ISSN 2278 - 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

Dynamic Modeling, Simulation and Control of MIMO Systems

M.Bharathi, C.Selvakumar

HOD, Department of Electronics And Instrumentation, Bharath University Chennai – 600073, India

Prof & Head, St.Joseph's College Of Engineering, Chennai-119 India

ABSTRACT: Designing control systems for complete plants is the ultimate goal of a control designer. The problem is quite large and complex. It involves a large number of theoretical and practical considerations such as quality of controlled response; stability; the safety of the operating plant; the reliability of the control system; the range of control and ease of startup, shutdown, or changeover; the ease of operation; and the cost of the control system. The difficulties are aggravated by the fact that most of the industrial and chemical processes are largely nonlinear, imprecisely known, multivariable systems with many interactions. The measurements and manipulations are limited to a relatively small number of variables, while the control objectives may not be clearly stated or even known at the beginning of the control system design. Thus, the presence of process input-output time delay of different magnitude in multi-input-multi-output systems have drawn attention to research as the processes are difficult to control. Increase in complexity and interactions between inputs and outputs yield degraded process behavior.

KEYWORDS: multivariable systems, interaction, control system design, nonlinear

I.OBJECTIVE

In recent years all the methodologies adapted to solve for the parameters of individual controllers in which the loop interactions are taken into account have not guaranteed a solution. In addition, the extension for higher dimensional systems seems difficult because of the complicated and non-linear computation. It has been found that the independent design of decentralized controllers based on model based method is simple and effective only for low dimensional processes. For high dimensional processes this design has to be more conservative due to the inevitable modeling errors encountered in formulation.

To overcome all these drawbacks and to include interactions in the control design, a novel method based on the equivalent transfer function method (ETF) is proposed. By considering four combination modes of gain and phase changes for a particular loop when all other loops are closed, this equivalent transfer function can effectively approximate the dynamic interactions among loops. Consequently, the design of decentralized controller for MIMO processes can be converted to the design of single loop controllers. The method is simple, straightforward, easy to understand and implement. Several multivariable industrial processes with different interaction characteristics are employed to demonstrate the effectiveness and simplicity of the design method compared to the existing methods.

CONTROLLER DESIGN METHODOLOGIES:

There are three major controller designs that are available. They are mainly

a) Centralized controller
b) Decentralized controller and
c) Decoupler

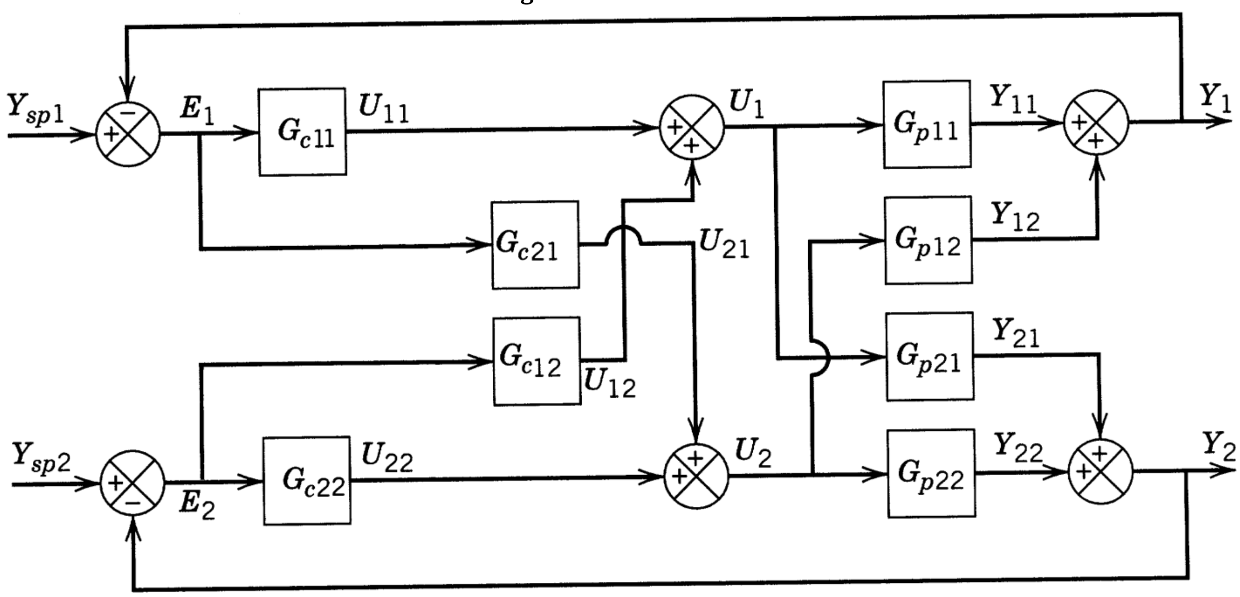
Of all the three configurations discussed above, the centralized controller is not used very widely because of the complexity and time constraints in computation. In addition to it the design is less transparent and can be damaging the entire plant during failures, thus not being highly reliable. The figure below shows the block diagram of a decentralized controller and with its representation.

