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Cantilever Beam and Torsion Rod Design Optimization Using Genetic Algorithm

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ABSTRACT- Any design optimization problem would be formulated as a multivariable, multi objective, constrained, non-linear programming problem. Various conventional optimization techniques are available for solving the above such problems. Due to the problem complexities and computational inefficiencies, better methods are needed. To achieve this goal, new techniques popularly known as intelligent techniques have been introduced. In this work it is proposed to implement Genetic Algorithm for solving the design optimization problem of Cantilever beam and Torsion rod. Cantilever beam and Torsion rod are used as structural elements in many Mechanical Engineering applications. The objective here is to design a cantilever beam and a Torsion rod for minimum weight in order to withstand the given working condition.

KEYWORDS- Optimization, Cantilever Beam, Torsion rod, Genetic Algorithm

I. INTRODUCTION

The goal of optimization is to minimize or maximize certain quantities such as life, mass, etc. In the mathematical model these goals are expressed as functions of certain variables. In a design problem those variables may be the various dimensions of the component. There are many methods available for solving these models towards minimization or maximization [1].

GA gives efficient result from multi-variable constrained non-linear programming problem. Genetic algorithms are global optimization methods based on several metaphors from biological evolution. The GA search for an optimum solution is from the population of candidate solution according to an objective function, which is used to establish the fitness of each candidate, as a solution.

The cantilever beam and Torsion rods are widely used as engineering components in various forms of applications and optimising the mass of these types of components becomes important. In this study, a problem has been formulated to minimize the mass of a cantilever beam and Torsion rod. A genetic algorithm has been used in this study to optimize the design parameters to achieve the minimum mass which satisfy various constraints.

Research has been carried out to develop a optimization algorithm based on both binary-coded and real-coded genetic algorithms [2] to solve mechanical component design problems like gear train design, Belleville spring design, welded beam design.

A method is proposed to prove the efficiency and ease of application of genetic algorithms by solving four different mechanical design problems [3].

A genetic algorithm based approach is developed for preliminary cam shape design and optimization based on predictive computer simulation of cam mechanism [4].The aim of this work is to describe a method that can be efficiently used for automatic cam shape determined and systematic seeking for optimum cam shaped.

New approach has been proposed to optimize the structure optimization with frequency constraints [5].

Work has been done to optimize tolerance design for gear train and over running clutch using genetic algorithm [6]. The objective of this work is to develop a genetic algorithm based optimization procedure.

The machine element coil spring, hollow shaft, pulley drive design has been optimized using the other non-traditional optimization algorithm like Simulated annealing [7]. The objective of this work is to imply that the simulated annealing algorithm technique can be applied efficiently and effectively for any type of design problem.

II. GENETIC ALGORITHM

Genetic algorithm simulates the survival of the fittest among individuals over consecutive generation for solving a problem. Each generation consists of a population of character .Each individuals represents a point in a search space and a possible solution. The individual in the population are then made to go through a process of evolution [8].The basic concept of genetic algorithm is to encode a potential solution to a problem as a series of parameters. A single set of parameters value is termed as the genome of an individual solution. An initial population of individuals is generated randomly or statically. In every generation the individuals in the current population are decoded according to a fitness function. The chromosome with the highest population fitness is selected for mating. The genes of the two parameters are allowed to exchange to reproduce off springs. These children then replace their parents in the next generation. Thus, the old population is discarded and the new population becomes the current population [9]. The current population is checked for acceptability or solution .The iteration is stopped after the completion of maximal number of generations or on the attainment of the best results.

The various steps involved are briefly described as given below

i) *Start:* Random population of 'n' chromes (suitable solution for the problem) are generated.

ii) *Fitness:* The fitness function of each chromosome in the population is evaluated

iii) *New Population*: A new population is created by repeating following steps.

iv) *Selection*: Two parent chromosomes are selected from the population according to their fitness, bigger the chance to be selected. In this work Tournament selection is used. [Melanie Mitchell,1998]

v) *Cross-Over*: Two parents are crossed to form a new off spring with a cross over probability.

vi) *Child Ness:* If no cross over is performed off-spring is an exact copy of parents.

vii) *Mutation:* New off springs are mutated with a mutation probability.

viii) *Accepting:* The new off spring are placed in a new population.

ix) *Replace:* Newly generated population is used for a further run of algorithm that is individuals from old population are killed and replaced by the new ones.

x) *Test*: The generation is stopped, if the end condition is satisfied and returns the best solution in the current population.

