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A Mathematical Model of the Propagation of Plane Electromagnetic Waves in a Fractal Tropospheric Medium

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Abstract: The purpose of this work is to construct a mathematical model for the propagation of plane electromagnetic waves (signals) in the tropospheric atmosphere of line of sight, taking into account the fractality of turbulence and interference.

Keywords: Fractal; Model; Propagation; Electromagnet; Troposphere; Waveguide; Turbulence

I. INTRODUCTION

In the history of science, there is often a situation where a study of the likeness of a single crystal, which has fallen into a saturated solution, instantly causes a huge flow of work. This flow, spilling over, captures large areas.
This was the case, for example, when the theory of automata developed rapidly. Approximately the situation in the first stage in the creation of programming languages and the theory of fuzzy sets looked like. To the American scientist B.B. Mandelbrot is due to the emergence of another analogous situation.
The main idea of Mandelbrot is that natural objects of coarsened forms cannot be a kind of Euclidean forms, i.e. the dimensions of these objects are not integral, but fractional, and they were introduced by the fractal theory (from the lot Frangeze-break and fractus-fractional) [1]. Fractal object models [1] are built on the basis of various mathematical algorithms using modern computer graphics [2,3]. So, what is a fractal? A simple example of a natural fractal is a tree whose trunk is divided into two branches, which in turn branch into two smaller branches, etc. We can say that the tree branches follow fractal scaling, or the hypothesis itself is similar. In this case, each branch with its own branches is similar to the whole tree in a qualitative sense. Therefore, the type of the fractal structure of the object does not change significantly with scale transformations in a certain range.

1.1 There is a Very Similarity of Space and Time

We take a bounded region of the Euclidean plane. If it is crushed into a ball, then the resulting figure is not two-dimensional, but there will not be a three-dimensional one. Because of its folds, the dimension will be greater than two, but less than three. Therefore they say, and it is mathematically proven that the resulting body has a fractional fractal dimension and the function describing its shape is not differentiable. Fractal dimension describes how an object fills its space. Our crumpled object does not completely fill the three-dimensional space, which has a topological dimension, or the dimension of the embedding. Therefore, the characteristic of a fractal object is the presence of its own dimension, which is not equal to the dimension of the embedding. Classical random distributions, in particular white noise, do not have this characteristic [4].
At present, there is clearly a lack of traditional physical models. In other words, a complete description of the processes of modern signal processing and fields is impossible with the help of formulas of classical mathematics obtained on the basis of representation of signals in the space of an integer measure and smooth functions. Today it is quite obvious that the application of the ideas of fractal theory and fractal synthesis in radio physics, radio engineering, radar, electronics and modern information technologies opens up great potential opportunities and new prospects in the processing of multidimensional signals in related scientific and technical fields.
As is known, from the standpoint of system analysis, modern means and devices of radio engineering for the transmission and processing of signals in aggregate represent a large and complex system, i.e. consisting of a set of interconnected and interacting subsystems, each of which performs a certain function. Successful and reliable operation

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of the whole system is due to the functioning of individual subsystems. When transmitting information (signal) through the atmosphere (air environment), the atmosphere is one of the subsystems of the communication channel. Unlike other subsystems, it has subsystems of natural origin and parameters that determine its functioning cannot be controlled.

Propagation of a signal (electric waves) in a long subsystem can be investigated only with the help of mathematical modules of the subsystem. A huge number of publications of both theoretical and experimental research have been published on this issue. A lot of mathematical models are suggested, which are a consequence of Maxwell's equations, and they describe the processes of signal propagation (electromagnetic waves) in a subsystem to some extent satisfactorily. Since, a review of the research on this issue was not part of the task of our work; we found it inappropriate to cite a list of these publications.

The purpose of this work is to construct a mathematical model for the propagation of plane electromagnetic waves (signals) in the tropospheric atmosphere of line of sight, taking into account the fractality of turbulence and interference.

