ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 7, July 2014

A Two Stage Batch Arrival Queue with Compulsory Server Vacation and Second Optional Repair

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ABSTRACT: This paper deals with a single server batch arrival queue, two stages of heterogeneous service with different (arbitrary) service time distribution subject to random breakdowns followed by a repair and compulsory server vacations with general (arbitrary) vacation periods. After first stage service the server must provide second stage service. However after the completion of each second stage service the server will take compulsory vacation. The system may breakdown at random and it must be send to repair process immediately. If the server could not be repaired with first essential repair, subsequent repairs are needed for the restoration of the server. Both first essential repair and second optional repair times follow exponential distribution The steady state solutions have been found by using supplementary variable technique. Also the mean queue length and the mean waiting time are computed.

KEYWORDS: Batch Arrivals, Breakdowns, Steady state, First essential repair, Second optional repair, Mean queue length, Mean waiting time.

I.INTRODUCTION

Vacation queues have been studied by numerous authors including Levy and Yechiali [8] Doshi [4,5] and Keilson and Servi[7] due to their wide applications in manufacturing and telecommunication systems. Vacation queues with c servers have been studied by Tian et al. [11]. Choudhury and Borthakur [3] have studied vacation queues with batch arrivals. Baba [2] employed the supplementary variable technique for deriving the transform solutions of waiting time for batch arrival with vacations. Multiple vacations have been studied by Tian and Zhang [12]

In real life situations, a queueing system might suddenly breakdown and hence the server will not be able to provide service unless the system is repaired. Madan and Maraghi[9] have studied batch arrival queueing system with random breakdowns and Bernoulli schedule server vacation having general vacation time. They have obtained steady state results in terms of the probability generating functions for the number of customers in the queue. Thangaraj and Vanitha [10] analysed a queueing system with compulsory server vacation and random breakdown.

The most realistic aspect in modelling of an unreliable server is multi optional repair. When server could not be repaired or restored by the first essential repair, subsequent repairs are needed to restore the server. Queues with multi optional repairs were considered by many others. Hsieh et al.[6] studied a queueing model in which the server is subject to several types of breakdowns and each type has two possible stages of repair. William Gray et al.[13]studied a queueing model with multiple types of server breakdown requires a finite random number of stages of repair. Ayyappan and shyamala[1] investigated batch arrival queue with second optional repair.

In this paper we consider a batch arrival queue where each arriving customer has to undergo two stages of service provided by a single server, one by one in succession. As soon as the second stage of a customer's service is complete, the server will go for compulsory vacation. The system may breakdown at random with breakdown rate α >0. As soon as the system is break down, it is immediately sent for repair wherein the repairman or repairing apparatus provides the first essential repair (FER). After the completion of (FER), the server may opt for second optional repair (SOR) with probability p or may join the system with complementary probability 1-p to render the service to the customers. Both first essential repair and second optional repair times follow exponential distribution. After the completion of the

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required repair, the server resumes its work immediately. And once the system breakdown, it enters repair process and the customer whose service interrupted goes back to the head of the queue.

II.MATHEMATICAL MODEL

1.Customers arrive at the system in batches of variable size in a compound Poisson process. Let $\lambda c_i \Delta t$ (i=1, 2...) be the first order probability that a batch of i customers arrives at the system during a short interval of time ($t, t+\Delta t$), where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches.

2.The server provides two stages of heterogeneous service one after the other in succession. An arrival batch shall receive the service offered at two stages one by one in succession, defined as the first stage (FS) and second stage (SS) service respectively. The service discipline is assumed to be on a first come first served basis (FCFS). The service time of the two stages follow general distribution with distribution function $B_j(v)$ and the density function $b_j(v)$, j=1,2

3.Let $\mu_j(x)dx$ be the conditional probability of the jth stage service during the interval $(x, x+dx]$ given that elapsed service time is x, so that

$$\mu_j(x) = \frac{b_j(x)}{1-B_j(x)} \qquad j=1, 2 \tag{1}$$

and therefore,

$$b_j(v) = \mu_j(v) e^{-\int_0^v \mu_j(x)dx} \qquad j=1, 2 \tag{2}$$

4.Once the second stage service of a unit is complete, the server will take compulsory vacation.