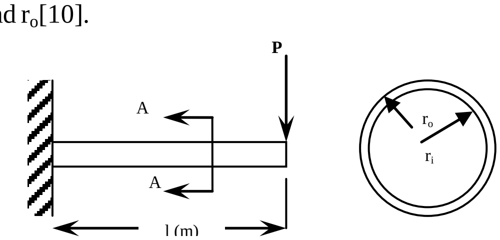
xi) *Loop:* If the termination criteria are not met, the loop is repeated from the fitness step again as reported above.

III. OPTIMIZATION MODEL

A. *Problem definition*

Case 1: Cantilever Beam

Minimise the mass of a cantilever beam shown in Fig. (1). It shows a cantilever beam arrangement which is subjected to a load P. Due to this load P stress and deflection are produced in the beam. The objective of the problem is to find the optimal values of the cross section parameters like r_i and r_o, such that the mass of the beam will be minimum within the given range of r_i and r_o[10].



Section A-A
Outer radius - r_o
Inner radius - r_i

Fig. 1 Cantilever Beam

1) Design variables-The chosen cross section here is hollow circular cross section. In this design problem the variables are inner and outer radius of the cross section r_i, r_o.

2) Objective function-The objective of this work is to minimise the mass f(x) of the cantilever beam.

$$\text{Minimise } f(x) = \rho\, l\, \Pi\, (r_o^2 - r_i^2)$$

Taking the constraints in to account a penalty factor is introduced and the new objective function

$$f(x) = \rho\, l\, \Pi\, (r_o^2 - r_i^2) + \text{Penalty factor}$$

The penalty factor is the value which corresponds to any violation of the given constrain.

3) *Stress constraints*-The maximum allowable stress which is developed in the beam due to the load P .Let σ_b be the bending stress and τ be the shear stress developed with the given load P

Let σ_a & τ_a be the allowable bending and shear stress respectively. Then the stress constraints are $\sigma_b \leq \sigma_a$ & $\tau \leq \tau_a$

$$\sigma_b = \frac{1.27 * \rho * l * r_o}{\left(r_0^{\ 4} - r_i^{\ 4}\right)} \leq \sigma_a \,(165 \text{ MPa})$$

$$\tau = \frac{0.424 * P * (r_o^{\ 2} - r_0 r_i + r_i^{\ 2})}{\left(r_o^{\ 4} - r_i^{\ 4}\right)} \leq \tau a \,(50\text{MPa})$$

4) Dimensional Constraints-The cross sectional area must be non-negative (i.e.) $A \geq 0$.

$r_i < r_o$ and $r_o - r_i \neq 0$. The range of these r_i and r_o should be selected accordingly.
i.e. thickness $t = r_o - r_i$
Therefore, $r_i = r_o - t$
And $t \neq 0$ and $t_{max} < r_{o\,min}$

B. Implementation of GA

Data for the Problem

Allowable bending stress	σ_a=165 MPa
Allowable shear stress	τ_a = 50 MPa
Density	ρ =7850 kg/m^3
Length	l =10 mts
Load	p=14 KN

The upper and lower limits of the design variables are selected as follows considering the dimensional constrains.

$$3.5 \text{ cm} \leq r_o \leq 6 \text{ cm}$$
$$1 \text{ cm} \leq t \leq 5 \text{ cm}$$

C. Selection of GA parameters

The GA parameters Mutation probability and cross over probability values were selected by making various runs with different combination of these parameters as tabled in table-I. The results were plot in Fig-2 and from which it is selected that the CP as 0.9 and the MP as 0.4.

