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# A COMBINED METHOD USING FUZZY CLUSTERING AND MGGVF SNAKE MODEL FOR BRAIN TUMOR SEGMENTATION ON MRI IMAGES

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*Abstract* - The active contours or snakes and region based methods are not used separately for effective segmentation of tumor region on brain MRI images. Active contours or snakes are problems with initialization, poor convergence to boundary concavities and difficulties in forcing a snake into long, thin boundary indentations. Region based methods do not include shape and boundary information. In this paper, we combine the region-based fuzzy clustering method called Enhanced Possibilistic Fuzzy C-Means (EPFCM) and Modified Generalized Gradient vector flow (MGGVF) snake model having diffusion in the normal direction for segmenting tumor region on MRI images effectively. The EPFCM method is used for initial segmentation of tumor then result of that is used to provide initial contour for MGGVF snake model. Then it is used to determine the final contour for exact tumor boundary. The experimental results on tumor MRI images reveal that our method is more robust and accurate for brain tumor segmentation.

Keywords - Snake model; FCM; Segmentation; MRI image; Edge map; GVF

#### INTRODUCTION

The precise and automatic segmentation of brain tumor on MRI image is of great interest for assessing tumor growth and treatment responses, enhancing computer-assisted surgery, planning radiation therapy, and constructing tumor growth models. This is very difficult task in existing methods. The existing methods are divided into regionbased and contour-based methods. Region-based methods [1-4] seek out clusters of pixels that share some measure of similarity. These methods reduce operator interaction by automating some aspects of applying the low level operations, such as threshold selection, histogram analysis and classification. They can be supervised or nonsupervised. In general these methods take advantage of only local information for each pixel and do not include shape and boundary information. Contour-based methods [5-8] rely on the evolution of a curve, based on internal forces and external forces, such as image gradient, to delineate the boundary of brain structure or pathology. These methods can also be supervised or non-supervised. In general these methods suffer from the problem of determining the initial contour and leakage in imprecise edges.

In this paper we propose a method that is a combination of region-based fuzzy clustering method called Enhanced Possibilistic Fuzzy C-Means (EPFCM) and Modified Generalized Gradient vector flow (MGGVF) snake model having diffusion in the normal direction to remove the problems using the capabilities of each one. For example a region-based method can solve the problem of the initialization of a contour-based method and a contour-based method is able to improve the quality of region-based segmentation at the boundary of objects.

So the proposed method has two main phases for tumor segmentation on MRI brain images namely, initial segmentation, which is done by a region-based EPFCM method and final segmentation that is performed by a boundary-based MGGVF snake model. We discuss these approaches in the following sections.

#### REGION-BASED ENHANCED POSSIBILISTIC FUZZY C-MEANS (EPFCM)

In this proposed Enhanced Possibilistic Fuzzy C-Means (EPFCM) method, distance metric  $D_{ij}$  in PFCM [9] is modified in such a way that it includes membership, typicality and both local, non-local spatial neighbourhood information to overcome the noise effect in MRI brain images. This modified distance metric is incorporated into objective function of PFCM. Then resultant algorithm is called Enhanced Possibilistic Fuzzy C-means (EPFCM) is obtained for enhanced segmentation results. Therefore the objective function of EPFCM is defined as follows,

$$J_{m}(U, V, T; X) = \sum_{i=1}^{c} \sum_{j=1}^{n} (a\mu_{ij}^{m} + bt_{ij}^{\eta}) D_{ij}^{2} + \sum_{i=1}^{c} \gamma i \sum_{j=1}^{n} (1 - t_{ij})^{\eta}$$
(1)

Where, the modified distance metric is given by

$$D^{2}(x_{j}, v_{i}) = D^{2}_{ij} = (1 - \lambda_{j})d^{2}_{l}(x_{j}, v_{i}) + \lambda_{j}d^{2}_{nl}(x_{j}, v_{i})$$
(2)

 $0\!\!\leq\!\!\mu_{ij}\!,\ t_{ij}\!\!\leq\!\!1 \ \text{and} \ a\!>\!0,\ b\!>\!0,\ m\!>\!1,\ \!\eta\!>\!1$ 

The membership function:  $\mu_{ij} = \left[\sum_{k=1}^{c} \left(\frac{D_{ij}}{D_{kj}}\right)^{2/m-1}\right]^{-1}$  (4)

(3)

(6)