5. The vacation time also follows a general (arbitrary) distribution with distribution function $V(r)$ and density function $v(r)$ and $E(V^n)$ is the nth moment (n=1,2,…) of vacation time. Let $\gamma(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx]$ given that the elapsed vacation time is x, so that

$$\gamma(x) = \frac{v(x)}{1-V(x)} \tag{3}$$

and therefore,

$$v(r) = \gamma(r)\, e^{-\int_0^r \gamma(x)dx} \tag{4}$$

6.On returning from vacation the server instantly starts serving the customer at the head of the queue if any.

7.The system may breakdown at random and breakdowns occur according to Poisson stream with mean breakdown rate $\alpha > 0$. The customer receiving service during breakdown returns back to the head of the queue.

8.Once the system breaks down, it is immediately sent for repair wherein the repairman or repairing apparatus provides the first essential repair (FER). After the completion of (FER), the server may opt for second optional repair (SOR) with probability r or may join the system with complementary probability 1-r to render the service to the customers.

9.The repair process provides two types of repair in which the first type of repair is essential and the second type of repair is optional. Both exponentially distributed with mean $1/\beta_1$ and $1/\beta_2$ .After the completion of the required repair, the server provides service with the same efficiency as before failure according to FCFS discipline.

10. Various stochastic processes involved in the system are assumed to be independent of each other.

III. DEFINITIONS AND NOTATIONS

We define

$P_n^{(j)}(x,t) =$ Probability that at time t, server is active providing jth stage $(j=1,2)$ service and there are $n(\ge 0)$ customers in the queue excluding the one customer in jth stage (j=1,2) being served and the elapsed service time for

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

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this customer is x. Consequently, $P_n^{(j)}(x)=\int_0^\infty P_n^{(j)}(x,t)dx$ denotes the probability that at time t there are n customers in the queue excluding the one customer in j^th stage (j=1,2) service irrespective of the value x.

$V_n(x,t)$ =Probability that at time t the server is under vacation with elapsed vacation time x and there are $n(\geq 0)$ customers waiting in the queue for service.Consequently, $V_n(t)=\int_0^\infty V_n(x,t)dx$ denotes the probability that at time t there are n customers in the queue and server is under vacation irrespective of the value x.

$R_n^{(1)}(t)=$ Probability that at time t, the server is inactive due to breakdown and the system is under first essential repair while there are n (≥ 0) customers in the queue.

$R_n^{(2)}(t)=$ Probability that at time t, the server is inactive due to breakdown and the system is under second optional repair while there are n (≥ 0) customers in the queue.

$Q(t)=$ Probability that at time t, there are no customers in the system and the server is ideal but available in the system.

IV STEADY STATE CONDITION

Let $\lim_{t\to\infty} A_n(x,t)=A_n(x) \quad \lim_{t\to\infty} A_n(t)=\lim_{t\to\infty}\int_0^\infty A_n(x,t)dx=A_n \quad \lim_{t\to\infty}\frac{dA_n(t)}{dt}=0, \quad n\geq 0$

Where A= $P^{(1)}$, $P^{(2)}$, V.

$\lim_{t\to\infty}$ Q(t)=Q $\quad \lim_{t\to\infty} R_n^{(1)}(t)=R_n^{(1)} \quad \lim_{t\to\infty} R_n^{(2)}(t)=R_n^{(2)}, \quad n\geq 0$

denote the corresponding steady state probabilities.