TABLE I- GA PARAMETER

Cantilever	Total 20 digits
Sample Size	40
Selection Operator	Tournament Selection
Cross-over Probability (Pc)	0.7, 0.8, 0.9
Mutation Probability (Pm)	0.001, 0.005, 0.010, 0.020, 0.030, 0.040, 0.050
No. of generations in each run	100
No. of runs	21

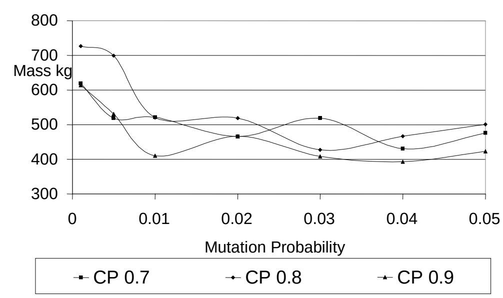
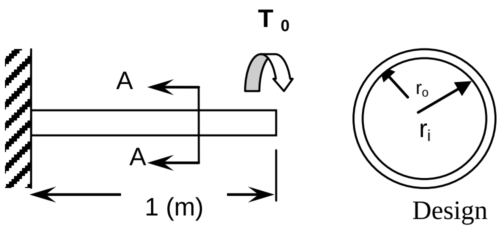


Fig. 2 Minimum Objective Function of Cantilever Beam

D. Problem definition

Case 2: Torsion Rod



Section A-A, Outer radius - r_o, Inner radius -ri
Fig. 3 Torsion Rod

Design the hollow torsion rod as shown in Fig 3 to satisfy the following requirements [10].

1) Design variables- The chosen cross section here is hollow circular cross section. In this design problem the variables are inner and outer diameter of the cross section di, d_0.

2) Objective function-The objective of this work is to minimise the mass f(x) of the cantilever beam.

Minimise $f(x) = (\Pi /4)\rho l(d_0^2 - d_i^2)$

Taking the constraints in to account a penalty factor is introduced and the new objective function

$f(x) = (\Pi /4)\rho l(d_0^2 - d_i^2)$+Penolty factor

The penalty factor is the value which corresponds to any violation of the given constrain.

3) Stress constraints-Calculated shear stress shall not exceed the allowable shear stress under normal operating torque T_o (N m).

The calculated angle of twist θ_c shall not be exceeding the allowable twist θ_a (radians).
The member shall not buckle under a short duration of T_{max} (N m)

$$\theta_c \approx \frac{l}{GJ} \mathrm{T}_0 \le \theta_a$$

$$\tau \approx \frac{C}{J} \mathrm{T}_0 \mathrm{P}_a \le \tau_a$$

$$\mathrm{T}_{cr} \approx \frac{\Pi d_0 \quad E}{12\sqrt{2}\left(1-v^2\right)^{.75}} \left(1 - \frac{d_i}{d_0}\right)^{2.5} \mathrm{N}m \ge \mathrm{T}_{Max}$$

4) Dimensional Constraint-

x_1 = Outside diameter d_0
x_2 = Inside diameter / Outside diameter

$$0.02m \le d_o \le 0.5m$$

$$0.6 \le {d_i}/{d_o} \le 0.999$$

'E. Implementation of GA

Data for the Problem

Five different materials were tried at three different lengths as 0.5m, 0.75m and 1m.The rod requirements, materials and properties are given in the following tables II and IV.

TABLE II- ROD REQUIREMENTS

Torsion rod	Length l (m)	Normal torque T_0 (kN m)	Max Tmax (kN m)	Allowable twist θa (degrees)
1	0.50	10	20	2
2	0.75	15	25	2
3	1.00	20	30	2

F. Selection of GA parameters

The GA parameters were selected as in the case 1 and the MP as 0.05 and CP as 0.8 from Fig-4.The parameters are tabled in table -III.