Typicality:  

$$t_{ij} = \frac{1}{1 + \left(\frac{b}{\gamma_i} D_{ij}^2\right)^{\frac{1}{\eta-1}}}$$
(5)

Clustercentre:  
$$v_{i} = \frac{\sum_{j=1}^{n} (a\mu_{ij}^{m} + bt_{ij}^{\eta})x_{j}}{\sum_{i=1}^{n} (a\mu_{ij}^{m} + bt_{ij}^{\eta})}$$

n

The following equation is suggested to compute  $\gamma_i$ :

$$\gamma_{i} = \frac{K \sum_{j=1}^{n} \mu_{ij}^{m} D_{ij}^{2}}{\sum_{j=1}^{n} \mu_{ij}^{m}} , K > 1$$
(7)

#### Importance of modified distance metric term $(D_{ij})$ :

The modified distance metric or dissimilarity measure is rewritten from Equation (2) as follows,

$$D^{2}(x_{j}, v_{i}) = D^{2}_{ij} = (1 - \lambda_{j})d^{2}_{i}(x_{j}, v_{i}) + \lambda_{j}d^{2}_{nl}(x_{j}, v_{i})$$
(8)

Where,  $d_l$  is the distance metric influenced by local spatial information. This added local spatial neighbourhood term is similar to the one which is used in [10] to incorporate the neighbourhood effects in the classic FCM. The local spatial constraint is evaluated by the feature difference between neighbouring pixels in the image.

 $d_{nl}$  is the distance measurement influenced by non- local spatial information. This added non local term is obtained from the non local means (NL-means) algorithm [11] for image de-noising. The non-local constraint determined by all points whose neighbourhood configurations look like the neighbourhood of the pixel of interest.

 $\lambda_j$  is the weighting factor controlling the trade-off between local and non-local spatial information. It varies from zero to one.

#### Importance of local distance metric $(d_l)$ :

Let  $N_j$  denote a chosen local neighbourhood configuration of fixed size with respect to a centre pixel  $x_j$ . If the value of a pixel  $x_k$  in  $N_j$  is close to the centre pixel, then  $x_j$  should be influenced greatly by it, otherwise, its influence to  $x_j$  should be small. According to the above description, the distance measurement influenced by local information  $d_l$  is given by

$$d_{l}^{2}(x_{j}, v_{i}) = \frac{\sum_{x_{k} \in N_{j}} \omega_{l}(x_{k}, x_{j}) d^{2}(x_{k}, v_{i})}{\sum_{x_{k} \in N_{j}} \omega_{l}(x_{k}, x_{j})}$$
(9)

where  $d^2(x_k, v_i) = || x_k - v_i ||^2$  is the Euclidean distance metric measure the similarity between pixel pixel  $x_k$  and cluster centroid  $v_i, \omega_l(x_k, x_j)$  is the weight of each pixel  $x_k$  in  $N_j$  and is given by

 $\omega_l(x_k, x_j) = e^{-\frac{|x_k, x_j|^2}{\sigma^2}}$  (10) Where,

 $\sigma^2$  is the variance of  $N_i.$  It specifies the steepness of the sigmoid curve.

#### Importance of non -local distance metric $(d_{nl})$ :

The distance measurement influenced by non-local information  $d_{nl}$  is computed as a weighted average of all the pixels in the image I,  $x_k \in I$ 

$$d_{nl}^{2}(x_{j}, v_{i}) = \sum_{x_{k} \in I} \omega_{nl}(x_{k}, x_{j}) d^{2}(x_{k}, v_{i})$$
(11)

Where, the family of weight  $\omega_{nl}(x_k, x_j)$ ,  $x_k \in I$  depends on the similarity between the pixel  $x_k$  and  $x_j$ , and satisfies the usual conditions  $0 \le \omega_{nl}(x_k, x_j) \le 1$  and  $\sum_{x_k \in I} \omega_{nl}(x_k, x_j) = 1$ .