V. EQUATIONS GOVERNING THE SYSTEM

According to the mathematical model mentioned above, the system has the following set of differential-difference equations

$$\frac{\partial}{\partial x}P_n^{(1)}(x)+(\lambda+\mu_1(x)+\alpha)P_n^{(1)}(x)=\lambda\sum_{i=1}^{n-1}c_iP_{n-i}^{(1)}(x), \qquad n\geq 1 \tag{5}$$

$$\frac{\partial}{\partial x}P_0^{(1)}(x)+(\lambda+\mu_1(x)+\alpha)P_0^{(1)}(x)=0 \tag{6}$$

$$\frac{\partial}{\partial x}P_n^{(2)}(x)+(\lambda+\mu_2(x)+\alpha)P_n^{(2)}(x)=\lambda\sum_{i=1}^{n-1}c_iP_{n-i}^{(2)}(x), \qquad n\geq 1 \tag{7}$$

$$\frac{\partial}{\partial x}P_0^{(2)}(x)+(\lambda+\mu_2(x)+\alpha)P_0^{(2)}(x)=0 \tag{8}$$

$$\frac{\partial}{\partial x}V_n(x)+(\lambda+\gamma(x))V_n(x)=\lambda\sum_{i=1}^{n-1}c_iV_{n-i}(x), \qquad n\geq 1 \tag{9}$$

$$\frac{\partial}{\partial x}V_0(x)+(\lambda+\gamma(x))V_0(x)=0 \tag{10}$$

$$(\lambda+\beta_1)R_n^{(1)} = \lambda\sum_{i=1}^{n} c_i R_{n-i}^{(1)} + \alpha\int_0^\infty P_{n-1}^{(1)}(x)dx + \alpha\int_0^\infty P_{n-1}^{(2)}(x)dx, \qquad n\geq 1 \tag{11}$$

$$(\lambda+\beta_1)R_0^{(1)} = 0 \tag{12}$$

$$(\lambda+\beta_2)R_n^{(2)} = \lambda\sum_{i=1}^{n} c_i R_{n-i}^{(2)} + p\beta_1 R_n^{(1)}, \qquad n\geq 1 \tag{13}$$

$$(\lambda+\beta_2)R_0^{(2)} = p\beta_1 R_0^{(1)} \tag{14}$$

$$\lambda Q = (1-p)\beta_1 R_0^{(1)} + \beta_2 R_0^{(2)} + \int_0^\infty V_0(x)\gamma(x)dx \tag{15}$$

The above equations are to be solved subject to the following boundary conditions

$$P_0^{(1)}(0) = c_1\lambda Q + (1-p)\beta_1 R_1^{(1)} + \beta_2 R_1^{(2)} + \int_0^\infty V_1(x)\gamma(x)dx \tag{16}$$

$$P_n^{(1)}(0) = c_{n+1}\lambda Q + (1-p)\beta_1 R_{n+1}^{(1)} + \beta_2 R_{n+1}^{(2)} + \int_0^\infty V_{n+1}(x)\gamma(x)dx, \qquad n\geq 1 \tag{17}$$

$$P_n^{(2)}(0) = \int_0^\infty P_n^{(1)}(x)\mu_1(x)dx, \qquad n\geq 1 \tag{18}$$

$$V_n(0) = \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx, \qquad n\geq 0 \tag{19}$$

VI.QUEUE SIZE DISTRIBUTION AT A RANDOM EPOCH

Defining the following probability generating functions

$$A_q(x,z) = \sum_{n=0}^{\infty} z^n \mathrm{A_n}(x) \qquad A_q(z) = \sum_{n=0}^{\infty} z^n \mathrm{A_n} \qquad \text{Where } \mathrm{A= P^{(1)},\ P^{(2)},\ V.}$$

$$R_q^{(1)}(z) = \sum_{n=0}^{\infty} z^n\, \mathrm{R_n^{(1)}}, \qquad R_q^{(2)}(z) = \sum_{n=0}^{\infty} z^n\, \mathrm{R_n^{(2)}}, \qquad C(z) = \sum_{n=1}^{\infty} c_n z^n \tag{20}$$

Now multiplying the equation (5) by z^n and summing over n from 1 to ∞, adding to equation (6) and using the generating functions defined in (20) we obtain

$$\frac{\partial}{\partial x} P_q^{(1)}(x,z) + (\lambda - \lambda C(z) + \mu_1(x) + \alpha) P_q^{(1)}(x,z) = 0 \tag{21}$$