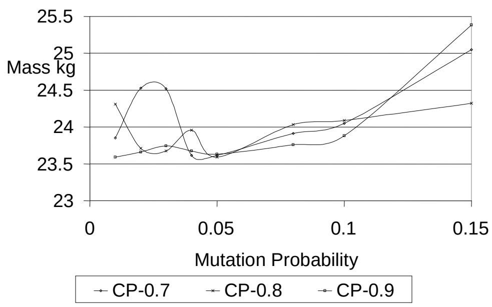


Fig (4) Minimum Objective function of Torsion Rod

TABLE III- GA PARAMETER

Cantilever	Total 20 digits
Sample Size	40
Selection Operator	Tournament Selection
Cross-over Probability (CP)	0.7, 0.8, 0.9
Mutation Probability (MP)	0.001, 0.02, 0.03, 0.04, 0.5, 0.8, 0.1, 0.15.
No. of generations in each run	100

IV. RESULTS AND DISCUSSION

Case 1:

Fig 5 shows the solution history of the GA run. The minimum objective function is achieved during the generation number 46 as 392.788 kg.

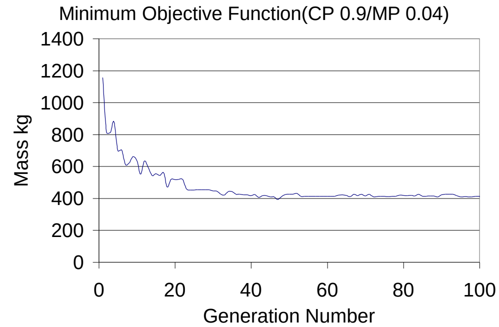


Fig (5) Solution history of GA Run

TABLE IV- MATERIALS AND PROPERTIES

Material	Density Kg/m3	Allowable shear stress τ_a (M Pa)	Elastic Modulus E (G pa)	Shear modulus G(G pa)	Poissions ratio ν
1. 4140 alloy steel	7850	275	210	80	0.30
2. Al alloy 24 ST 4	2750	165	75	28	0.32
3. Mg alloy A261	1800	90	45	16	0.35
4. Beryllium	1850	110	300	147	0.02
5.Titanium	4500	165	110	42	0.30

Case 2:

The optimum mass value for the torsion rod with different materials and with different lengths were found through GA and listed in table V.

The figures from 6 to 20 shows the solution history of those GA runs and found the curves get converged.

TABLE V- OPTIMUM MASS VALUE OF TORSION ROD

Material	Length m	Min Mass kg
4140 alloy steel	0.50	23.717
	0.75	50.272
	1.00	78.939
Aluminium Alloy 24 ST 4	0.50	10.572
	0.75	21.336
	1.00	35.922
Mg Alloy A261	0.50	6.207
	0.75	12.409
	1.00	20.670
Beryllium	0.50	8.577
	0.75	15.618
	1.00	25.876
Titanium	0.50	14.144
	0.75	28.673
	1.00	52.949