II. FRACTAL MODEL OF FLUCTUATIONS IN THE PERMITTIVITY OF THE TROPOSPHERIC ATMOSPHERE

Usually, with the traditional approach, the solution of the problem of propagation of electromagnetic waves (signals) in a turbulent atmosphere is the permittivity of the medium - $\varepsilon(x,y,z,t)$- are assumed to be a random variable, a-scalar differential equation describing wave propagation by a random process. There is no systematic solution of the equation. In order to solve the problem, known laws for the distribution of the probabilities of a random variable and a random process (for example, the normal law of distribution of the Poisson distribution law, Rayleigh, etc.) are preliminarily assumed, then using numerical or simulation methods (the Monte Carlo method) evaluate the statistical characteristics of the wave field characteristics of the signal propagated). We will further consider the propagation of plane electromagnetic waves in a medium with weak fluctuations in the permittivity. We set $\varepsilon = < \varepsilon > + \varepsilon_1$, where $< \varepsilon >$ - the averaged value of ε and $\varepsilon_1 = \varepsilon - < \varepsilon >$ is the fluctuating part. Obviously, by the definition of $<\varepsilon> = 0$. The smallness of the fluctuations means that $0 <| \varepsilon_1 |> << <\varepsilon>$. This condition is satisfied with great accuracy in the troposphere, where $<\varepsilon>$ is of order 1, $\omega <| \varepsilon_1 |> \approx$10-5 ÷ 10-6. The condition $<| \varepsilon_1 |> << < \varepsilon >$ can be violated in the ionosphere near the layer, where $<\varepsilon>$ vanishes, this case is not considered by us. We confine ourselves to the case $< \varepsilon >$ =const.

In this case we consider simply $< \varepsilon > = 1$. Assuming that $\varepsilon = 1 + \varepsilon_1$ and assuming $0 <| \varepsilon_1 |> << 1$. In this case, ε_1 directly enter into the wave equation. By virtue of this, the random wave equation is transformed into a stochastic equation. We have already noted the shortcomings of the traditional methods of specifying ε_1 in determining the estimation of the stochastic characteristics of the wave field. More preferable for describing the fluctuations in the dielectric constant of the medium ε is the use of the fractal concept of scale invariance or scaling. One example of a scale-invariant fractal curve is the Weierstrass-Mandelbrot fractal function. W (t) defined by the relation [5].

$$W(t) = \sum_{n=-\infty}^{\infty} \frac{(1 - e^{ib^n t})e^{i\varphi_n}}{b^{(2-D)n}} \quad (1)$$

Where, b> 1-parameter, 1 <D <2 - fractal dimension of the function (curve), φ_n arbitrary phase. The cosine fractal Weierstrass-Mandelbrot function is the real part of the function W(t):

$$C(t) = \mathrm{Re}W(t) = \sum_{n=-\infty}^{\infty} \frac{(1 - \cos b^n t)}{b^{(2-D)n}} \quad (2)$$

It is customary to assume that this function is fractal with dimension D. Modeling of fluctuations of the refractive index n of the troposphere by the Weierstrass function was considered in [6]. In the one-dimensional case, this model looks like this [6]:

$$n_1(z) = P_1 \frac{\{2 < n_f^2 > \left[1 - b^{2(D-2)}\right]\}^{1/2}}{\{1 - b^{2(D-2)(N+1)}\}^{1/2}} \sum_{n=0}^{N} b^{2(D-2)n} \cos(2\pi b^{n_{z/L}} + \varphi_n) \quad (2)$$

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Where, $\{2 < n_f^2 > [1\text{-}b^{2(D\text{-}2)}]\}^{1/2}\ [1 - b^{2(D-2)(N+1)}]^{-1/2}$ – coefficient of normalization; P_1 – determines the ratio of fluctuations $<n_f^2>$ and fluctuations $<n_1^2>$ in an inertial dispersion; b>1 - b>1 – space-frequency scaling parameter; D – fractal dimension, applying a small value of 5/3 for one-dimensional fluctuations; N+1 – the number of scales, or intervals in logarithmic different; φ_n – an arbitrary phase of the uniform distribution of the non-interval [0,2π]. In that work, a three-dimensional model of the coefficient of tropospheric application of the Weierstrass function is given. We used a fractal model of the per molecular permeability of the tropospheric atmosphere depending on two applications $\mathcal{E}_1$(z,t) The Weierstrass function of the following form is adopted:

$$\varepsilon_1(z,t) = \sqrt{2}\tau \frac{[1-b^{2(D-3)}]^{1/2}}{[1-b^{2(D-3)(N+1)}]^{1/2}} \sum_{n=1}^{N-1} b^{(D-3)n} \sum_{m=1}^{M}\sum_{m=1}^{M} \sin\left\{K_0 b^N \left[Z\cos\left(\frac{2\pi m}{M}\right) + t\sin\left(\frac{2\pi m}{M}\right)\right] + \varphi_{nm}\right\} \quad (3)$$

Where, C is the standard deviation, b> 1 is the parameter of the space-frequency scaling, 2 <D <3 is the fractal dimension, K_0 – wave number, N and M - the number of harmonics, φ_{nm} – arbitrary phase, t - time, z - spatial coordinate. Expression (3) completely describes the fluctuations in the permittivity of the tropospheric atmosphere.