ISSN 2278 - 8875

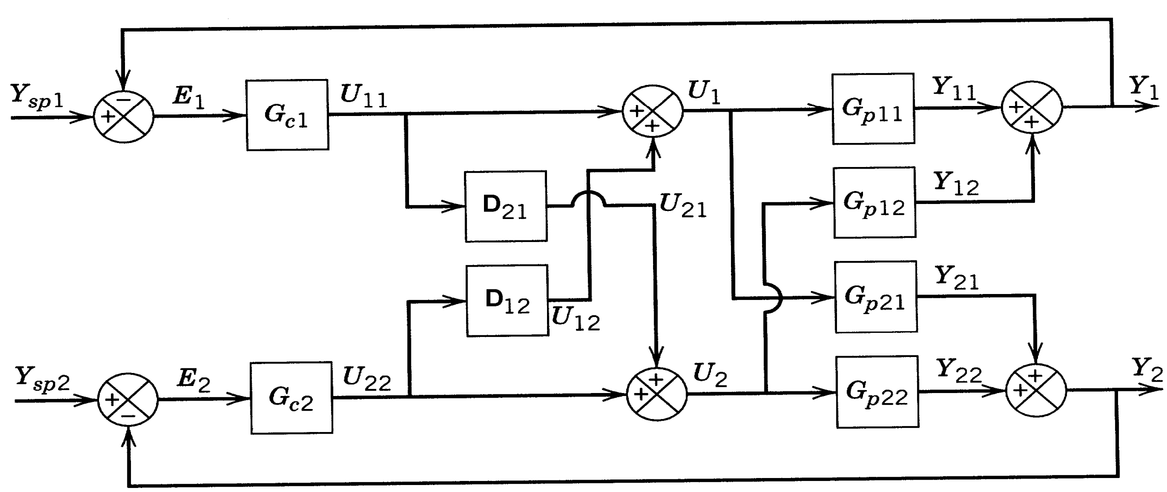
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

$$\underline{\underline{G_c}}(s) = \begin{bmatrix} G_{C11} & G_{C12} & \cdots & G_{C1N} \\ G_{C21} & G_{C22} & \cdots & G_{C2N} \\ \cdots & \cdots & \cdots & \cdots \\ G_{CN1} & G_{CN2} & \cdots & G_{CNN} \end{bmatrix}$$

Figure 1: Centralized Control



The decoupler though profitable and realistic is also very complex and degrades the load rejection. It has to be applied carefully and is often recommended only for the servo operations. The Figure 2 shows the block diagram for the decoupler.

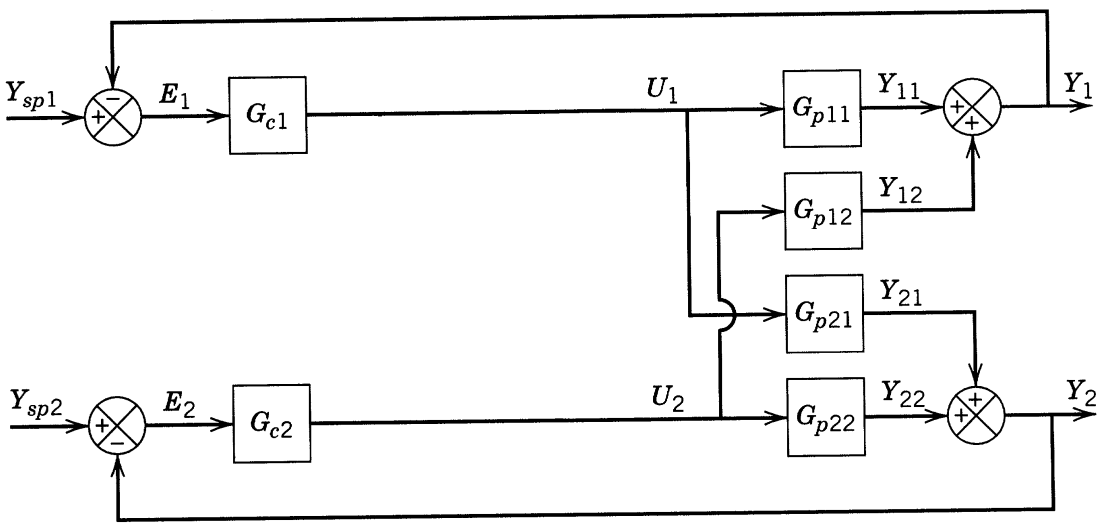


ISSN 2278 - 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

The Decentralized controllers are widely used because of their simplicity in hardware, design and tuning simplicity, flexibility in operation and maintenance.
The block diagram of the decentralized controller is shown in figure 3. The decentralized controller is represented as

$$\begin{bmatrix} G_{C1} & 0 & \cdots & 0 \\ 0 & G_{C2} & 0 & \cdots \end{bmatrix}$$



All the three controller configurations are being shown for a 2x2 system with interactions. The decentralized controllers consist of multi loop SISO controllers with one control variable paired with one manipulated variable. The major idea in this design approach is that the SISO controllers should be tuned simultaneously with the interactions in the process taken into account.