The similarity between two pixels  $x_k$  and  $x_j$  depends on the similarity of the intensity gray level vector  $v(N_k)$  and  $v(N_j)$ , where  $N_k$  denotes a square neighbourhood of fixed size and centred at a pixel  $x_k$ . This similarity is measured as a decreasing function of the weighted Euclidean distance  $\|v(N_k) - v(N_j)\|_{2,a}^2$ , where a > 0 is the standard deviation of the Gaussian kernel. The pixels with a similar gray level neighbourhood to  $v(N_j)$  have larger weights in the average. These weights are defined as,

$$\omega_{nl}(x_k, x_j) = \frac{1}{Q(x_j)} S(x_k, x_j)$$
 (12)

 $S(x_k, x_j)$  is the exponential form of the similarity, and  $Q(x_i)$  is the normalizing constant. These terms are defined

as,  $\| (\alpha_i) - \alpha_i \|^2$ 

$$\mathbf{S}(\mathbf{x}_{k}, \mathbf{x}_{j}) = e^{\frac{\|\mathbf{v}(\mathbf{x}_{k}) - \mathbf{v}(\mathbf{x}_{j})\|_{2,a}}{h^{2}}}$$
(13)

$$Q(x_{j}) = \sum_{x_{k} \in I} e^{-\frac{\|v(x_{k}) - v(x_{j})\|_{2,a}}{h^{2}}}$$
(14)

The parameter h acts as a degree of filtering. It controls the decay of the exponential function and therefore the decay of the weights as a function of the Euclidean distance.

#### Importance of trade-off parameter $(\lambda_j)$ :

For computational purpose, the search of the similar neighbourhood configuration always be restricted in a larger "search window" denoted by  $\Omega_i$ . Let  $x_j$  be the pixel under consideration. For each pixel  $x_k$  in the search window of size S×S, calculate its exponential similarity to  $x_j$  using Equation (13). The trade-off parameter of  $x_j$  is then defined as

$$\lambda_{j} = \frac{1}{m} \sum_{i=1}^{m} S_{i}(x_{k}, x_{j})$$
(15)

Where,  $S_i$  represents the i<sup>th</sup> exponential similarity term in the search window and choose m=S-1.The parameter  $\lambda_j$  decides the trade-off between local and non-local spatial information.

#### ALGORITHM FOR PROPOSED EPFCM METHOD

Finally the algorithm for carrying out our proposed EPFCM for tumor segmentation of MRI brain images can now be stated from the following steps,

- a. Select the number of clusters 'C' and fuzziness factor 'm'
- b. Select initial class centre prototypes  $v = \{v_i\}; i=1,2...,C$ , randomly and  $\varepsilon$ , a very small number
- c. Select the neighbourhood size and search window size
- Calculate modified distance measurement D<sub>ij</sub><sup>2</sup> using the Equation (2)
- e. Update membership function  $\mu_{ij}$  using  $D_{ij}^2$

- f. Update  $\gamma_{i;i}=1, 2..., C$ , using Equation (7)
- g. Update typicality using the Equation (5)
- h. Update cluster centre using equation (6)
- i. Repeat steps 4 to 8 until termination. The termination criterion is as follows,

 $\|v_{t+1}-v_t\| \le \varepsilon$ , where 't' is the iteration steps,  $\|.\|$  is the Euclidean distance norm.

We applied this proposed algorithm to segment tumor on MRI images. In this case, we segmented the brain image into five classes: namely, CSF, WM, GM, tumor and background .Due to some classification errors, there are undesired additional pixels in the tumor class. To remove misclassified these components, several binary morphological operations are applied to the tumor class after users defined segmentation classes are obtained (number of clusters). An opening operation is first used to disconnect the components. Then we select the largest connected component, which proved to always correspond to the tumor, even if it has a small size. Here the elementary neighbourhood of the morphological operations corresponds to 6-connectivity. The result of this algorithm gives segmented tumor class .This output is the initial contour for the MGGVF Model discussed in the next section.

# MODIFIED GENERALIZED GRADIENT VECTOR FLOW FOR ACTIVE CONTOUR (MGGVF)

The conventional Active contours or Snake model [12-13] is a curve X(S) = [x(s), y(s)], s  $\varepsilon$  [0, 1], that move within the image to minimize the energy function. The curve dynamically changes the shape of an initial contour in response to internal and external forces. The internal forces provide the smoothness of the contour. While the external forces push the curve move toward the desired features, such as boundaries. The object contour will be got when the energy function is minimized. The energy is defined as:

$$E = \int \frac{1}{2} \left| \mathbf{x}'(s) \right|^2 + \beta \left| \mathbf{x}''(s) \right|^2 + E_{ext}(\mathbf{x}(s)) ds$$
(16)