III. FRACTAL INTERFERENCE MODEL

As is known in the propagation of radio waves under atmospheric conditions (in the troposphere in particular), they are prone to all kinds of extraneous interference. They are random variables or functions. Usually when analysing and synthesizing signals, interference is treated as white noise whose statistical characteristics are normally distributed. The traditional approach to the analysis of random signals is based on the spectral-correlation theory with the Wiener-Khinchin fundamental theorem. However, if the random process is not Gaussian, then a complete statistical description of the signals requires evaluation of higher order moments with allowance for multipoint correlations, which does not always justify itself. An alternative approach is to estimate the fractal dimensions of various geometric objects associated with the process. An example of a random process possessing fractal properties is the classical Wiener process of Brownian motion. The trajectory of a Wiener process has the property of scale invariance, or scaling.

We consider a Gaussian random process with independent step values {ξ}. The increment of the coordinate of the Brownian particle is determined by the expression for any pair of instants t and t_0.

$$X(t) – X(t_0) \sim \xi\, |t-t_0|^{1/2},\ t \geq t_0 \quad (4)$$

From (4) we can determine the coordinate X (t) from the coordinate $X(t_0)$, choosing a random number ξ from the Gaussian distribution, multiplying it by the degree of increment of time $|t-t_0|$ and adding the result to a known coordinate $X(t_0)$. Thus expression (4) describes a classical Brownian motion, or a random function.

On the basis of a Wiener Brownian process, Mandelbrot introduced the concept of a generalized Brownian motion [1] by replacing the exponent in the formula [4] by any real number in the interval 0<H<1. Happening H=1/2 corresponds to independent increments and describes the classical Brownian motion. The indicator H is called the Hurst exponent; information on it can be obtained, for example, from [5].

Usually, in the statistical analysis of signals with allowance for interference, it is believed that they are independent of spatial coordinates, and depends only on time, i.e. N(z,t)=N(t).

In terms of physical content, this approach is justified in practice. Thus, interference in the tropospheric atmosphere is described by a function that depends on one variable of time t. To approximate the interference N (t) (random function). With the generalized Brownian motion $B_H(t)$ we use the self-affinity of the fractal Brownian function. Self-affine functions include the Weierstrass-Mandelbrot fractal function, which we considered earlier expression (1). Then, the hindrance can be approximated with the help of the following expression.

$$N(t) = \sqrt{2}\delta \frac{[1-b^{(2d-4)}]^{1/2} \sum_{n=0}^{N} b^{(D-2)n} \sin(2\pi S b^n t + \varphi_n)}{[1-b^{(2d-4)(N+1)}]^{1/2}} \quad (5)$$

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where δ – standard deviation, b,S – parameters of space-frequency scaling, D fractal dimension, N + 1 - number of harmonics, φ_n – phase, distribution randomly into segments [0,2π], t - time.

3.1 Phase, Distribution, Randomly into Segments [0,2π], t - Time

As we have already stated, we are considering the propagation of a plane wave. Let the plane electromagnetic wave U (z, t) propagate in the tropospheric atmosphere. In the process of propagation, it is subject to all kinds of random perturbations, which adversely affect the course of propagation. They can be reduced to the following basic types: fluctuations in the permittivity of the medium Ɛ (z, t) and noise (noise), they are shown in Figure 1.

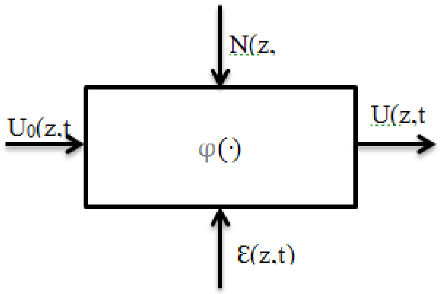

Figure 1: The tropospheric atmosphere subsystem with perturbations applied.