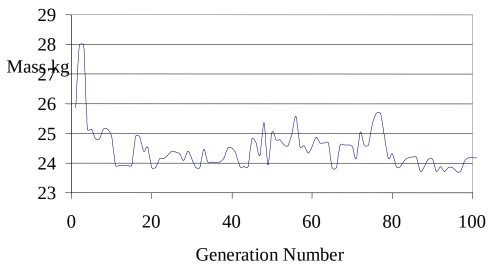


Fig (6) Minimum Objective Function 4140 Alloy Steel/ L 0.5m

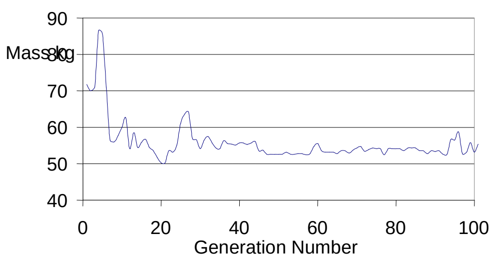


Fig (7) Minimum Objective Function4140 Alloy Steel/ L 0.75 m

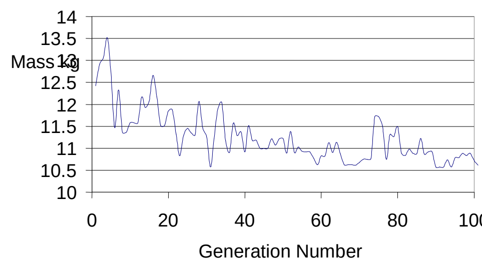


Fig (9) Minimum Objective Function Aluminium Alloy 24 ST4/L 0.5 m

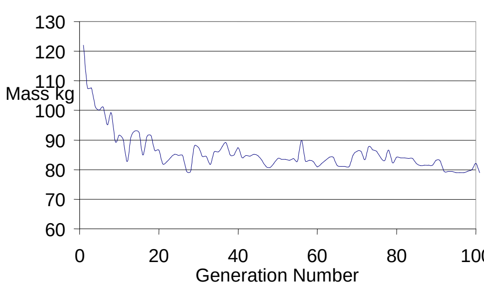


Fig (8) Minimum Objective Function 4140 Alloy Steel/ L 1 m

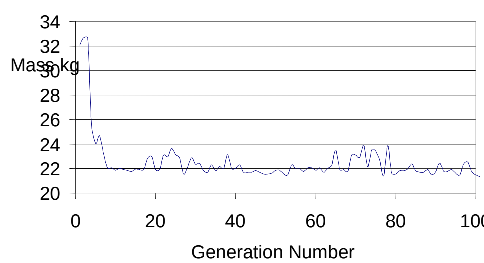


Fig (10) Minimum Objective Function Aluminium Alloy 24 ST4/L 0.75m

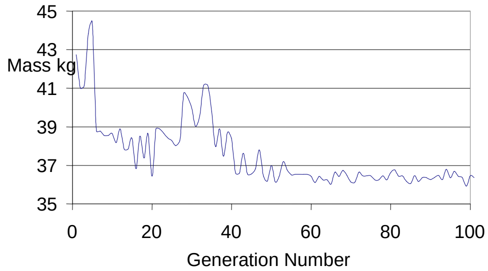


Fig (11) Minimum Objective Function Aluminium Alloy 24 ST4/L 1 m

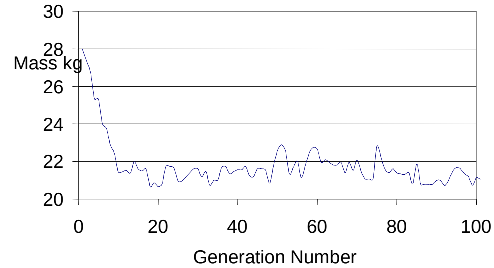


Fig (14) Minimum Objective Function Magnesium Alloy A261 /L 1 m

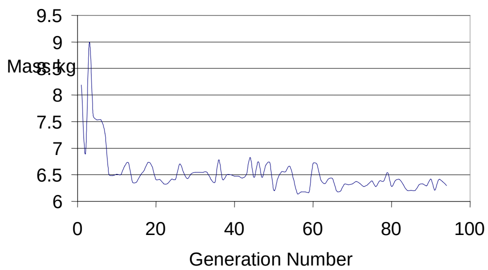


Fig (12) Minimum Objective Function Magnesium Alloy A261 /L 0.5 m

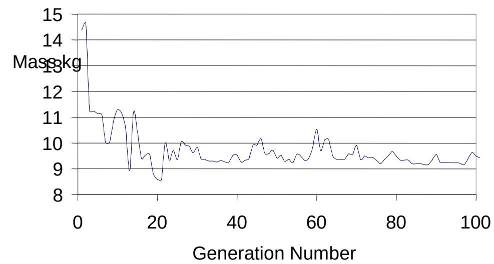


Fig (15) Minimum Objective Function Beryllium/L 0.5 m

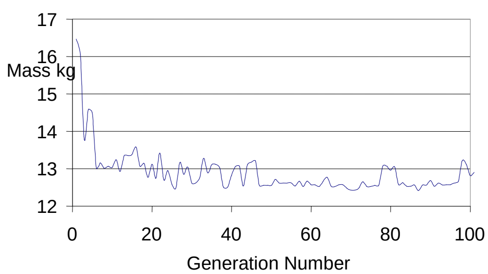


Fig (13) Minimum Objective Function Magnesium Alloy A261 / L 0.75 m

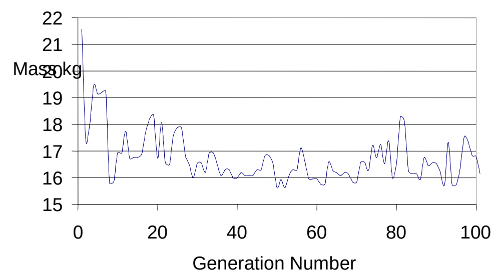


Fig (16) Minimum Objective Function Beryllium/L 0.75 m

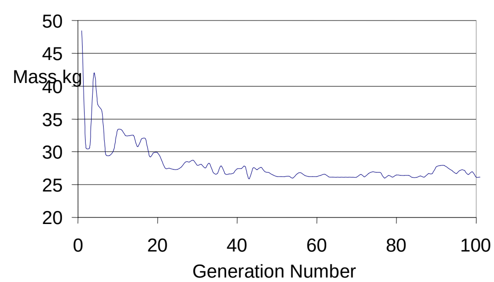


Fig (17) Minimum Objective Function Beryllium/L 1 m

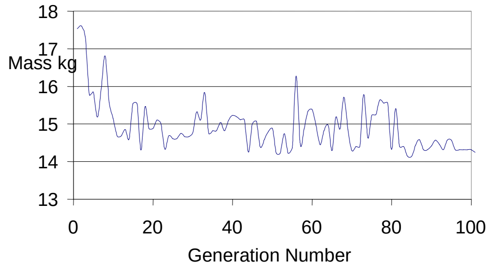


Fig (18) Minimum Objective Function Titanium/L 0.5 m

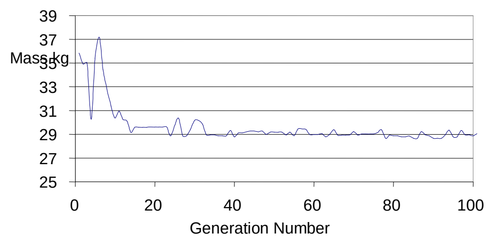


Fig (19) Minimum Objective Function Titanium/ L 0.75 m