X'(S) and X''(S) are first and second derivatives of X(S) with respect to s. The parameter α controls the tension of the curve and β controls its rigidity.  $E_{ext.}$  is the external energy which is calculated from the image data. This external force has a short capture range, poor convergence to boundary concavities and difficulties in forcing a snake into long, thin boundary indentations. To overcome these problems, Generalized Gradient vector flow snake was proposed by Xu and Prince [12], in which the external force term  $-\nabla E_{ext}$  is replaced with a new field of the form  $E_{ext} = V(x, y) = [u(x, y) v(x, y)]$  (17)

$$F_{ext} = V(x,y) = [u(x,y), v(x,y)]$$
 (17)

Where, u and v are the grey value changes on x-axis and yaxis of the image respectively and V(x,y) minimizes the following new variational formulation with varying weighting function and decreasing the smoothing effect near strong gradients:

$$\varepsilon = \iint g(|\nabla f|) |\nabla v|^2 + h(|\nabla f|) |v - \nabla f|^2 dx dy \qquad (18)$$

The first term on the right is referred to as the smoothing term since this term alone will produce a smoothly varying vector field. The second term is referred as the data term since it encourages the vector field V to be close to  $\nabla f$ computed from the data. The weighting functions g (.) and h (.) apply to the smoothing and data terms, respectively. Since these weighting functions are dependent on the gradient of the edge map which is spatially varying, the weights themselves are spatially varying, in general. Since we want the vector field V to be slowly varying (or smooth) at locations far from the edges, but to conform to  $\nabla f$  near the edges, g(.) and h(.) should be monotonically nonincreasing and non-decreasing functions of  $|\nabla \mathbf{f}|$ respectively.f(x,y) is the edge map which is derived from image I(x,y) using canny edge detection operator[14]. Following are the weighting functions for GGVF:  $-(|\nabla \mathbf{f}|/\mathbf{K})$ 

$$g(|\nabla \mathbf{f}|) = e^{-\langle \nabla \mathbf{f} | \mathbf{X} \rangle}$$

$$b(|\nabla \mathbf{f}|) = 1 - \alpha(|\nabla \mathbf{f}|)$$
(19)

$$\Pi(|v_1|) = 1 - g(|v_1|) \tag{20}$$

The GGVF field computed using this pair of weighting functions will conform to the edge map gradient at strong edges, but will vary smoothly away from the boundaries. The specification of K determines to some extent the degree of trade-off between field smoothness and gradient conformity. Using the calculus of variations, it can be showed that the GGVF can be found by solving the following Euler equations:

$$g(|\nabla f|)\nabla^2 u - h(|\nabla f|)(u - \nabla f) = 0$$
(21)

$$g(|Vf|)V^{2}v - h(|Vf|)(v - Vf) = 0$$
(22)

Where,  $\nabla^2$  is the Laplacian operator.

In equation (21-22) laplacian operator  $\nabla u$ ,  $\nabla v$  are diffusion terms, has very strong isotropic smoothing properties and does not preserve edges while the second term is data attraction terms only preserve the edge map. The laplacian terms are decomposed using the local image structures, that is, the tangent and normal directions to the isophote lines (15).Further, we rewrite Laplacian terms as

$$\nabla^2 \mathbf{u} = \mathbf{u}_{tt} + \mathbf{u}_{nn} \tag{23}$$

$$u_{tt} = \frac{1}{|\nabla u|^2} (u_x^2 u_{yy} + u_y^2 u_{xx} - 2u_x u_y u_{xy})$$
(24)

$$u_{nn} = \frac{1}{|\nabla u|^2} (u_x^2 u_{xx} + u_y^2 u_{yy} + 2u_x u_y u_{xy})$$
(25)

Where,  $u_{tt}$  and  $u_{nn}$  denote the second derivatives of u in the tangent direction and normal direction. The equations (23-24) also satisfy the interpolation functions. Therefore laplacian operator is both a diffusion operators and an interpolation operator. The image interpolation and diffusion of force field is the diffusion process that an edge map spreads out. Since equation(23) is better interpolation operator than Laplacian operator, it should be a better diffusion operator than Laplacian operator, we can throw out the tangent direction part and only remain as well as stress the normal direction part of the Laplace in order to improve its ability of diffusion and interpolation to improve the Laplace diffusions term .The new force field for active contour is obtained by replacing the laplacian diffusion

terms of equation (21-22) with equation (23) and throw out the tangent direction part. The obtained new force field is called Modified Generalized Gradient vector flow (MGGVF) expressed as follows,

$$g(|\nabla f|) \cdot \lambda u_{nn} - h(|\nabla f|)(u - \nabla f) = 0$$
(26)

$$g(|\nabla f|) \cdot \lambda v_{nn} - h(|\nabla f|)(v - \nabla f) = 0$$
<sup>(27)</sup>

Where,  $\lambda$  is the positive integer.

