

Analysis of the Gaussian Beam on a Corrugated Dielectric Interface

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ABSTRACT: This paper presents the analysis of the Gaussian light beam at a planar and a corrugated dielectric interfaces using Beam Propagation Method (BPM). The Reflected and Transmitted fields are investigating at total reflection. The results obtained through that method are in accordance with the theory of the non-specular phenomena of the electromagnetic field interaction at the same situation. Additionally, they are in agreement with theoretical predictions and significant improvement over the previously published results.

KEYWORDS: Beam Propagation Method, Gaussian Beam, Goos-Hänchen shift, the non-specular phenomena of Gaussian Beam at a planar interface, and Gaussian Beam at a corrugated interface.

I. INTRODUCTION

Many previous studies for the Gaussian light beam interactions on a dielectric interfaces, while the interface may be planar [1] or corrugated interface [2], are investigated. The Gaussian light beam shifts from geometric optics predications. These are known as the non specular shifts. These shifts investigated by many researchers from the beginning of the last century till now. Furthermore, many applications based on these shifts have very interesting practical implementations.

It is well known that, if the light beam is incidence from the denser medium on the corrugated interface it will be diffracted as Fig.1a demonstrates. The diffraction orders depend on the wavelength and the angle of incidence. On the other hand, in a planar interface case, the incident filed will give reflected field that is propagating in denser medium while the transmitted field propagates in rare medium as illustrated in Fig. 1b. There have been numerous researchers who are concerned with the same type of problems. Mostly of them has arrived to analytical solutions. In carrying out their task, there are several steps that are followed to reach the final results. Firstly, the incident beam is formulated in specular (Fourier) representation. Secondly, the longitudinal wave vector k_z represented as a function of the transversal wave vector k_x around the centered point k_{xi} using Taylor series. Finally, the Fresnel reflection coefficient $\rho(k_x)$ must be calculated in order to get an expression for the reflected field using numerical techniques. Consequently, the transmission coefficient $\tau(k_x)$ can be evaluated, following the same steps as in the case of reflection coefficient, and an expression for the transmitted field can be formulated. These steps lead to formulate the incident, reflected and transmitted fields. By comparing these fields with those predicted by geometric optics, it yields to some shifts which are called the nonspecular shifts of the field. These shifts include: longitudinal shift "Goos-Hänchen shift", transversal shift "Impert -Fedrove shift" (in case of circular or elliptic polarization), focal shift, angular displacement, and beam waist modification. In the case of Gaussian beam there are additional modifications in the beam profile (radius of curvature and beam divergence angle) as well as the complex form for the phase and magnitude of the resulted amplitude.

In this paper we present studding the Gaussian light beam on corrugated interface using BPM, and for more insights, we give the results at planar and corrugated cases of interfacing. BPM gives the total field after each propagation step, which means that; no need for calculating the reflection and transmission coefficients as in the case of the analytical studies. To achieve our object, some numerical processing techniques are used to represent the analysis of the Gaussian light beam at a corrugated interface. In other words, our analysis can be easily used to carry out a comparison between the interactions of the same incident field with different types of interfaces in order to check the validity of using BPM

as a numerical tool in analysing the Gaussian beam.