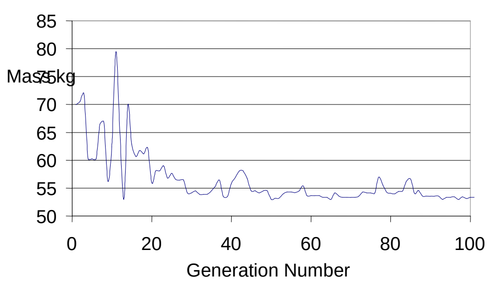


Fig (20) Minimum Objective Function Titanium/L 1 m

V. CONCLUSION

A standardised computer program using C language was developed that can be easily manipulated to synthesis the optimal design for the components like cantilever beam and torsion rod. The proposed procedure using GA has the capability to achieve minimum mass for the given component. Considerable reduction in computational effort is achieved by this proposed procedure.

Different traditional techniques have been reported earlier to optimise the above kind of components. In this work, it is demonstrated that a GA-based procedure can be applied to a variety of design optimisation problems.

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APPENDIX

NOMENCLEATURE: *Symbols*

M	= Mass of the rod (kg)
r_i	= Inside diameter of the beam
r_o	= Outside diameter of the beam
d_0	= Outside diameter of the rod (m)
d_i	= Inside diameter of the rod (m)
P	= Mass density of material (kg/m^3)
L	= Length of the rod (m)
T_0	= Normal operating torque (N m)
C	= Distance from rod axis to extreme fibre (m)
J	= Polar moment of inertia (m^4)
θ	= Angle of twist (radians)
G	= Shear modulus (Pa)
T_{cr}	= Critical buckling torque (N m)
E	= Modulus of elasticity (pa)
ν	= Poisson's ratio
T_{Max}	= Maximum Torque (N m)

σ_b = Bending stress
σ_a = Allowable bending stress
τ = Shear stress
τ_a = Allowable shear stress