This MGGVF snake model is first initialized by the tumor class output of EPFCM method, then move towards the final tumor boundary.

### **RESULTS AND DISCUSSION**

Initially MRI brain image is segmented for tumor class using EPFCM method, which is then initial contour for MGGVF snake. Then the contour attracted towards final tumor boundary by edge map derived from the image using canny edge detector. We set parameter h=500,search window size is 7×7,neighborhood window size is  $3\times3,m=2,a=5,b=3$  and  $\eta =2$  for EPFCM method to have proper segmentation result. We have chosen  $\alpha$  and  $\beta$  values between 0.1 and 0.2, K value between 0.15 and 0.2 and  $\lambda=2$ for MGGVF snake model to have final tumor boundary. By the application of our method to 5 contrasts enhanced T1weighted images shows the better tumor segmentation result. The results of three cases are as shown in Fig 1.

The evaluation of segmentation performance is also carried out quantitatively by employing three volume metrics [16] namely, the similarity index(S), false positive volume function (FPVF) and false negative volume function (FNVF) in our experiment. For a given image, suppose that  $A_i$  and  $B_i$  represent the sets of pixels belong to class *i* in manual segmentation and in segmentation result.  $|A_i|$ denotes the number of pixels in  $A_i$ .  $|B_i|$  denotes the number of pixels in  $B_i$ .

The similarity index is an intuitive and clear index to consider the matching pixel between  $A_i$  and  $B_i$ , and defined as

$$\mathbf{S} = \frac{2|\mathbf{A}_i \cap \mathbf{B}_i|}{|\mathbf{A}_i| + |\mathbf{B}_i|} \tag{28}$$

The value of similarity index S > 70% indicates an excellent similarity.

The false positive volume function (FPVF) represents the error due to the misclassification in class i and the false negative volume function (FNVF) represents the error due to the loss of desired pixels of class i, they are defined as follows,

$$FPVF = \frac{|\mathbf{B}_i| - |\mathbf{A}_i \cap \mathbf{B}_i|}{|\mathbf{A}_i|}$$
(29)

$$FNVF = \frac{|A_i| - |A_i \cap B_i|}{|A_i|}$$
(30)

Higher value of S, and lower value of FPVF, FNVF gives better segmentation result.

The result of three volume metrics for our method, applied on a five CE-T1w images, is as shown in Table 1 and figure 1(only three cases are shown). From these results, we can see that an average similarity metrics(S) of our method is 95. 8% that is, the overlap degree between segmentation result and the manual segmentation is higher. The average FPVF and FNVF values are equal to 0.64% and 0.41%. It shows misclassification and loss of desired tumor pixels is reduced in great degree. The comparison of results with work such as in [1, 2, 3] shows that our method has a better tumor segmentation performance.



- Figure1. (a) First column: Original CE-TIW enhanced tumor images (b) Second column: Manual segmentation result (c) Third column: EPFCM result showing tumor class (d) Fourth column: Final deformable contour region (Blue curve) shows tumor region (e) Fifth column: Segmentation of tumor class using our combined approach (EPFCM and MGGVF)
- Table1. Evaluation of the segmentation results of full enhanced tumors by our approach on a five CE-T1w images (FET denotes the full enhanced tumor).

MRI modality	Type of			
type	tumor	S	FPVF	FNVF
CE-T1w	FET1	98.5	98.5	98.5
CE-T1w	FET2	97.2	97.2	97.2
CE-T1w	FET3	93.8	93.8	93.8
CE-T1w	FET4	94.7	94.7	94.7
CE-T1w	FET5	94.8	94.8	94.8
Average		95.8	0.64	0.41

#### CONCLUSION

We have presented in this paper a method for tumor segmentation which combine both region based fuzzy clustering method called EPFCM and MGGVF snake .We verified our method with brain tumor MRI images. The results obtained are quantitatively verified with other existing method shows that our combined approach provides better result.

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