U0(z,t) – falling predetermined plane electromagnetic wave; Ɛ (z, t) - fluctuations in the permittivity of the propagation medium; N (z, t) is noise (noise); U (z, t) is a plane electromagnetic wave at the point of reception; φ (·) is the transformation operator. Let us assume that a flat monochromatic EMW (Electromagnetic wave) is incident on the tropospheric atmosphere, i.e. U0(z,t)=U_m cos(ωt-K_0 z+φ_m). In the process of propagation through the troposphere, this EMW is subjected to perturbations Ɛ (z, t) and N (z, t), which are determined by relations (3) and (5), respectively. In this case, the process of propagation of a flat EMW in the troposphere can be described by the following wave equation. Mobile Ad Hoc Networks (MANETs) consists of a collection of mobile nodes which are not bounded in any infrastructure.

$$\frac{\partial^2 U}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} = U_0(z,t) + N(z,t) \quad (6)$$

Where, v – the velocity of propagation of a plane wave in the troposphere. If we take into account $v = \frac{c}{\sqrt{\varepsilon\mu}}$ и $\mu = 1$, $\varepsilon = 1 + \varepsilon_1$, then (6) can be rewritten in the form:

$$\frac{\partial^2 U}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = \varepsilon_1(z,t)\frac{\partial^2 U}{\partial t^2} + U_0(t,t) + N(z,t)(t) \quad (7)$$

In equation (7)

$$\frac{\partial^2 U}{\partial t^2} = U_m\omega^2\cos(\omega t - v - \varphi_m)$$

Then we finally have:

$$\frac{\partial^2 U}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 U}{\partial z^2} = U_0(z,t) - U_m\omega^2\cos(\omega t - \varphi_m) x \varepsilon_1(z,t) + N(z,t) \quad (8)$$

Where, ω^2- circular frequency, k_0- wave number, φ_m – phase, c - the speed of light.
The initial and boundary conditions from the physical meaning of the problem are taken as follows:

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$$\begin{cases} U(z,0)=0 \quad U(z,0)=0 \qquad U(0,t)=U_0 cos(\omega t+\varphi_0) \\ \dfrac{\partial U}{\partial t}=0 \qquad\qquad\qquad U(z,t)=U_1 cos(\omega t-K_0 z+\varphi_1) \end{cases} \quad (9)$$

Equations (8) can be solved by means of the Laplace transform with respect to the variable t for zero initial conditions. After the formation of (8) with respect to the variable t, we obtain a new equation.

$$\frac{d^2U(z,p)}{dz^2}-\frac{p^2}{c^2}U(z,p)=U_0(z,p)-\ \varepsilon_1(z,p)+N(z,p) \quad (10)$$

And boundary conditions:

$$U_0(z,p)=0 \;\; U(0,p)=U_{m_0}\frac{P^2cos\varphi_{m_0}-\omega Psin\varphi_{m_0}}{P^2+\omega^2}$$

$$\frac{dU(z,0)}{dt}=0 \;\; U(z,p)=U_1\frac{P^2\cos(K_0 z+\varphi_1)+\omega Psin(K_0 z+\varphi_1)}{P^2+\omega^2} \quad (11)$$

The Laplace transform of the terms on the right-hand side of equation (10) is equal to:

$$U_0(z,p)=U_{mo}\frac{P^2cos\varphi_{mo}-\omega Psin\varphi_{mo}}{p^2+\omega^2} \quad (12)$$

$$\varepsilon_1(z,p)=\frac{A_1U_{m0}\omega^2}{2}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left\{\begin{aligned}&\frac{P}{P^2+(A_4-\omega)^2}[(A_4-\omega)\cos(A_3z+K_0z+\varphi_{mn}+\varphi_{m0})+\\&+Psin(A_3z+K_0z+\varphi_{mn}+\varphi_{m0})]+\frac{P}{P^2+(A_4-\omega)}+\\&\left[(A_4-\omega)\cos(A_3z+K_0t+\varphi_{mn}+\varphi_{m0})+Psin(A_3z-K_0z+\varphi_{mn}+\varphi_{m0})\right]\end{aligned}\right\} \quad (13)$$