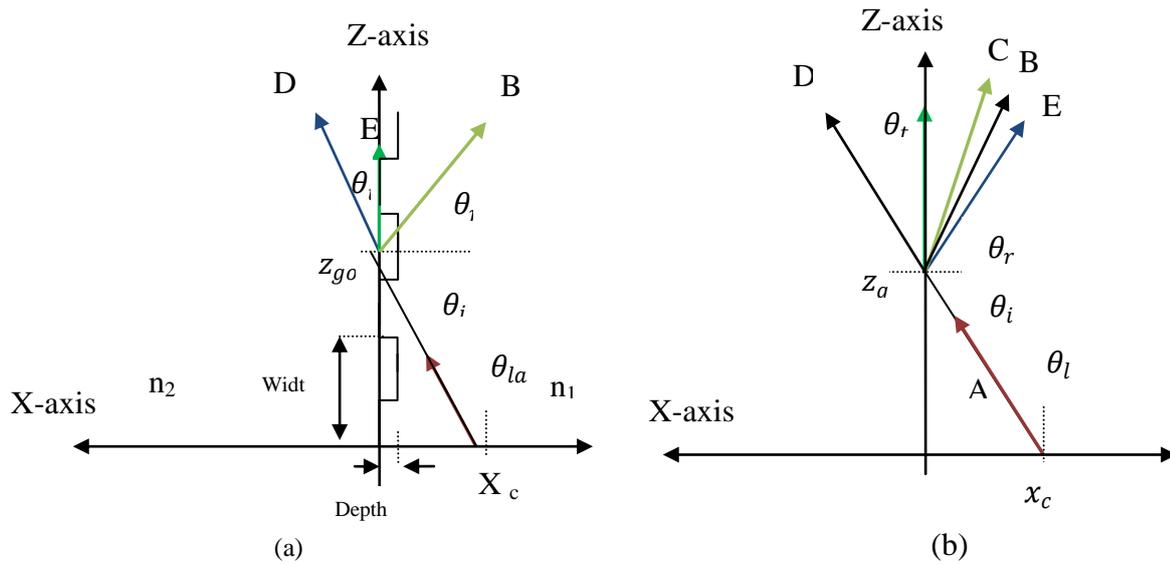


Fig.1: Ray tracing in the simulation (a) with corrugated interface (b) with a planar interface.

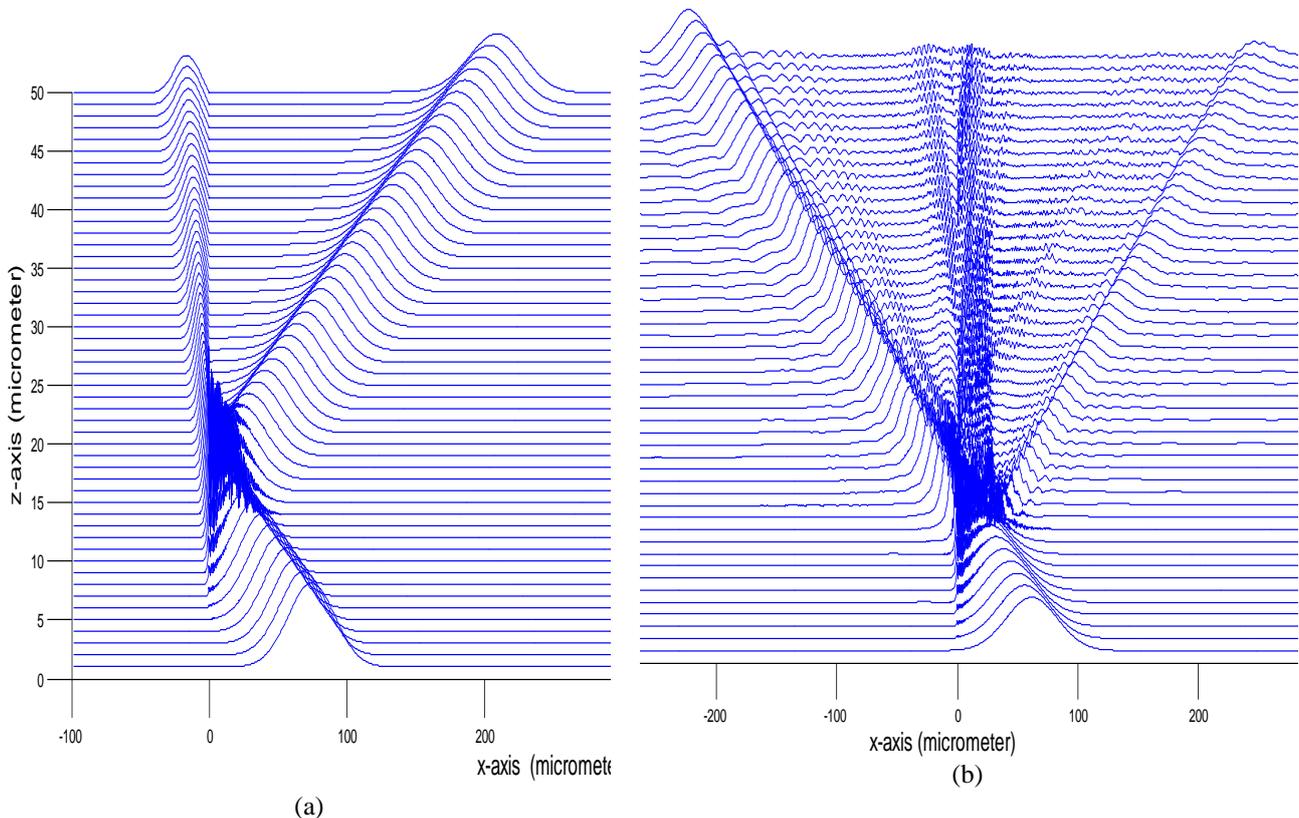


Fig 2: The total field overview at the incident critical angle (a) in planar interface case (b) in corrugated interface case.