Performing similar operations on equations (7) to (14) we get

$$\frac{\partial}{\partial x} P_q^{(2)}(x,z) + (\lambda - \lambda C(z) + \mu_2(x) + \alpha) P_q^{(2)}(x,z) = 0 \tag{22}$$

$$\frac{\partial}{\partial x} V_q(x,z) + (\lambda - \lambda C(z) + \gamma(x)) V_q(x,z) = 0 \tag{23}$$

$$(\lambda - \lambda C(z) + \beta_1) R_q^{(1)}(z) = \alpha z\int_0^\infty P_q^{(1)}(x,z)dx + \alpha z\int_0^\infty P_q^{(2)}(x,z)dx \tag{24}$$

$$(\lambda - \lambda C(z) + \beta_2) R_q^{(2)}(z) = p\beta_1 R_q^{(1)}(z) \tag{25}$$

For the boundary conditions we multiply both sides of equation (17) by z^{n+1}, sum over n from 1 to ∞, multiply both sides of (16) by z, add the two results and using the probability generating functions in equation (20) we obtain

$$zP_q^{(1)}(0,z) = (\lambda C(z) - \lambda)Q + (1-p)\beta_1 R_q^{(1)}(z) + \beta_2 R_q^{(2)}(z) + \int_0^\infty V_q(x,z)\gamma(x)dx \tag{26}$$

Now multiply equation (18) by z^n and sum over n from 0 to ∞ and using equation (20) we get

$$P_q^{(2)}(0,z) = \int_0^\infty P_q^{(1)}(x,z)\mu_1(x)dx \tag{27}$$

Performing similar operation on equation (19) we obtain

$$V_q(0,z) = \int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx \tag{28}$$

Integrating equations (21) to (23) from 0 to x we get

$$P_q^{(1)}(x,z) = P_q^{(1)}(0,z)e^{-(\lambda-\lambda C(z)+\alpha)x - \int_0^x \mu_1(t)dt} \tag{29}$$

$$P_q^{(2)}(x,z) = P_q^{(2)}(0,z)e^{-(\lambda-\lambda C(z)+\alpha)x - \int_0^x \mu_2(t)dt} \tag{30}$$

$$V_q(x,z) = V_q(0,z)e^{-(\lambda-\lambda C(z))x - \int_0^x \gamma(t)dt} \tag{31}$$

Again integrating equations (29) to (31) by parts with respect to x yields

$$P_q^{(1)}(z) = P_q^{(1)}(0,z)\left[\frac{1-B_1^*(\lambda-\lambda C(z)+\alpha)}{\lambda-\lambda C(z)+\alpha}\right] \tag{32}$$

$$P_q^{(2)}(z) = P_q^{(2)}(0,z)\left[\frac{1-B_2^*(\lambda-\lambda C(z)+\alpha)}{\lambda-\lambda C(z)+\alpha}\right] \tag{33}$$

where $B_j^*(\lambda-\lambda C(z)+\alpha) = \int_0^\infty e^{-(\lambda-\lambda C(z)+\alpha)x} dB_j(x)$ is Laplace- stieltjes transform of the j^{th} (j=1,2) stage service time $B_j(x)$.

$$V_q(z) = V_q(0,z)\left[\frac{1-V^*(\lambda-\lambda C(z))}{\lambda-\lambda C(z)}\right] \tag{34}$$

where $V^*(\lambda-\lambda C(z)) = \int_0^\infty e^{-(\lambda-\lambda C(z))x} dV(x)$ is Laplace- stieltjes transform of the vacation time $V(x)$.