Here, to reduce the notation, we introduce the following notation:

$$A_1=\sqrt{2}\,\frac{[1-b^{2(D-3)}]^{1/2}}{[1-b^{2(D-3)(N-1)}]^{1/2}}$$

$$A_2=b^{(D-3)n}$$

$$A_3=K_0b^N\cos(\frac{2\pi m}{M})$$

$$A_4=K_0b^N\sin(\frac{2\pi m}{M})$$

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$$N(z,p) = B_1 \sum_{n=0}^{N} B_2 \left[\frac{P}{P^2 + B_3^2} \left(B_3 \cos\Psi_n + P \sin\Psi_n \right) \right] \quad (14)$$

Where the abbreviations are introduced:

$$B_1 = \sqrt{2}\sigma \frac{[1 - b^{(2D-4)}]^{1/2}}{[1 - b^{(2D-4)(N+1)}]^{1/2}}$$

$$B_2 = b^{(D-2)n}$$

$$B_2 = 2\pi S b^n$$

The general solution of equation (10) consists of the sum of a homogeneous and inhomogeneous (particular solution) of an ordinary differential equation, i.e.

$$U(z,p) = C_1 e^{\frac{P}{c}z} + C_2 e^{\frac{P}{c}z} + U_0^r(z,p) - E_0^r(z,p) + N^r(z,p) \quad (15)$$

$$U_0^z(z,p) = -\frac{C^2\left(P^2 \cos\varphi_{m0} - \omega p \sin\varphi_{m0}\right)}{P^2\left(P^2 + \omega^2\right)} \quad (16)$$

$$E_1^r(z,p) = -\frac{A_1 U_{m0}\omega^2}{2\left[\left(A_3 + K_0\right)^2 + \frac{P^2}{C^2}\right]} \sum_{n=0}^{N} A_2 \sum_{m=1}^{M} \left[\begin{array}{l} \frac{P\left(A_4 - \omega\right)}{P^2 + \left(A_4 - \omega\right)^2} \cos\left(A_3 z + K_0 z + \varphi_{mn} + \varphi_{mo}\right) \\ + \frac{P^2}{P^2 + \left(A_4 - \omega\right)^2} \sin\left(A_3 z + K_0 z + \varphi_{mn} + \varphi_o\right) \end{array} \right] - \frac{A_1 U_{m0}\omega^2}{2\left[\left(A_3 - K_0\right)^2 + \frac{P^2}{C^2}\right]} \sum_{n=0}^{N} A_2 \sum_{m=1}^{M} \left[\begin{array}{l} \frac{P\left(A_1 + \omega\right)}{P^2 + \left(A_2 + \omega\right)^2} \cos\left(A_3 z - K_0 z + \varphi_{mn} + \varphi_m\right) \\ + \frac{P^2}{P^2 + \left(A_4 + \omega\right)^2} \sin\left(A_3 z - K_0 z + \varphi_{mn} + \varphi_{m0}\right) \end{array} \right]$$

(17)

$$N^2(z,p) = -\frac{C^2}{P^2} B_1 \sum_{n=0}^{N} B_2 \left[\frac{P}{P^2 + B_3^2} \left(B_3 \cos\Psi_m + P \sin\Psi_n \right) \right] \quad (18)$$

Using the boundary conditions, we define the unknowns C_1 andC_2, then substituting them into (15) we find solutions of the problem in Laplace's inventions. Omitting the cumbersome calculations, we give the final solutions.

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$$\mathrm{U(z,p)} = \frac{\left(1-e^{\frac{p}{c}z}\right)C^2}{\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)P^2(P^2+\omega^2)}(P^2\cos\varphi_{m0}-\omega\mathrm{P}\sin\varphi_{m0})U_0(\mathrm{z,p})$$

$$+\left\{\frac{\left(1-e^{\frac{p}{c}z}\right)A_1U_{m0}\omega^2}{2\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)\left[(A_3+K_0)^2+\frac{P^2}{C^2}\right]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4-\omega)}{P^2+(A_4-\omega)^2}\cos(A_3z+K_0z+\varphi_{mn}+\varphi_0)\right.\right.$$