II. PROCEDURE FOR DECENTRALISED CONTROLLER DESIGN

Most industrial control systems use the multi loop SISO diagonal control structure.
It is the most simple and understandable structure. Operators and plant engineers can use it and modify it when necessary. It does not require an expert in applied mathematics to design and maintain it. In addition, the performance of these diagonal controller structures is usually quite adequate for process control applications. In fact, there has been little quantitative unbiased data showing that the performances of the more sophisticated controller structures are really any better! The slight improvement is seldom worth the price of the additional complexity and engineering cost of implementation and maintenance.
A number of critical questions must be answered in developing a control system for a plant. What should be controlled? What should be manipulated? How should the controlled and manipulated variables be paired in a multivariable plant? How do we tune the controllers? The procedure discussed in this chapter provides a practical approach to answering these questions. It was developed to provide a workable, stable, simple SISO system with only a modest amount of engineering effort. The resulting diagonal controller can then serve as a realistic benchmark, against which the more complex multivariable controller structures can be compared. The limitations of the procedure should be pointed out. It does not apply to open loop-unstable systems. It also does not work well when the time constants of the transfer functions are quite different, i.e., some parts much faster than others.
The fast and slow sections should be designed separately in such a case. The procedure has been tested primarily on realistic distillation column models.

This choice was deliberate because most industrial processes have similar gain, dead time, and lag transfer functions. Undoubtedly, some pathological transfer functions can be found that the procedure cannot handle. But we are interested in a practical engineering tool, not elegant, rigorous, all-inclusive mathematical theorems.

The steps in the procedure are summarized below. Each step is discussed in more detail in later sections of this chapter.
1. *Select controlled variables.* Use primarily engineering judgment based on process understanding.
  1. 2. *Select manipulated variables.* Find the set of manipulated variables that gives the largest minimum singular value of the steady-state gain matrix.
3. *Eliminate unworkable variable pairings.* The pairing can be done with RGA ERGA or using NI indices.
4. *Find the best pairing from the remaining sets.*
a. Tune all combinations using a efficient tuning methodology.
b. Select the pairing that gives the lowest-magnitude closed loop regulator transfer function.

III.EFFECTIVE TRANSFER FUNCTION

COMPUTATION OF ETF

Consider an open loop stable multivariable system within inputs and n outputs as shown in Fig. 1, where r_i, i =1, 2, . . . ,n, are the reference inputs; u_i, i = 1,2,. . . ,n, are the manipulated variables; y_i, i = 1,2,. . . ,n, are the system. Outputs, G(s) and $G_c(s)$ are process transfer function matrix
And decentralized controller matrix with compatible dimensions, expressed by

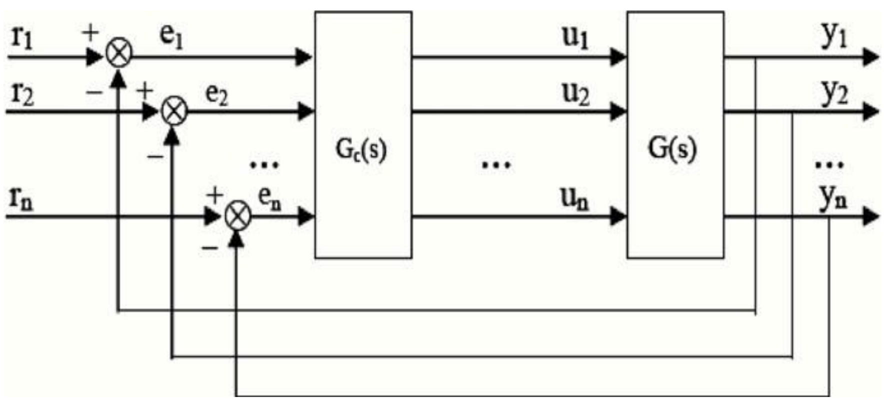


Figure 4: Closed-loop multivariable control system.

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & \dots & \dots & g_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & g_{n2}(s) & \dots & g_{nn}(s) \end{bmatrix}$$

And

$$G_c(s) = \begin{bmatrix} g_{c1}(s) & 0 & \dots & 0 \\ 0 & g_{c2}(s) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & g_{cn}(s) \end{bmatrix}$$

respectively.