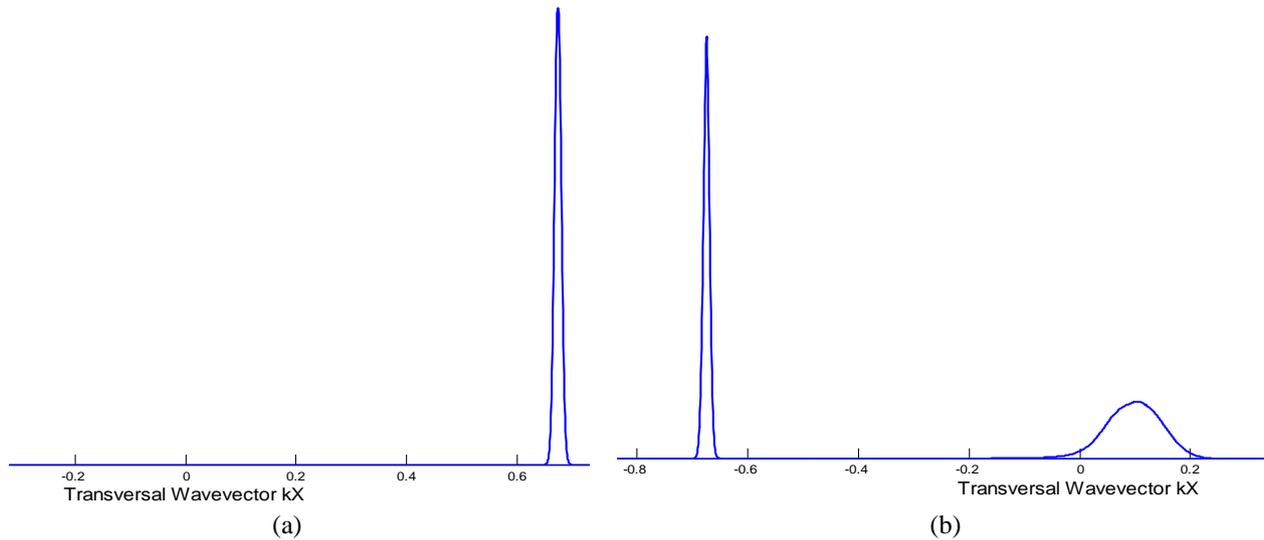


Fig.3: The spectrum for the field (a) input field spectrum, and (b) The output field spectrum in planar interface case.

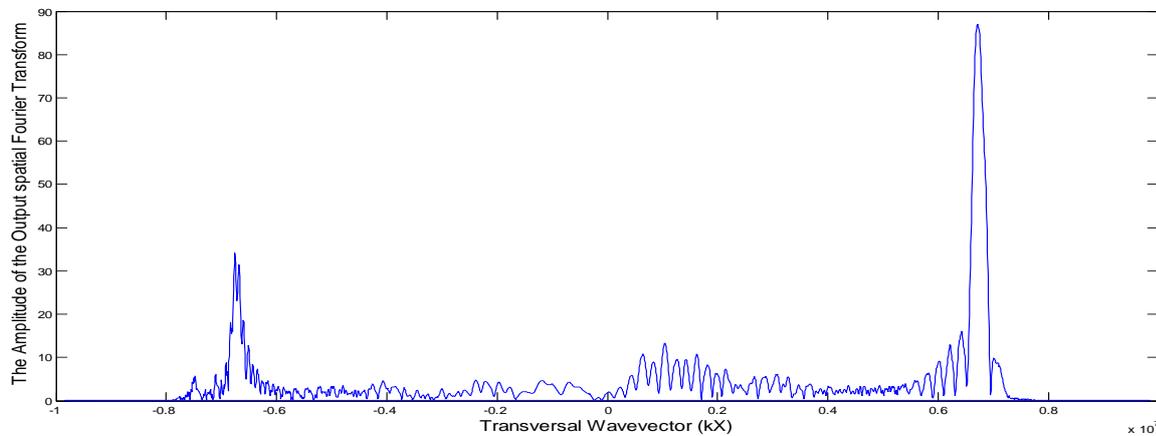


Fig.4: The output field spectrum in a corrugated interface case.