$$+\frac{P^2}{P^2+(A_4-\omega)^2}\sin(A_3z+K_0z+\varphi_{mn}+\varphi_{mi})$$

$$+\frac{\left(1-e^{-\frac{p}{c}z}\right)A_1U_{m0}\omega^2}{2\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)\left[(A_3+K_0)^2+\frac{P^2}{C^2}\right]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4+\omega)}{P^2+(A_4+\omega)^2}\cos(A_3z-K_0z+\varphi_{mn}+\varphi_{mi})\right.$$

$$\left.\left.\left.+\frac{P^2}{P^2+(A_4+\omega)^2}\sin(A_3z-K_0z+\varphi_{mn}+\varphi_{m0})\right]\right]\right\}U_\varepsilon(\mathrm{z,p})$$

$$+\frac{\left(1-e^{\frac{p}{c}z}\right)C^2}{\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)P^2}B_1\sum_{n=0}^{N}B_2\left[\frac{P}{P^2+B_3^2}(B_3cos\Psi_m+Psin\Psi_n)\right]U_N(\mathrm{z,p})+\frac{e^{-\frac{p}{c}z}}{\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)}U_0P^2cos\varphi_0$$

$$-\ \omega Psin\varphi_0-\frac{1}{\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)}U_1P^2cos(K_0z+\varphi_1)+\frac{\omega Psin(K_0z+\varphi_1)}{P^2+\omega^2}$$

$$+\left(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}\right)\frac{C^2(P^2cos\varphi_{m0}-\ \omega Psin\varphi_{m0}}{P^2(P^2+\omega^2)}U_0(\mathrm{z,p})$$

$$+\left\{\left(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}\right)\frac{A_1U_{m0}\omega^2}{2\left[(A_3+K_0)^2+\frac{P^2}{C^2}\right]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4-\omega)}{P^2+(A_4-\omega)^2}\cos(A_3z+K_0z+\varphi_{mn}+\varphi_{mi})\right.\right.$$

$$+\frac{P^2}{P^2+(A_4-\omega)^2}\sin(A_3z+K_0z+\varphi_{mn}+\varphi_{m0})$$

$$+\frac{A_1U_{m0}\omega^2}{2\left[(A_3-K_0)^2+\frac{P^2}{C^2}\right]}\left(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}\right)\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4-\omega)}{P^2+(A_4+\omega)^2}\cos(A_3z-K_0z+\varphi_{mn}+\varphi_{mi})\right.$$

$$+\frac{P^2}{P^2+(A_4+\omega)^2}\sin(A_3z-K_0z+\varphi_{mn}$$

$$\left.\left.\left.+\ \varphi_{m0})\right]\right]\right\}U_\varepsilon(z,p)\left(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}\right)\frac{C^2B_1}{P^2}\sum_{n=0}^{N}B_2\left[\frac{P}{P^2+B_3^2}(B_3cos\Psi_m+Psin\Psi_n)\right]U_N(z,p)$$

$$+\frac{e^{\frac{p}{c}z}}{\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)}U_0\frac{P^2cos\varphi_0-\omega Psin\varphi_0}{P^2+\omega^2}-\frac{1}{\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)}U_1\frac{P^2cos(K_0z+\varphi_1)+\omega Psin(K_0z+\varphi_1)}{P^2+\omega^2}$$

$$-\frac{C^2(P^2cos\varphi_{m0}-\omega Psin\varphi_{m0})}{P^2(P^2+\omega^2)}U_0(z,p)$$

$$-\left\{\frac{A_1U_{m0}\omega^2}{2\left[(A_3+K_0)^2+\frac{P^2}{C^2}\right]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4-\omega)}{P^2+(A_4-\omega)^2}\cos(A_3z+K_0z+\varphi_{mn}+\varphi_{m0})\right.\right.$$

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$$+\frac{P^2}{P^2+(A_4-\omega)^2}sin(A_3z+K_0z+\varphi_{mn}+\varphi_{m0})\Bigg]$$

$$+\frac{A_1U_{m0}\omega^2}{2\left[(A_3-K_0)^2+\frac{P^2}{C^2}\right]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4+\omega)}{P^2+(A_4+\omega)^2}\cos(A_3z-K_0z+\varphi_{mn}+\varphi_{m0})\right.$$

$$\left.+\frac{P^2}{P^2+(A_4+\omega)^2}sin(A_3z-K_0z+\varphi_{mn}+\varphi_{m0})\right]\Bigg\}U_\varepsilon(z,p)$$