Let $g_{ij}(j\omega) = k_{ij} g_{ij}^0(j\omega),$

Where k_{ij} and $g_{ij}^{0}(j\omega)$,are steady state gain and normalized transfer function of $g_{ij}(j\omega)$,i.e., $g_{ij}^{0}(0) = 1$, respectively. The interaction among individual loop is described by ERGA, the main result of ERGA is summarized as follows.
Define e_{ij} of a particular transfer function as

$$e_{ij} = k_{ij} \int_{0}^{\omega_{c,ij}} \left| g_{ij}^{0}(j\omega) \right| \, d\omega,$$

where $\omega_{c,ij}$for i,j = 1,2,. . . ,n are the critical frequency of the transfer function $g_{ij}(j\omega)$ and $|\blacksquare|$is the absolute value of $\blacksquare$. In order to calculate e_{ij}, the critical frequency can be defined in two ways:

$\omega_{c,ij} = \omega_{B,ij}$, where $\omega_{B,ij}$for i,j = 1,2,. . . ,n is the bandwidth of the transfer function $g_{ij}^{0}(j\omega)$ and determined by the frequency where the magnitude plot of frequency response reduced to 0.707 time, i.e.,
$\left|g_{ij}(j\omega_{B,ij})\right| = 0.707\left|g_{ij}(0)\right|$.

$\omega_{c,ij} = \omega_{u,ij}$, where $\omega_{u,ij}$for i,j = 1,2,. . . ,n is the ultimate of the transfer function $g_{ij}^{0}(j\omega)$ and determined by the frequency where the phase plot of frequency response across -π, i.e.,
$arg\left[g_{ij}(j\omega_{u,ij})\right] = -\pi$.

For transfer function matrices with some elements without phase crossover frequencies, such as first order or second order without time delay, it is necessary to use corresponding bandwidths as critical frequencies to calculate e_{ij} .However, it is worth to point out that the phase crossover frequency information, i.e., ultimate frequency $(\omega_{u,ij})$ is recommended if applicable for calculation of e_{ij}, since it is closely linked to system dynamic performance and control system design. Without loss of generality, we will use $\omega_{u,ij}$ as the bases for the following development.

For the frequency response of $g_{ij}(j\omega)$ as shown in Fig. 5, $e_{ij}$is the area covered by $g_{ij}(j\omega)$ up to $\omega_{u,ij}$. Since$\left|g_{ij}^{0}(j\omega)\right|$ represents the magnitude of the transfer function at various frequencies, $e_{ij}$is considered to be the energy transmission ratio from the manipulated variable $u_j$to the controlled variable y_i.
Express the energy transmission ratio array as

$$E = \begin{bmatrix} e_{11} & e_{12} & \dots & e_{1n} \\ e_{21} & \dots & \dots & e_{2n} \\ \dots & \dots & \dots & \dots \\ e_{n1} & e_{n2} & \dots & e_{nn} \end{bmatrix}$$

To simplify the calculations, we approximate the integration of e_{ij} by a rectangle area, i.e.,

$$e_{ij} \approx k_{ij}\omega_{u,ij} \quad \text{i,j} = 1,2,\dots\dots,n.$$

Then, the effective energy transmission ratio array is given as:

$$E = G(0) \otimes \Omega,$$

Where the operator $\otimes$ is the Hadamard product, and

$$G(0) = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & \dots & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{bmatrix}$$

