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A Simple Design Approach In Yaw Plane For Two Loop Lateral Autopilots

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Abstract: The present work is an attempt towards achieving an alternative design algorithm for a two-loop autopilot which would involve simpler and less complicated relations, requiring lesser computational efforts and to some extent be flexible in achieving different deflection, deflection rate and body rate demands in specific flight conditions. The design approach in [2] is based on a two-loop lateral autopilot configuration in pitch plane developed as an equivalent flight path rate demand autopilot system. The key idea of this design approach is to achieve a preliminary set of control gains (K_p, K_q) on the basis of a simplified lateral autopilot in yaw plane using an ideal actuator system. This developed approach has been verified by evaluating the performance of the designed autopilots at a typical operating point. In this approach simulation of body rate, fin deflection and fin deflection rate has been observed. An attempt has been made to determine the effect of the different values of control gains on such body rate, fin deflection and fin deflection rate. In this approach sensitivity of the control gains have been studied. It has been studied how the change in control gains affect the frequency domain performance.

Keywords: Autopilot, Control Gain, Critical Gain Margin, Critical Phase Margin.

I. INTRODUCTION

Autopilot is an automatic control mechanism for keeping the spacecraft in desired flight path. An autopilot in a missile is a close loop system and it is a minor loop inside the main guidance loop. If the missile carries accelerometer and rate gyros to provide additional feedback into the missile servos to modify the missile motion then the missile control system is usually called an autopilot. When the autopilot controls the motion in the pitch and the yaw plane, they are called lateral autopilot. For a symmetrical cruciform missile pitch and the yaw autopilots are identical. The guidance system detects whether the missile is flying too high or too low, or too much to the left or right. It measures the deviation or errors and sends signals to the control system to minimize the errors. The lateral autopilot of a guided missile is a servo system delivering lateral acceleration (latax) according to the demand from the guidance computer. For aerodynamically controlled skid to run missile the autopilot activates to move the control surfaces suitably for orienting the missile body with respect to the flight path. This action generates angle of attack and consequently latax for steering the missile in the desired path[1].Missile autopilot design techniques have been dominated by classical control methods over the past several decades and a number of autopilots with two-loop and three-loop control configuration have been designed with their own merits and limitations. G.Das et al has suggested distinct approaches of designing two loop lateral autopilot[2,3].In [2] controller parameters are obtained quickly from a set of parametric equations involving performance measures, the aerodynamic characteristics of the plant and the available actuator. In [2] two design situations have been considered .The first design situation is the selection of actuator bandwidth for a given performance specifications of the controlled missile and in the second design, salient airframe parameters are derived for the given closed loop specifications. In [2] G.Das et al has suggested a systematic design methodology to design the control gains of two-loop lateral autopilot based on frequency domain performance specifications. Expressions are derived for GM, PM, GCF and PCF in parametric form are involving both aerodynamic and other sub-system parameters. For any operating point on the missile flight path envelope, the two desired control gain can be achieved using a developed algorithm by an iterative approach. The resulting design achieves the desired stability margins demanding specific fin deflections, fin rate and body rate demands. The design algorithm however, does not provide any explicit means to achieve some control over control surface deflections, their rates and the missile body rate which might be important in realistic flight conditions to meet the available system hardware constraints of to make a better or the best of the hardware resources implemented in the flight control system. The present work is a modest attempt towards achieving an alternative design algorithm for a two-loop autopilot which would involve simpler and less complicated relations, requiring lesser computational efforts and to some extent be flexible in achieving different deflection, deflection rate and body rate demands in specific flight conditions. The design approach in [2] is based on a two-loop lateral autopilot configuration in pitch plane developed as an equivalent flight path rate demand autopilot system. The aerodynamic transfer functions used are in parametric form and all force and moment coefficients etc are in the semi non-dimensional form in