Using equations (32) and (33) in equation (24) we get

$$R_q^{(1)}(z) = \frac{\alpha z P_q^{(1)}(0,z)}{(\lambda-\lambda C(z)+\beta_1)}\left[\frac{1-B_1^*(\lambda-\lambda C(z)+\alpha)}{\lambda-\lambda C(z)+\alpha}\right] + \frac{\alpha z P_q^{(2)}(0,z)}{(\lambda-\lambda C(z)+\beta_1)}\left[\frac{1-B_2^*(\lambda-\lambda C(z)+\alpha)}{\lambda-\lambda C(z)+\alpha}\right] \tag{35}$$

Using equation (35) in equation (25) we have

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$R_q^{(2)}(z) = \left(\frac{\alpha z P_q^{(1)}(0,z)}{(\lambda - \lambda C(z) + \beta_1)} \left[\frac{1 - B_1^*(\lambda - \lambda C(z) + \alpha)}{\lambda - \lambda C(z) + \alpha} \right] + \frac{\alpha z P_q^{(2)}(0,z)}{(\lambda - \lambda C(z) + \beta_1)} \left[\frac{1 - B_2^*(\lambda - \lambda C(z) + \alpha)}{\lambda - \lambda C(z) + \alpha} \right] \right) \times \left[\frac{p\beta_1}{(\lambda - \lambda C(z) + \beta_2)} \right] \quad (36)$$

Multiplying both sides of equations (29) to (31) by $\mu_1(x)$, $\mu_2(x)$ and $\gamma(x)$ respectively and integrate with respect to x we obtain

$$\int_0^\infty P_q^{(1)}(x,z)\mu_1(x)dx = P_q^{(1)}(0,z)B_1^*(\lambda - \lambda C(z) + \alpha) \quad (37)$$

$$\int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx = P_q^{(2)}(0,z)B_2^*(\lambda - \lambda C(z) + \alpha) \quad (38)$$

$$\int_0^\infty V_q(x,z)\gamma(x)dx = V_q(0,z)V^*(\lambda - \lambda C(z)) \quad (39)$$

Let us take

$$\lambda - \lambda C(z) + \alpha = m, \quad \lambda - \lambda C(z) = k, \lambda - \lambda C(z) + \beta_1 = n, \lambda - \lambda C(z) + \beta_2 = l$$

Using equations (37) to (39) in equations (26) to (28) we obtain

$$zP_q^{(1)}(0,z) = -kQ + (1-p)\beta_1 R_q^{(1)}(z) + \beta_2 R_q^{(2)}(z) + V_q(0,z)V^*(k) \quad (40)$$

$$P_q^{(2)}(0,z) = P_q^{(1)}(0,z)B_1^*(m) \quad (41)$$

$$V_q(0,z) = P_q^{(1)}(0,z)B_1^*(m)B_2^*(m) \quad (42)$$

Using equation (41) in equations (33) , (35) and (36) we get

$$P_q^{(2)}(z) = \left[\frac{1 - B_2^*(m)}{m}\right] B_1^*(m)P_q^{(1)}(0,z) \quad (43)$$

$$R_q^{(1)}(z) = \frac{\alpha z}{nm}\left[(1 - B_1^*(m)) + B_1^*(m)(1 - B_2^*(m))\right]P_q^{(1)}(0,z) \quad (44)$$

$$R_q^{(2)}(z) = \frac{p\beta_1 \alpha z}{nml}\left[(1 - B_1^*(m)) + B_1^*(m)(1 - B_2^*(m))\right]P_q^{(1)}(0,z) \quad (45)$$

Using equation (42) in equation (34) we obtain

$$V_q(z) = \left[\frac{1 - V^*(k)}{k}\right] B_1^*(m)B_2^*(m)P_q^{(1)}(0,z) \quad (46)$$

Now using equations (42), (44) and (45) in equation (40) and solving for $P_q^{(1)}(0,z)$

$$P_q^{(1)}(0,z) = \frac{-kmnlQ}{Dr} \quad (47)$$

Where $Dr = mnl(z - B_1^*(m)B_2^*(m)V^*(k)) - \left[l(1-p) + p\beta_2\right](1 - B_1^*(m)B_2^*(m))z\alpha\beta_1$ (48)