II. RELATED WORK

The beam propagation method is a powerful method that deals with bounded beams [6]. It can develop in many techniques [7, 8]. Here, we use Fourier Transform Beam Propagation Method (FT-BPM) numerical technique in analyzing the underlined problem. This technique represents the electromagnetic field in its angular spectrum representation, and its results give the total field after each step of discreteness of the path length. Briefly, the theory of the beam propagation method is based on operator splitting “Barker-Hausdroff theory [8]”. The spatial space is desecrating into small elements; each of length Δx , in transversal direction and Δz in longitudinal direction. Each component of beam spectrum propagates a distance of $\Delta z/2$ then it suffers from phase retardation, after that the field propagates the other distance $\Delta z/2$ to give the final field after each step Δz . we repeat these processes until the beam exit from the waveguide (the waveguide length = $\Delta z * \text{number of propagation steps}$).

BPM gives the results of the total field for each step of propagation. BPM takes in its account any modifications for the field until the propagations. As indicated earlier, no need to calculate the reflected or transmitted coefficient. With this

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in mind, the BPM has the simplicity of giving the results and more efficient in computation representation from our knowledge. We expect that BPM will help in studying the interaction of any type of fields such as: Gaussian beam, Cauchy beam, Lorentz beam..., with any type of media; homogenous or not, and the interface may be planer, corrugated, between two dielectric media, or between metallic and dielectric. Furthermore, BPM doesn't take into account any back reflection of the field. Consequently, it neglects the back reflection component for the field through the propagation. As a solution to this problem, the bidirectional BPM algorithm must be used. In our treating the beam analysis, we use very large waveguide dimension, relative to the beam parameters, in order to avoid the problem of back reflection.

III. RESULTS AND DISCUSSION

In this section, we are concerned with illustrating the benefits of using our technique over the BPM algorithm. We will present some numerical results for the same previously parameter values to show to what extent our method can improve the beam analysis relative to the BPM. The input parameters are chosen as: $n_1 = 1.94$ and $n_2=1.0$ (Fused Quartz-air interface). These refractive index values will lead to a critical angle at 31.0285° . It is assumed that the beam axis makes an angle θ_i with respect to the z-axis which corresponds to the critical angle, i.e. $\theta_i = 90-\theta_c= 58.9715^\circ$, as recommended from [6]. In addition, the Gaussian beam waist $w = 12.4 \mu\text{m}$, the beam position at $x_c = 6 w$, and the lanching beam wavelength in free space is $\lambda_0=1.55 \mu\text{m}$. In order to achieve our proposed condition, the wave guide width is taken as $2048 * 0.4\mu\text{m}$ (very large compared to the Gaussian beam width). Moreover, the surface of interfacing is assumed to have two forms: planer and corrugated. In the case of corrugated, the corrugation is square pitch; each pitch depth is $30 \mu\text{m}$ while the pitch depth is $60 \mu\text{m}$.

The initial Gaussian beam can be treated as a bundle of plane waves propagating at angles centered at θ_i . The wave vector k_x is the projection of the plane wave component of the spectrum, while k_{xi} is the transverse wave number corresponding to the centered plane wave component. The beam centered component angle corresponds to θ_c . The simulated BPM results can be summarized as follows:

A. THE INCIDENT FIELD

The Gaussian field is incident at an angle which corresponds to the critical. Consequently, in the case of planar interface, this field will split into two ones: the first is the refracted field that propagates in the rare medium near to the interface as Fig. 2a illustrates. The second one is the reflected field in the same medium of incidence. The input field spectrum takes the form [6]:

$$E_i(k_{xi}, 0) = \frac{1}{\cos \theta_i} e^{-\left[\frac{w(k_x - k_{xi})}{2 \cos \theta_i}\right]^2 + i(k_x - k_{xi})x_d} \tag{1}$$

The centered component value can be calculated numerically from the relation $k_{xi} = k_1 \sin \theta_i$. The substitution of the parameters in this relation gives $k_{xi} = 6.7388 \mu\text{m}^{-1}$. From Eq. (1), the width of the initial beam is $\Delta k_{xi} = 4 \cos \theta_i / w = 0.16628 \mu\text{m}^{-1}$, according to our proposed values. However, in corrugated interface case, the field intersects with the interface and diffracted in many orders, each order of diffraction depends on the incident angle and wavelength. Since, the pitch depth and width are widely assumed, the reflected and transmitted fields are collected smoothed. Fig. 2b shows the total field at the corrugated interface while Fig. 3a illustrates the spectrum of the incident field. As shown in this figure, the incident spectrum attains its peak at $k_{xi} = 6.739 \mu\text{m}^{-1}$ which matched with the calculated one. The input field spectrum is Gaussian and this insures that the incident field is Gaussian. Accordingly, the input spectrum is symmetric and centered around k_{xi} .