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the pitch plane. For the purpose of design and analysis, the present work uses an equivalent two-loop flight path rate demand autopilot system in the yaw plane. This model has been derived using the equations of motion for a missile as presented in[1] and with same control surface conventions and using all aerodynamic derivatives in parametric form in the yaw plane. Design situations considered is similar to that in [2] for a specified actuator system (given ω_a, ζ_a). The key idea of this design approach is to achieve a preliminary set of control gains (K_p, K_q) on the basis of a simplified lateral autopilot in yaw plane using an ideal actuator system with the help of much simplified relations developed in parametric form involving aerodynamic parameters only and necessitating less computational effort in evaluating the stability margin values. Accordingly, parametric expressions have been derived for GM, PM, GCF and PCF of autopilot with ideal actuator. Again parametric expressions have been derived for GM, PM, GCF and PCF of autopilot with actuator, having same control gains (K_p, K_q). There will be a loss of GM and PM. This loss of GM and PM should be compensated by varying the control gains (K_p, K_q) so as to meet the desired performance specifications. This developed approach has been verified by evaluating the performance of the designed autopilots at a typical operating point for four different types of missile with aerodynamic data. In all four cases value of K_p is obtained from parametric equations where as K_q is chosen in iterative process so as to designed autopilot meet the desired performance specifications. In this approach simulation of body rate, fin deflection and fin deflection rate has been observed. An attempt has been made to determine the effect of the different values of control gains on such body rate, fin deflection and fin deflection rate. In this approach sensitivity of the control gains have been studied. It has been studied how the change in control gains affect the frequency domain performance.

II. MISSILE AUTOPILOT OVERVIEW

Guided missiles have assumed much importance in recent years. A guided missile is one which receives steering commands from the guided system to improve its accuracy. Guided action for guided missiles may be defined as the process of gathering information concerning the flight of a missile towards a given objective or target and utilising this information to develop manoeuvring commands to the control system of the missile. Guidance system functions by comparing the actual path of the missile with the desired path and providing commands to the control system which will result in manoeuvring the missile to its desired path. Guidance system actually gives command to the autopilot to activate the controls to achieve the correction necessary. Autopilots are closed loop system and these are minor loops inside the main guidance loop. An autopilot may be defined as the missile control system which modify the missile motion according to accelerometer and / or gyros feedback which provide information about the missile acceleration and body rate respectively. A lateral autopilot receives guidance command from the guided system of the missile to produce desired missile acceleration in lateral planes to follow the guidance path needed to the target. The autopilot responses to guidance system demand by deflecting the control surfaces of the missile for aerodynamic controlled missiles. The deflections in control surfaces produce change in missile angle of incidence. If the incidence angle is changed, the forces acting on the missile body changes and it results in change in missile acceleration. The two loop autopilot system uses two loops to feedback information of missile motion to the forward path of the autopilot. One loop is involved with body rate information which is fed back using one rate gyro. The other is the missile acceleration, sensed using accelerometer and provides the main feedback. The autopilot system results in change in missile motion. So, modelling of missile airframe dynamics is an important part of configuring an autopilot system. Missile dynamics is of non linear type. For configuring missile dynamics in transfer function form the missile airframes are trimmed and then linearized.

III. PROCESS MODEL

The reference axis system standardized for Guided Missile Industry is centered at the Centre of Gravity (c.g.) of the missile and is fixed in the body as shown in Fig 1. The *X-axis* is called the *"Roll Axis"*, which defined as positive in forward direction. The *Y-axis* is called the *"Pitch Axis"*, which is defined positive outwards and to the right of the Roll Axis, when viewed from the rear end of the missile.The *Z-axis* is called the *"Yaw Axis"*, which is defined positive downwards in the plane of symmetry & forms a right handed orthogonal system with the other two axes.

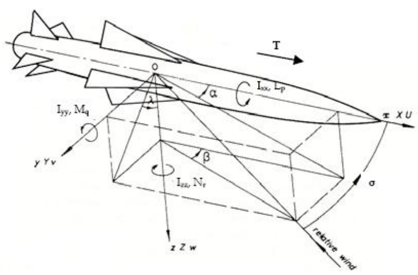

Fig.1: Missile Autopilot with Motion variable notations

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Block diagram of two-loop lateral autopilot in yaw plane will be as follows