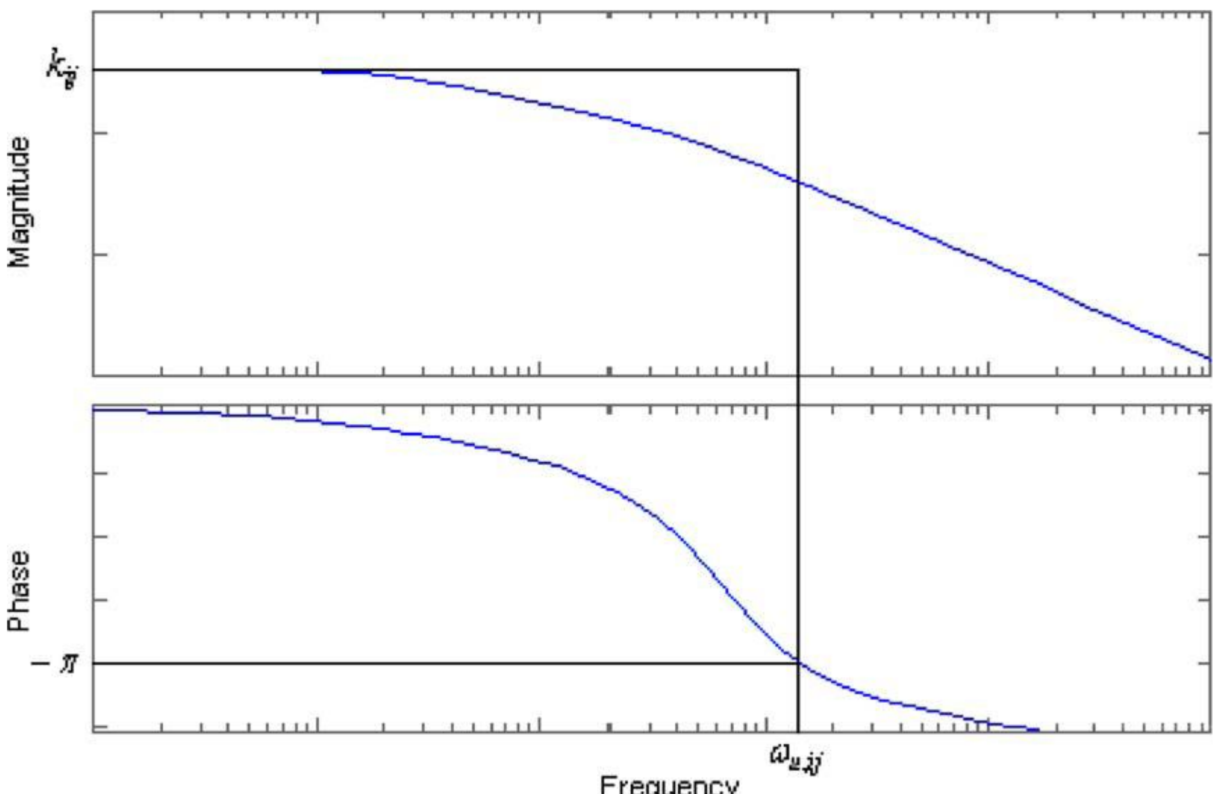
And

ISSN 2278 - 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

$$\Omega = \begin{bmatrix} \omega_{u,11} & \omega_{u,12} & \cdots & \omega_{u,1n} \\ \omega_{u,21} & \cdots & \cdots & \omega_{u,2n} \\ \cdots & \cdots & \cdots & \cdots \\ \omega_{u,n1} & \omega_{u,n2} & \cdots & \omega_{u,nn} \end{bmatrix}$$

Are the steady state gain and the critical frequency array, respectively. Since e_{ij} is an indication of energy transmission ratio for loop $y_i - u_{j,}$ the bigger the e_{ij} value is, the more dominant of the loop will be.



Similar to the definition of relative gain , the effective relative gain,ϕ_{ij} between output variable y_i and input variable $u_{j,}$is define as the ratio of two effective energy transmission ratio:

$$\phi_{ij} = \frac{e_{ij}}{\widehat{e_{ij}}}$$

where $\widehat{e_{ij}}$ is the effective energy transmission ratio between output variable y_i and input variable $u_{j,}$ when all other loops are closed. When the effective relative gains are calculated for all the input/output combinations of a multivariable process, it results in an array, ERGA, which can be
Calculated by

$$\phi = E \otimes E^{-T} = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \cdots & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}$$

The introduction of energy transmission ratio is to mathematically represent the effectiveness of a control loop which is affected by two key factors, i.e., the steady state gain of the transfer function reflecting the effect of the manipulated variable $u_{j,}$ to the controlled variable y_i and the response speed reflecting the sensitivity of the controlled variable y_i to the manipulated variable $u_{j,}$ and, consequently, the ability to reject the interactions from other loops. Since ERGA is a relative measure, using the multiplication of the two parameters to approximate the energy transmission ratio in ϕ_{ij} can simplify the calculation while captures the key elements in a multivariable control system.

The introduction of energy transmission ratio is to mathematically represent the effectiveness of a control loop which is affected by two key factors, i.e., the steady state gain of the transfer function reflecting the effect of the manipulated variable $u_{j,}$ to the controlled variable y_i and the response speed reflecting the sensitivity of the controlled variable y_i to the manipulated variable $u_{j,}$ and, consequently, the ability to reject the interactions from other loops. Since ERGA is a relative measure, using the multiplication of the two parameters to approximate the energy transmission ratio in ϕ_{ij} can simplify the calculation while captures the key elements in a multivariable control system.
The ERGA is used to determine the best variable paring. In the following sections, we will employ this interaction measure to develop effective transfer functions (ETFs) under decentralized control structure