$$-\frac{C^2B_1}{P^2}\sum_{n=0}^{N}B_2\left[\frac{P}{P^2+B_3^2}(B_3cos\Psi_m+Psin\Psi_n)\right]U_N(z,p)$$

$$U_{E_1}(z,p)+\frac{\left(1-e^{\frac{p}{c}z}\right)C^2}{\left(e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}\right)P^2}B_1\sum_{n=0}^{N}B_2\left[\frac{P}{P^2+B_3^2}a(B_3\cos\Psi_m+P\sin\Psi_n)\right]U_N(z,p)$$

$$+\frac{e^{\frac{P}{c}z}}{e^{\frac{P}{c}z}+e^{-\frac{P}{c}z}}U_0\frac{P^2cos\varphi_0-\omega Psin\varphi_0}{P^2+\omega^2}$$

$$-\frac{1}{e^{\frac{P}{c}z}-e^{-\frac{P}{c}z}}U_1\frac{P^2\cos(K_0z+\varphi_1)+\omega Psin(K_0z+\varphi_1)}{P^2+\omega^2}$$

$$+\left(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}\right)\frac{c^2(P^2cos\varphi_{m0}-\omega Psin\varphi_{m0})}{P^2(P^2+\omega^2)}U_0(z,p)$$

$$+\{\left(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}\right)\frac{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}{2\left[(A_3-K_0)^2+\frac{P^2}{C^2}\right]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_3-\omega)}{P^2+(A_4-\omega)^2}\cos(A_3z+K_0z+\varphi_{mn}\right.$$

$$+\varphi_{m1})+\frac{P^2}{P^2+(A_4-\omega)^2}\sin(A_3z+K_0z+\varphi_{mn}+\varphi_{m0})$$

$$+\frac{A_1U_{m0}\omega^2}{2\left[(A_3-K_0)^2+\frac{P^2}{C^2}\right]}\left(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}\right)\Bigg]\sum_{n=0}^{N}A_2\sum_{m=1}^{M}[\frac{P(A_4+\omega)}{P^2+(A_4+\omega)^2}\cos(A_3z-K_0z+\varphi_{mn}$$

$$+\varphi_{m1})+\frac{P^2}{P^2+(A_4+\omega)^2}\sin(A_3z-K_0z+\varphi_{mn}+\varphi_{m0})]]\}U_{\varepsilon_1}(z,p)$$

$$+(\frac{e^{\frac{p}{c}z}-1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}})\frac{C^2B_1}{P^2}\sum_{n=0}^{N}B_2\left[\frac{P}{P^2+B_3^2}(B_3\cos\Psi_m+P\sin\Psi_n)\right]U_N(z,P)$$

$$+\frac{e^{-\frac{p}{c}z}}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}U_0\frac{P^2cos\varphi_0-\omega Psin\varphi_0}{P^2+\omega^2}$$

$$--\frac{1}{e^{\frac{p}{c}z}-e^{-\frac{p}{c}z}}U_1\frac{P^2\cos(K_0z+\varphi_1)+\omega Psin(Kz_0+\varphi_1)}{P^2+\omega^2}-$$

$$\frac{C^2(P^2cos\varphi_{m0}\text{-}\omega Psin\varphi_{m0})}{P^2(P^2+\omega^2)}U_0(z,P)\text{-}\Big\{A_1U_m\omega^2$$

Expressions (19) after simplifications and transformations are expressed in terms of transfer functions along the channels of the incident wave, fluctuations in the permittivity of the troposphere and interference (Figure 2).

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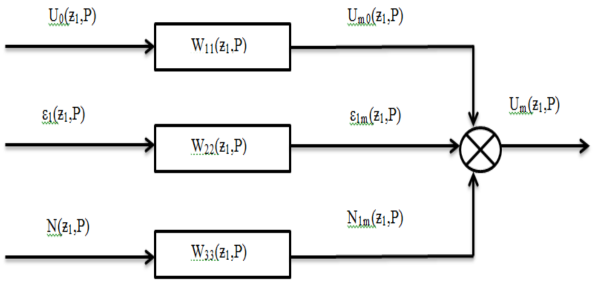

Figure 2: Structural diagram of the process of propagation of a plane electromagnetic wave in a fractal tropospheric atmosphere.