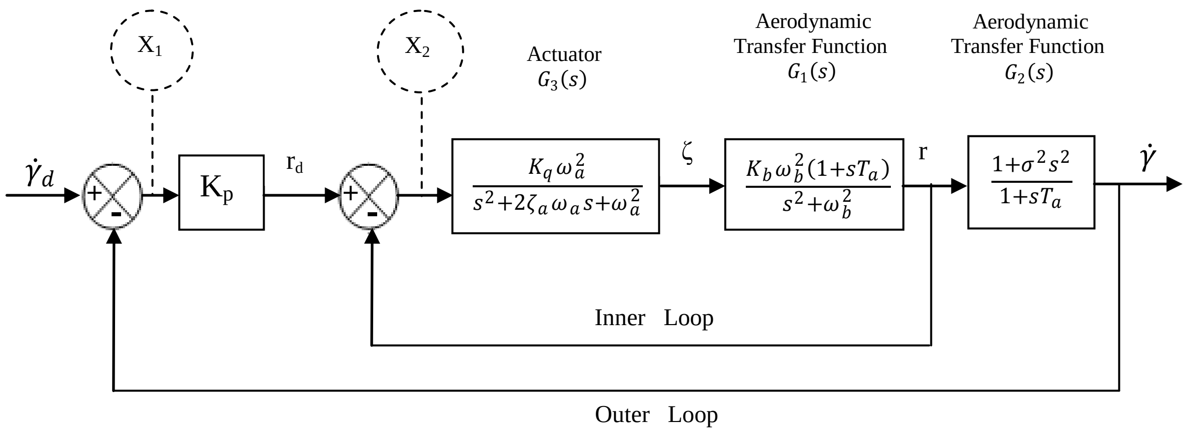

Fig.2: Flight Path Rate Demand Autopilot in Yaw Plane

IV. Mathematical Model Of Two-Loop Autopilot In Yaw Plan

The open loop transfer function of the autopilot for loop opened at X_1 is given by,

$$GX_1(s) = \frac{K_pG_1(s)G_2(s)G_3(s)}{1+G_1(s)G_3(s)}$$
$$= \frac{K_pK_qK_b\omega_a^2\omega_b^2(1+\sigma^2s^2)}{(s^2+\omega_b^2)(s^2+2\zeta_a\omega_a s+\omega_a^2)+K_qK_b\omega_a^2\omega_b^2(1+sT_a)}$$

Magnitude of $GX_1(j\omega)$is

$$|GX_1(j\omega)| = \frac{\frac{K_p\omega_a^2\omega_c}{T_a}(1+\sigma^2\omega^2)}{\sqrt{\left[\frac{\omega_a^2\omega_c}{T_a}-(\omega^2-\omega_b^2)(\omega_a^2-\omega^2)\right]^2+[\omega_c\omega_a^2\omega-2\zeta_a\omega_a\omega(\omega^2-\omega_b^2)]^2}}$$

Phase angle of $GX_1(j\omega)$ is

$$\angle GX_1(j\omega) = \theta_1 = -\tan^{-1}\left[\frac{\omega_a^2\omega_c\omega-2\zeta_a\omega_a\omega(\omega^2-\omega_b^2)}{\frac{\omega_a^2\omega_c}{T_a}-(\omega^2-\omega_b^2)(\omega_a^2-\omega^2)}\right]$$

Condition for Phase crossover frequency$(\omega = \omega_{px1})$:

$$\omega = \omega_{px1} = \sqrt{\frac{\omega_a\omega_c}{2\zeta_a}+\omega_b^2}$$

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Gain Margin :

$$GMX_1 = \frac{1}{|GX_1(j\omega)|}\bigg|_{\omega=\omega_{px1}}$$

$$= \frac{\sqrt{\left[\frac{\omega_a^2\omega_c}{T_a} - (\omega^2-\omega_b^2)(\omega_a^2-\omega^2)\right]^2 + [\omega_c\omega_a^2\omega - 2\zeta_a\omega_a\omega(\omega^2-\omega_b^2)]^2}}{\frac{K_p\omega_a^2\omega_c}{T_a}(1+\sigma^2\omega^2)}$$

Condition for Gain crossover frequency$(\omega = \omega_{gx1})$:

$$K_p = \frac{\sqrt{\left[\frac{\omega_a^2\omega_c}{T_a} - (\omega^2-\omega_b^2)(\omega_a^2-\omega^2)\right]^2 + [\omega_c\omega_a^2\omega - 2\zeta_a\omega_a\omega(\omega^2-\omega_b^2)]^2}}{\frac{\omega_a^2\omega_c}{T_a}(1+\sigma^2\omega^2)}$$

Phase Margin:

$$PMX_1 = 180^\circ - \tan^{-1}\left[\frac{\omega_a^2\omega_c\omega - 2\zeta_a\omega_a\omega(\omega^2-\omega_b^2)}{\frac{\omega_a^2\omega_c}{T_a} - (\omega^2-\omega_b^2)(\omega_a^2-\omega^2)}\right]$$