IV.EFFECTIVE TRANSFER FUNCTION

Suppose that the best loop configuration has been determined and the best pair is diagonally placed in the transfer function matrix as shown in Fig. 7. Similar to the open loop gain, we let the effective energy transmission ratio, $\hat{e}_{ij}$ when all other loops are closed be $\hat{e}_{ij} = \hat{g}_{ij}(0)\hat{\omega}_{u,ij}$ i,j= 1; 2; . . . ; n, where $\hat{g}_{ij}(0)$ and $\hat{\omega}_{u,ij}$ are the steady state gain and ultimate frequency between output variable y_i
and input variable $u_{j,}$ when all other loops are closed, respectively. Then, from Eq.(1)

$$\hat{g}_{ij}(0)\hat{\omega}_{u,ij} = \frac{g_{ij}(0)\omega_{u,ij,}}{\phi_{ij}} \qquad (2)$$

By the definition of RGA, we have

$$\hat{g}_{ij}(0) = \frac{g_{ij}(0)}{\lambda_{ij}}, \qquad (3)$$

Substitute Eq.(3) into (2) and rearrange to result

$$\frac{\phi_{ij}}{\lambda_{ij}} = \frac{\omega_{u,ij,}}{\hat{\omega}_{u,ij}} \equiv \gamma_{ij} \qquad (4)$$

where γ_{ij} represents the critical frequency change of loop $i - j$ when other loops are closed, defined as relative critical frequency. When the relative frequencies are calculated for all the input/output combinations of a multivariable process, it results in an array, i.e., relative frequency array (RFA).
Since control loop transfer functions when other loops closed will have similar frequency properties with when other loops open if it is well paired , we can let the ETFs have the same structures as the corresponding open loop transfer functions but with different parameters.

$$\hat{g}_{ij}(s) = \hat{g}_{ij}(0)g_{ii}^{r}(s)e^{-\hat{d}_{ii}s} \qquad (5)$$

Where $g_{ii}^{r}(s)$ is defined by,

$$g_{ii}^{r}(s) = g_{ii}^{0}(s)\, e^{-\hat{d}_{ii}s}$$

And $\hat{d}_{ii}$ is the time delay of the ETF.

As the change in ultimate frequency of a control loop is generally affected by changes in both time constant and time delay when other loops are closed, and they are exchangeable by linear approximation, it is reasonable to change only time delay to reflect the phase changes.

In Eq. (5), $g_{ii}^{0}(0)$ can be determined by using Eq. (3), while by the definition of the ultimate frequency,

$$\widehat{-\mathrm{d}}_{ii}\hat{\omega}_{u,ii} + \angle g_{ii}^{r}(j\hat{\omega}_{u,ii}) = -d_{ii,}\omega_{u,ii,} + \angle g_{ii}^{r}(\omega_{u,ii,}) = -\pi.$$

$d_{ii,}$ can be easily determined by

$$d_{ii,} = \frac{\pi + \angle g_{ii}^{r}(j\hat{\omega}_{u,ii})}{\hat{\omega}_{u,ji}} = \gamma_{ij} \times \frac{\pi + \angle g_{ii}^{r}(j\hat{\omega}_{u,ii})}{\omega_{u,ij,}} \qquad (6)$$

ISSN 2278 - 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

Notice that $g_{ii}^{r}(s)$ is usually low order transfer functions, their contribution to the phase change at low frequency range are small and can be equivalently represented by the additional time delay term.
In many decentralized control system designs, such as gain and phase margin method, an individual loop is tuned around the critical frequency region of each control loop. Accurate estimation of overall variation is required around the critical frequency, not who contribute to the change.

By letting $g_{ii}^{r}(j\widehat{\omega}_{u,ii}) \approx g_{ii}^{r}(j\widehat{\omega}_{u,ii})$, we can make further simplification to Eq. (6) as

$$\widehat{-d}_{ii}\widehat{\omega}_{u,ii} \approx -d_{ii,}\omega_{u,ii,}$$

which results by considering Eq. (4)

$$\hat{d}_{ii} \approx \frac{\omega_{u,ij,}}{\widehat{\omega}_{u,ij}} d_{ii,} = \gamma_{ii} d_{ii,} \qquad (7)$$

This is the practical formula which will be used to derive the ETFs. Even though Eq. (7) is less accurate than Eq. (6), several simulation results have showed that the control system performances are comparable by the two approximations, but Eq. (7) is much more straightforward ,easier explainable and understandable than Eq. (6). Since it is necessary that the controlled system possesses integrity property; that is, the overall control system remained to be stable regardless put in and/or taken out of other control loops, $\widehat{g}_{ii}(0)$ and $\hat{d}_{ii}$ in ETF must take different values for different combination of $\lambda_{ii}$and γ_{ii}. For the four different combinations of λ_{ij} andγ_{ii}, $\widehat{g}_{ii}(s)$ may take different modes shown in Figs. 6-9, and are discussed below:

Case 1: $\lambda_{ii} \leq 1, \gamma_{ii} \leq 1$
In this case, $\left(\frac{1}{\lambda_{ii}} - 1\right) \geq 0$ and$(\gamma_{ii} - 1) \leq 0$. According to Eqs. (3) and (7), we have $\widehat{g}_{ii}(0) \geq g_{ii}(0)$ and $\hat{d}_{ii} \leq d_{ii,}$.