Substituting $P_q^{(1)}(0,z)$ in equations (32), (43),(44),(45) and (46) we obtain

$$P_q^{(1)}(z) = \frac{-knl(1 - B_1^*(m))Q}{Dr} \quad (49)$$

$$P_q^{(2)}(z) = \frac{-knlB_1^*(m)(1-B_2^*(m))Q}{Dr} \tag{50}$$

$$V_q(z) = \frac{-mnlB_1^*(m)B_2^*(m)(1-V^*(k))Q}{Dr} \tag{51}$$

$$R_q^{(1)}(z) = \frac{-kl\alpha z[1-B_1^*(m)B_2^*(m)]Q}{Dr} \tag{52}$$

$$R_q^{(2)}(z) = \frac{-k\alpha zp\beta_1[1-B_1^*(m)B_2^*(m)]Q}{Dr} \tag{53}$$

Let $W_q(z)$ denote the probability generating function of the queue size irrespective of the state of the system. Then adding equations (49) to (53) we get

$$W_q(z) = P_q^{(1)}(z) + P_q^{(2)}(z) + V_q(z) + R_q^{(1)}(z) + R_q^{(2)}(z)$$

$$W_q(z) = \frac{-Q\{mnlB_1^*(m)B_2^*(m)[1-V^*(k)] + [1-B_1^*(m)B_2^*(m)]k[nl + l\alpha z + p\alpha z\beta_1]\}}{Dr} \tag{54}$$

Where Dr is given in equation (48)

It is easy to verify that for z = 1, $W_q(z)$ is indeterminate of the form 0/0.Hence we apply L'Hopitals rule on equation (54) where $C(1)=1$, $C'(1)=E(I)$ is the mean of arriving batch of customers, $V^*(0)=1$ and $-V^{*'}(0)=E(V)$ is the mean vacation time. Thus

$$W_q(1) = \frac{\lambda QE(I)\{[\beta_1\beta_2 + \alpha\beta_2 + p\alpha\beta_1][1-B_1^*(\alpha)B_2^*(\alpha)] + \alpha\beta_1\beta_2B_1^*(\alpha)B_2^*(\alpha)E(V)\}}{dr} \tag{55}$$

Where

$$dr = \alpha\beta_1\beta_2B_1^*(\alpha)B_2^*(\alpha) - \lambda E(I)\{[\beta_1\beta_2 + \alpha\beta_2 + \alpha p\beta_1][1-B_1^*(\alpha)B_2^*(\alpha)] + \alpha\beta_1\beta_2B_1^*(\alpha)B_2^*(\alpha)E(V)\} \tag{56}$$

Therefore adding Q to equation (55) and equating to 1 and on simplifying, we get

$$Q = 1 - \lambda E(I)\left[\frac{p}{\beta_2B_1^*(\alpha)B_2^*(\alpha)} + \frac{1}{\alpha B_1^*(\alpha)B_2^*(\alpha)} + \frac{1}{\beta_1B_1^*(\alpha)B_2^*(\alpha)} - \frac{p}{\beta_2} - \frac{1}{\alpha} - \frac{1}{\beta_1} + E(V)\right] \tag{57}$$

And hence the utilization factor ρ of the system is given by

$$\rho = \lambda E(I)\left[\frac{p}{\beta_2B_1^*(\alpha)B_2^*(\alpha)} + \frac{1}{\alpha B_1^*(\alpha)B_2^*(\alpha)} + \frac{1}{\beta_1B_1^*(\alpha)B_2^*(\alpha)} - \frac{p}{\beta_2} - \frac{1}{\alpha} - \frac{1}{\beta_1} + E(V)\right] \tag{58}$$

where $\rho < 1$ is the stability condition under which the steady state exists. Equation (57) gives the probability that the server is idle. Substituting for Q from equation (57) into (54),we have completely and explicitly determined $W_q(z)$,the probability generating function of the queue size.