Autopilot Loop Opened at X_2 :

The open loop transfer function of the autopilot for loop opened at X_2 is given by,

Magnitude of $GX_2(j\omega)$

$$|GX_2(j\omega)| = \frac{\frac{\omega_c}{T_a}\omega_a^2\left[\sqrt{\left(1+K_p-K_p\sigma^2\omega^2\right)^2 + (\omega T_a)^2}\right]}{(\omega^2-\omega_b^2)\left[\sqrt{(\omega_a^2-\omega^2)^2 + (2\zeta_a\omega_a\omega)^2}\right]}$$

Phase angle of $GX_2(j\omega)$:

$$\angle GX_2(j\omega) = \theta_2 = -180^\circ + \tan^{-1}\left(\frac{\omega T_a}{1+K_p-K_p\sigma^2\omega^2}\right) - \tan^{-1}\left(\frac{2\zeta_a\omega_a\omega}{(\omega_a^2-\omega^2)}\right)$$

Condition for Phase Crossover Frequency$(\omega = \omega_{px2})$:

$$\omega = \omega_{px2} = \sqrt{\frac{\omega_a^2T_a - 2\zeta_a\omega_a(1+K_p)}{(T_a - 2\zeta_a\omega_aK_p\sigma^2)}}$$

$$K_p = \frac{T_a(\omega_a^2-\omega^2)}{2\zeta_a\omega_a(1-\sigma^2\omega^2)} - \frac{1}{(1-\sigma^2\omega^2)}$$

Gain Margin :

$$GMX_2 = \frac{1}{|GX_2(j\omega)|}\bigg|_{\omega=\omega_{px2}}$$

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$$= \frac{2\zeta_a\left[T_a(\omega_a^2 - \omega_b^2) - 2\zeta_a\omega_a\left(1 + K_p - K_p\sigma^2\omega_b^2\right)\right]}{\omega_a\omega_c\left(T_a - 2\zeta_a\omega_a K_p\sigma^2\right)}$$

Condition for gain crossover frequency$\left(\omega = \omega_{gx2}\right)$:

$$K_p = \frac{T_a}{\omega_c\omega_a^2(1-\sigma^2\omega^2)}\sqrt{[(\omega^2-\omega_b^2)^2[(\omega_a^2-\omega^2)^2+(2\zeta_a\omega_a\omega)^2]-\omega_c^2\omega_a^4\omega^2]-\frac{1}{(1-\sigma^2\omega^2)}}$$

Phase Margin :

$$PMX_2 = \left[-180^\circ + \tan^{-1}\left(\frac{\omega T_a}{1+K_p-K_p\sigma^2\omega^2}\right) - \tan^{-1}\left(\frac{2\zeta_a\omega_a\omega}{\omega_a^2-\omega^2}\right)\right] + 180^\circ$$
$$= \tan^{-1}\left(\frac{\omega T_a}{1+K_p-K_p\sigma^2\omega^2}\right) - \tan^{-1}\left(\frac{2\zeta_a\omega_a\omega}{\omega_a^2-\omega^2}\right)$$

Design Objective: To obtain the control gains K_p and K_q for maximum possible gain cross over frequency (GCF) satisfying desired performance specifications.

Desired Specifications:

Critical gain margin $\geq$ 6 dB
Critical phase margin $\geq$ 40 degrees

Case	T_a , s	ω_b, rad/sec	σ^2 ,s^2	$K_b\, m_\eta$,s^{-2}	ω_a, rad/sec	ζ_a	K_b ,s^{-1}	U,m/s
Case-1	0.36	11.77	0.00029	-530	180	0.6	9.91	470
Case-2	2.85	5.6	0.00142	-12.84	180	0.6	0.1437	3000
Case-3	3.58	3.6	0.0067	-15.39	180	0.6	-0.3317	500
Case-4	1.37	8.59	0.00206	-128	157	0.6	1.265	200

Table1: Missile parameter values

Case	Control Gains		Loop opened at X_2							
			Without actuator				With actuator			
	Kp	Kq	GM (dB)	PM (deg)	GCF	PCF	GM (dB)	PM (deg)	GCF	PCF
Case-1	5.69	-0.07	14.94	60	49.37	Inf	7.56	40.16	50.16	120.49
Case-2	30	-1.72	9.63	51.02	29.48	Inf	7.71	40.97	84.3	26.25
Case-3	19.39	-0.72	7.89	46.86	16.02	Inf	6.5	40.7	59.37	16.03
Case-4	10.8	-0.15	10.16	52.91	26.8	Inf	6.98	40.9	75.37	26.98