- $\widehat{g}_{ii}(0) \geq g_{ii}(0)$, this means that the magnitude of the frequency response when the other loops closed is not less than that of when the other loops open. Since the retaliatory effect from the other loops magnifies the main effect of $u_{j,}$on y_i, we need to reduce the controller gain to assure system stability. In this case, the gain is by Eq. (3) $\widehat{g}_{ii}(0) = \frac{g_{ii}(0)}{\lambda_{ii}}$

- $\hat{d}_{ii} \leq d_{ii,}$.this means that the time delay when the other loops closed is not bigger than that of when other loops open. The reduced time delay will increase the phase margin. However, by considering the control system integrity, the time delay needs to be kept as before, i.e., $\hat{d}_{ii} = d_{ii,}$

ISSN 2278 - 8875
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

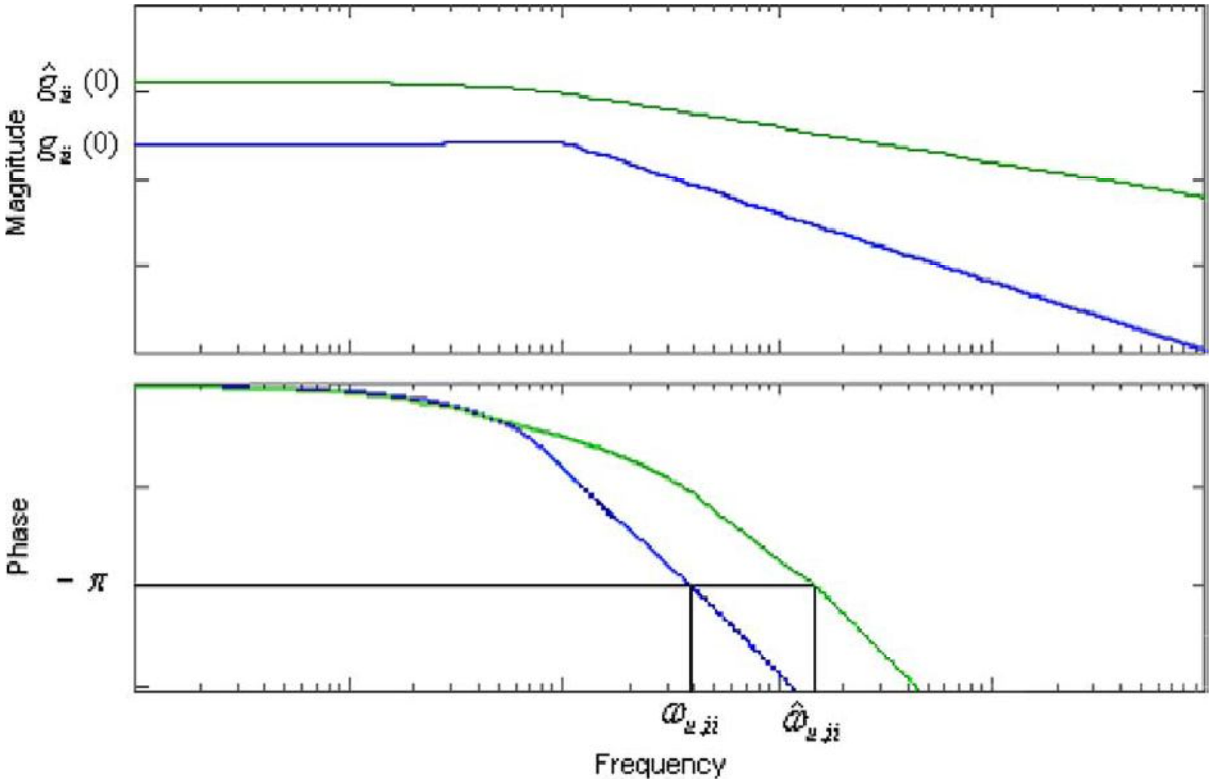


Figure 6: Interaction mode with case 1

Case 2: $\lambda_{ii} \leq 1, \gamma_{ii} > 1$

In this case $\left(\frac{1}{\lambda_{ii}} - 1\right) \geq 0$ and$(\square_{\square\square} - 1) > 0$. According to Eqs. (3) and (7), we have $\widehat{\square}_{\square\square}(0) \geq \square_{\square\square}(0)$ and $\widehat{\square}_{\square\square} > \square_{\square\square}$,.

