1}{2\pi sin^2\alpha sin^2\theta\sqrt{1+cot^2\alpha}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} \overline{G_{\alpha,\theta}(p,q,r,s)} \; F_{\alpha,\theta}[(uv)f(x,y,u,v)](p,q,r,s)dpdqdrds$

$= \frac{1}{2\pi sin^2\alpha sin^2\theta cosec\alpha} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} \overline{G_{\alpha,\theta}(p,q,r,s)} \; F_{\alpha,\theta}[(uv)f(x,y,u,v)](p,q,r,s)dpdqdrds$

$= \frac{1}{2\pi sin\alpha \; sin^2\theta} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} F_{\alpha,\theta}[(uv)f(x,y,u,v)](p,q,r,s) \; \overline{G_{\alpha,\theta}(p,q,r,s)} \; dpdqdrds$

Putting $g(x,y,u,v) = f(x,y,u,v)$

$F_{\alpha,\theta}(p,q,r,s) = G_{\alpha,\theta}(p,q,r,s)$

$\overline{F_{\alpha,\theta}(p,q,r,s)} = \overline{G_{\alpha,\theta}(p,q,r,s)}$

By using above result

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} f(x,y,u,v)\,\overline{g(x,y,u,v)}dxdydudv = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} F_{\alpha,\theta}(p,q,r,s)\,\overline{G_{\alpha,\theta}(p,q,r,s)}\,dpdqdrds$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} |f(x,y,u,v)|^2\,dxdydudv = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} |F_{\alpha,\theta}(p,q,r,s)|^2\,dpdqdrds$$

VI. CONCLUSION

In this paper we discussed on modulation and Parsvals Identity of two dimensional Fourier-Mellin transform.

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