Table2: Frequency domain study of two-loop autopilot in yaw plane

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V. DESIGN ALGORITHM

Step1: Considering ideal actuator system parametric expressions for GM, PM, GCF and PCF of autopilot and simple relations of K_p and K_q are derived.
Step2: Next autopilot in yaw plane has been analyzed deriving expressions GM, PM, GCF and PCF implementing the actuator dynamics (ω_a, ζ_a) in parametric form.
Step3: Loss in GM and PM value in presence of a given actuator with a desired specifications GM $\leq$ 6 dB and PM $\leq$ 40 degree have been utilized.
Step4: Set a value of PM as $PM_e = PM_d + PM_d/2$. PM_d is the desired PM.
Step5: Find out new GM, PM, GCF and PCF.
Step6: Check whether new PM is greater than or equal to PM_d. If new PM is less than PM_d then go to Step7, otherwise go to Step4.
Step7: Set $PM_{e1} = PM_e - 1$ and go to Step 5.
Step8: Print the values of K_p and K_q.

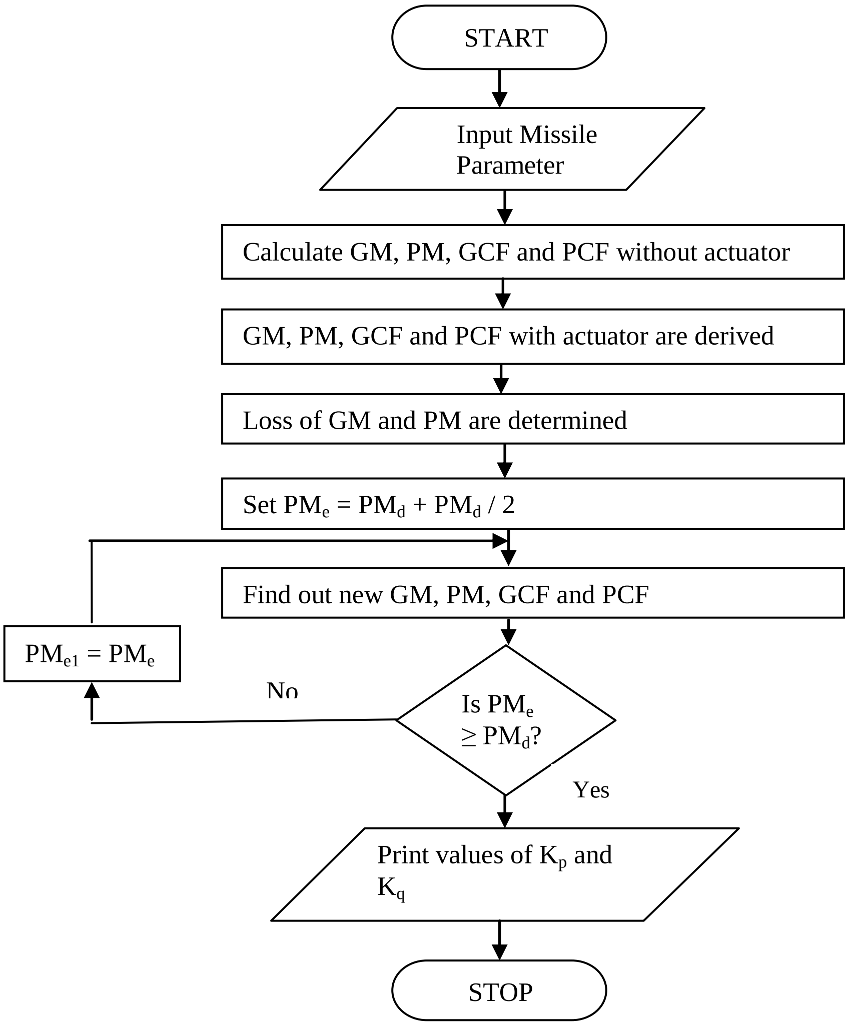

Fig. 3: Flow chart for Control Gain design

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VI. NATURE OF BODY RATE, FIN DEFLECTION, FIN DEFLECTION RATE FOR CONTROL GAINS