VII. THE MEAN NUMBER IN THE SYSTEM

Let L_q denote the mean number of customers in the queue under the steady state. Then

$$L_q = \lim_{z\to 1}\frac{d}{dz}W_q(z) \tag{59}$$

Since this formula gives 0/0 form, then we write $W_q(z)$ given in (54) as $W_q(z) = \frac{N(z)}{D(z)}$ where $N(z)$ and $D(z)$ are the numerator and denominator of the right hand side of (54) respectively. Then using the L'Hopital's rule twice we obtain

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

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$$L_q = \frac{D'(1)\,N''(1) - N'(1)\,D''(1)}{2(D'(1))^2} \quad (60)$$

Where primes and double primes in (60) denote first and second derivatives at z = 1. Carrying out the derivatives at z = 1, we have

$$N'(1) = \lambda E(I)Q\{(\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2) + B_1^*(\alpha)B_2^*(\alpha)[\alpha\beta_1\beta_2 E(V) - (\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2)]\} \quad (61)$$

$$N''(1) = 2Q[\lambda E(I)]^2\{[\frac{\alpha\beta_2 + \alpha p\beta_1}{\lambda E(I)} - (\alpha + \beta_1 + \beta_2)] + B_1^*(\alpha)B_2^*(\alpha)[\frac{\alpha\beta_2 + \alpha p\beta_1}{\lambda E(I)} - (\alpha + \beta_1 + \beta_2)$$

$$-\alpha\beta_1 E(V) - \beta_1\beta_2 E(V) - \alpha\beta_2 E(V) + \frac{1}{2}\alpha\beta_1\beta_2 E(V^2)] + B_1^{*'}(\alpha)B_2^*(\alpha)[(\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2) - \alpha\beta_1\beta_2 E(V)]$$

$$+ B_1^*(\alpha)B_2^{*'}(\alpha)[(\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2) - \alpha\beta_1\beta_2 E(V)]\}$$

$$+ \lambda Q E(I(I-1))\{(\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2) + B_1^*(\alpha)B_2^*(\alpha)[\alpha\beta_1\beta_2 E(V) \quad - (\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2)]\} \quad (62)$$

$$D'(1) = -\lambda E(I)[\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2] + B_1^*(\alpha)B_2^*(\alpha)\{\alpha\beta_1\beta_2 + \lambda E(I)[p\alpha\beta_1$$

$$+ \beta_1\beta_2 + \alpha\beta_2 - \alpha\beta_1\beta_2 E(V)]\} \quad (63)$$

$$D''(1) = 2[\lambda E(I)]^2\{[(\alpha + \beta_1 + \beta_2) - \frac{\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2}{\lambda E(I)}] + B_1^*(\alpha)B_2^*(\alpha)[-(\alpha + \beta_1 + \beta_2)$$

$$+ E(V)(\alpha\beta_1 + \beta_1\beta_2 + \alpha\beta_2) - \frac{1}{2}\alpha\beta_1\beta_2 E(V^2) - \frac{\alpha\beta_1(1-p)}{\lambda E(I)}]$$

$$+ B_1^{*'}(\alpha)B_2^*(\alpha)[-\frac{\alpha\beta_1\beta_2}{\lambda E(I)} - (\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2) + \alpha\beta_1\beta_2 E(V)]$$

$$+ B_1^*(\alpha)B_2^{*'}(\alpha)[-\frac{\alpha\beta_1\beta_2}{\lambda E(I)} - (\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2) + \alpha\beta_1\beta_2 E(V)]\}$$

$$+ \lambda E(I(I-1)\{-(\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2) + B_1^*(\alpha)B_2^*(\alpha)[\alpha p\beta_1 + \beta_1\beta_2 + \alpha\beta_2 - \alpha\beta_1\beta_2 E(V)]\} \quad (64)$$

where $E(V^2)$ is the second moment of the vacation time, E(I(I-1)) is the second factorial moment of the batch size of arriving customers. Using equations (61) to (64) into (60) , we obtain L_q in closed form where Q has been found in equation (57).If L denotes the mean number in the system including the one in the service, using Little's formula, we obtain,

$$L = L_q + \rho \quad (65)$$

VIII.THE MEAN WAITING TIME

Let W_q and W denote the mean waiting time in the queue and in the system respectively. Then using Little's formula, we obtain

$$W_q = \frac{L_q}{\lambda} \quad (66)$$

$$W = \frac{L}{\lambda} \quad (67)$$

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