B. THE REFLECTED FIELD

From the reflected field point of view, the incident field intersects with the interface at z_{go} point which denotes the geometric optics point between the incident field and the interface. Evidently, in case of planar interface, all plane waves which their incident angle greater than the critical one, will totally reflected, while all those of lower incident angle will partially reflected and partially refracted. The angle for the most significant component of the plane wave can be calculated from the relation $\theta_{max} = \sin^{-1} (k_{xmax} / k_1) = 60.1675^\circ$, which leads to a total angular spread of $\Delta \theta_i = 2(\theta_{max} - \theta_i) = 2.3919^\circ$. In other words, all plane waves at angles $(\theta_i + \Delta \theta_i / 2)$ will totally reflected, while those at angles

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$(\theta_i - \Delta\theta_i / 2)$ will partially reflected. Conversely, in the case of corrugated interface, each plane wave of the spectrum will get rise to different reflected and transmitted fields, Fig. 2b shows that there are reflected fields in different directions, and for more clarifications, the corrugation parameters can be optimized to get the orders and directions of the total reflected field. From numerical calculations, the reflected spectrum is to be localized at $k_{xr} = -6.7338 \mu\text{m}^{-1}$, where the negative sign indicates that the reflected field is in the opposite direction relative to the incident field. Fig. 3b gives the BPM results for the output field spectrum. As shown, the reflected field is located at $k_{xr} = -6.741 \mu\text{m}^{-1}$, and is nearly Gaussian as we predict which means that the results are very close to the numerical expectations.

C. THE TRANSMITTED FIELD

In planar interface case, the plane waves corresponding to angles less than the critical will partially transmitted as shown in Fig. 2a. This figure demonstrates that the total transmitted field lies in rare medium at constant z-planes while the transmitted field propagates parallel to the interface. Fig. 3b gives the transmitted spectrum peak located at $k_{xt} = 1.037 \mu\text{m}^{-1}$ which means that the transmitted peak spectrum angle is $\theta_t = \sin^{-1}(k_{xt} / k_2) = 14.822^\circ$. The angle corresponding to the normal to the interface is $\theta_n = 90 - 14.8220 = 75.1780^\circ$. From Snell's law, the angle of incidence corresponding to θ_n in the denser medium is equal to $\sin^{-1}((n_2 / n_1) \sin \theta_n) = 29.8884^\circ$ which is less than θ_c , and this answer the question why there are refracted fields even though the incident beam is centered angle at the critical angle. The highest significant value of the transmitted spectrum is located at $k_{xt \text{ max}} = 2.371 \mu\text{m}^{-1}$ which corresponds to an angle $\theta_{t \text{ max}} = 35.7961^\circ$ with respect to the z-axis. The angle with respect to the normal to the interface is equal to $\theta_n = 54.2039^\circ$. The angular spread of the transmitted field can be calculated as: $\Delta\theta_t = 2(\theta_{t \text{ max}} - \theta_t) = 41.9482^\circ$, the spectral width is $\Delta k_{xt} = 2(k_{xt \text{ max}} - k_{xt}) = 2.6680 \mu\text{m}^{-1}$, and the transmitted field ratio is $(\Delta\theta_t / \Delta k_{xt}) = 15.7227 \mu\text{m}^{-1}$. It is evident that this ratio isn't equal to the incident field ratio and consequently the transmitted field not completely Gaussian.

IV. CONCLUSION

This analysis describes the Gaussian beam interaction at planar and corrugated dielectric interfaces between two homogenous media. The BPM results are matched with the theory of the nonspecular phenomenon of the electromagnetic field interaction on a dielectric interface. The results are obtained for total reflection and at linear polarization in order to check the shifts which are previously published [1-3]. The results are in agreed with the nonspecular phenomenon of the Gaussian beam at total reflection. Accordingly, BPM is the simplest numerical method to represent the interaction of the electromagnetic field at dielectric interface, which is simple in implementation and accurate in results. Hence, it can be used in studying the interaction of any type of fields (like Gaussian or Cauchy or others). This method doesn't propose any conditions for the field to be symmetric around its axis or not.

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