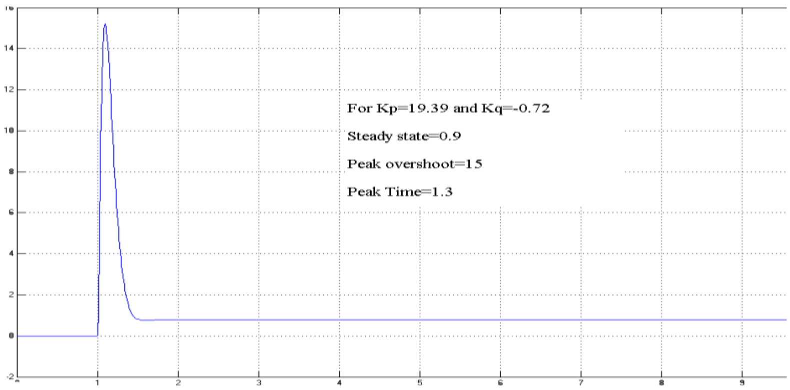

Fig. 4(a): Body rate for K_p =19.39 and K_q=-0.72

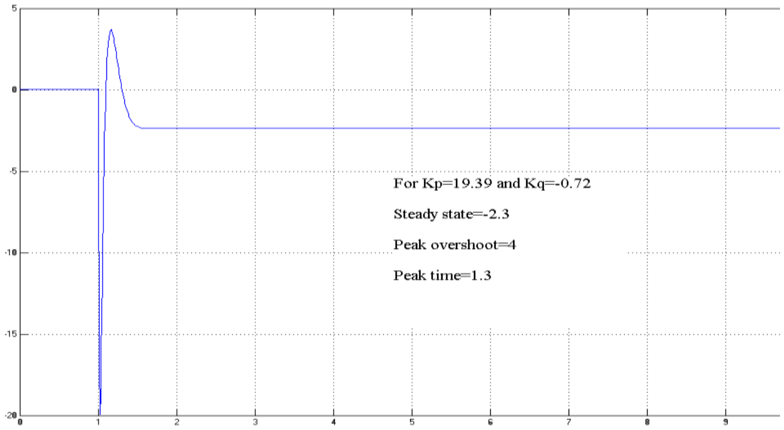

Fig. 4(b): Fin deflection for K_p =19.39 and K_q=-0.72

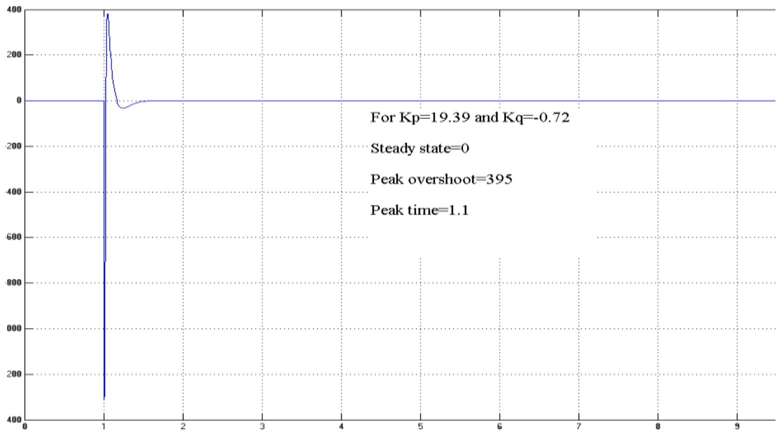

Fig. 4(c): Fin deflection rate for K_p =19.39 and K_q=-0.72

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VII. CONCLUSION

A simple design approach has been developed for designing two-loop missile autopilot in yaw plane In this work modeling of two-loop autopilot has been developed for tail-controlled system. Simple expressions in parametric form have been derived to find out the GM, PM, GCF and PCF. It was observed that this methodology gives results comparable to what had been obtained by G.Das et al in[2], but this approach of evaluation of control gains is comparatively simpler and requires less computational efforts. The proposed approach may be advantageously used to find out whether a combination of specified phase margin and gain margin pairs could be obtained for a given aerodynamic data set and actuator dynamics with the two-loop autopilot configuration. In the future this work may be extended, by taking system nonlinearities into consideration.

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Biography

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