U0(z,P) – incident plane wave; Ɛ1(z,t) – dielectric constant fluctuation; N(z,P) – interference in the tropospheric atmosphere.

$$U(z,P) = W_{11}(z,P)U_0(z,P) + W_{22}(z,P)\square_1(z,P) + W_{33}(z,P)N(z,P)$$

Где

$$W_{11}(z,P) = \frac{2P}{\left(e^{\frac{P}{c}z} - e^{-\frac{P}{c}z}\right)(P^2+\omega^2)}\left[U_0 e^{-\frac{P}{c}z}(Pcos\varphi_0 - \omega Psin\varphi_0) - U_1\left(Pcos(K_0z+\varphi_1) + \omega sin(K_0z+\varphi_1)\right)\right] - \frac{c^2(Pcos\varphi_{m0} - \omega sin\varphi_{m0})}{P(P^2+\omega^2)}$$

$$W_{11}(z,P) = \frac{2P}{\left(e^{\frac{P}{c}z} - e^{-\frac{P}{c}z}\right)(P^2+\omega^2)}\left[U_0 e^{-\frac{P}{c}z}(Pcos\varphi_0 - \omega Psin\varphi_0) - U_1\left(Pcos(K_0z+\varphi_1) + \omega sin(K_0z+\varphi_1)\right)\right] - \frac{c^2(Pcos\varphi_{m0} - \omega sin\varphi_{m0})}{P(P^2+\omega^2)}; \ (21)$$

$$W_{22}(z,P) = -\frac{A_1U_{m0}\omega^2}{2[(A_3+K_0)^2+\frac{P^2}{c^2}]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4-\omega)}{P^2+(A_4-\omega)^2}\cos(A_3z+K_0z+\varphi_{mn}+\varphi_{m0}) - \frac{P^2}{P^2+(A_4-\omega)^2}\sin(A_3z+K_0z+\varphi_{mn}+\varphi_{m0})\right] - \frac{A_1U_{m0}\omega^2}{2[(A_3+K)^2+\frac{P^2}{c^2}]}\sum_{n=0}^{N}A_2\sum_{m=1}^{M}\left[\frac{P(A_4-\omega)}{P^2+(A_4-\omega)^2}cos(A_3z-K_0z+\varphi_{mn}+\varphi_{m0}) - \frac{P^2}{P^2+(A_4-\omega)^2}sin(A_3z-K_0z+\varphi_{mn}+\varphi_{m0})\right]; \ (22)$$

$$W_{33}(z,P) = -\frac{c^2B_1}{P^2}\sum_{n=0}^{N}B_2\left[\frac{P}{P^2+B_3^2}\left(B_3cos\Psi_m + Psin\Psi_m\right)\right] \ (23)$$

The transfer functions (21), (22) and (23) of a completely sufficient power describe the process of propagation of plane electromagnetic waves (signals) in a fractal tropospheric medium.

IV. CONCLUSION

Expressions (21), (22) and (23) can be used to study and analyze the processes of propagation of fractal signals in the tropospheric atmosphere in line of sight. In statistical radio engineering, with optimal signal processing, the processed signal is treated as a mixture of useful signal and noise (noise). In relations (21) and (22), the transfer function (21) is the transmitted useful signal, and the transfer functions (22) and (23) are interference. Thus, at reception points (solution of the differential equation (20)), we fix a useful signal additive by fluctuations and noise, which can be subjected to further optimal processing by the next subsystem of the system.

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The obtained expressions (21), (22) and (23) also make it possible to determine the spectral characteristics of the transmitted signal at the observation point if the spectral characteristics of the useful source of electromagnetic waves, the fluctuations in the permittivity of the medium and noise are known.

V. REFFRENCES

1. Benoit M, The Fractal Geometry of Nature. WH Freeman and company 1982; 1-468.
2. Paitgen HO, Richter PH, Beauty of fractals. 1993; 1-176.
3. Armin B, Shlomo H, Fractals in Disordered Systems. Springer Verlag, 1995: 1-408.
4. Alekseevich PA, Fractals and Frozen Operators in the Processing of Information - The Fundamental Direction of Synergetics. Loose of SFU. Technical Sciences 2005; 1-847.
5. http://books.sernam.ru/book_fract.php
6. Y Kim, DL Jaggerd, Band-limited fractal model of atmospheric refractivity fluctuation. Journal of Optical Society 1988; 5: 475-480.