- $\widehat{\square}_{\square\square}(0) \geq \square_{\square\square}(0)$, same as in Case 1,

$$\widehat{\square}_{\square\square}(0) = \frac{\square_{\square\square}(0)}{\square_{\square\square}}$$

- $\widehat{\square}_{\square\square} > \square_{\square\square}$,., this means that the time delay when the other loops closed is bigger than that of when the other loops open. The enlarged time delay will reduce the phase margin. In this case, the time delay is determined by Eq. (7)

$$\widehat{\square}_{\square\square} = \square_{\square\square}\square_{\square\square}$$

ISSN 2278 - 8875
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

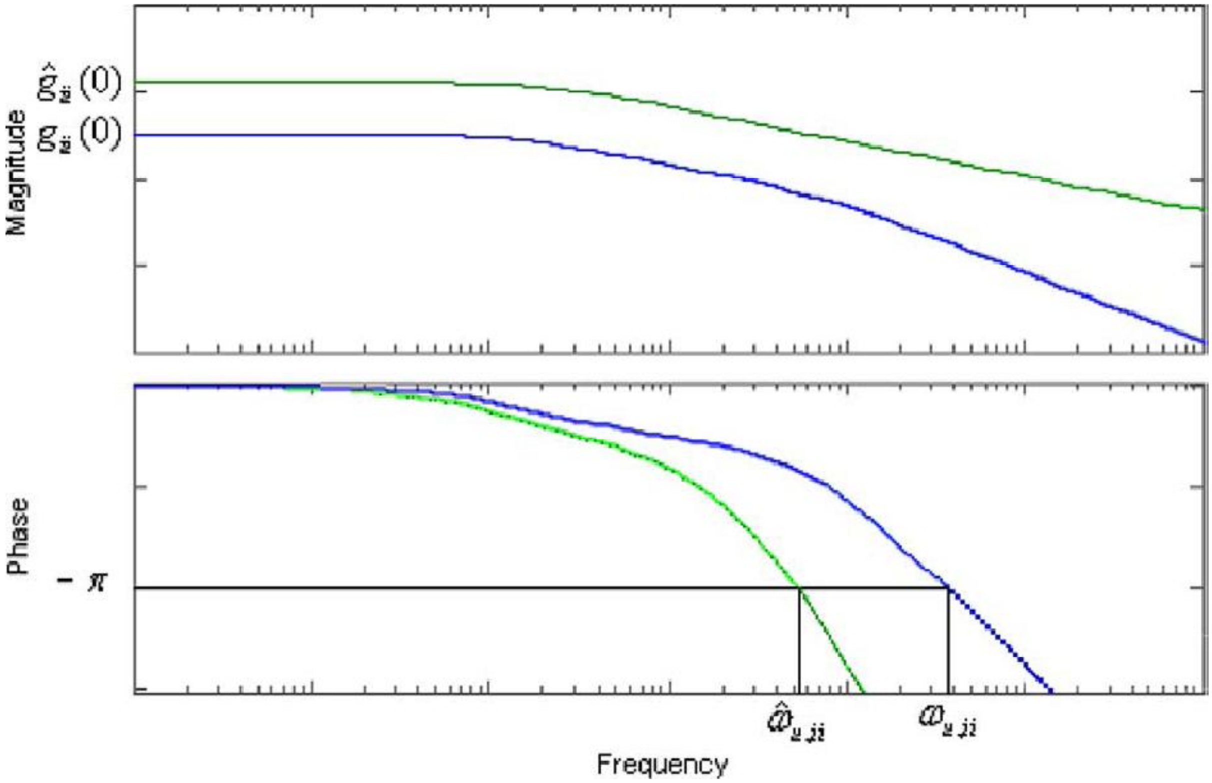


Figure 7: Interaction mode with case 2

Case 3: □□□ > 1, □□□ ≤ 1

In this case, $\left(\frac{1}{□□□} - 1\right) < 0$ and $(□□□ - 1) \leq 0$. According to Eqs. (3) and (7), we have $\widehat{□}_{□□}(0) < □_{□□}(0)$ and $\hat{□}_{□□} \leq □_{□□,}$.

- $\widehat{□}_{□□}(0) \geq □_{□□}(0)$ this means that the magnitude of the frequency response when the other loops closed is smaller than that of when the other loops open. Even if the retaliatory effect from other loops acts in opposition to the main effect of □□, on □□, we cannot enlarge the controller gain for better performance due to the system integrity consideration. Hence, the gain should be unchanged, i.e.,

$$\widehat{□}_{□□}(0) = □_{□□}(0).$$

- $\hat{□}_{□□} > □_{□□,}$., same as in case 1,
  $\hat{□}_{□□} = □_{□□,}$.,

ISSN 2278 - 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

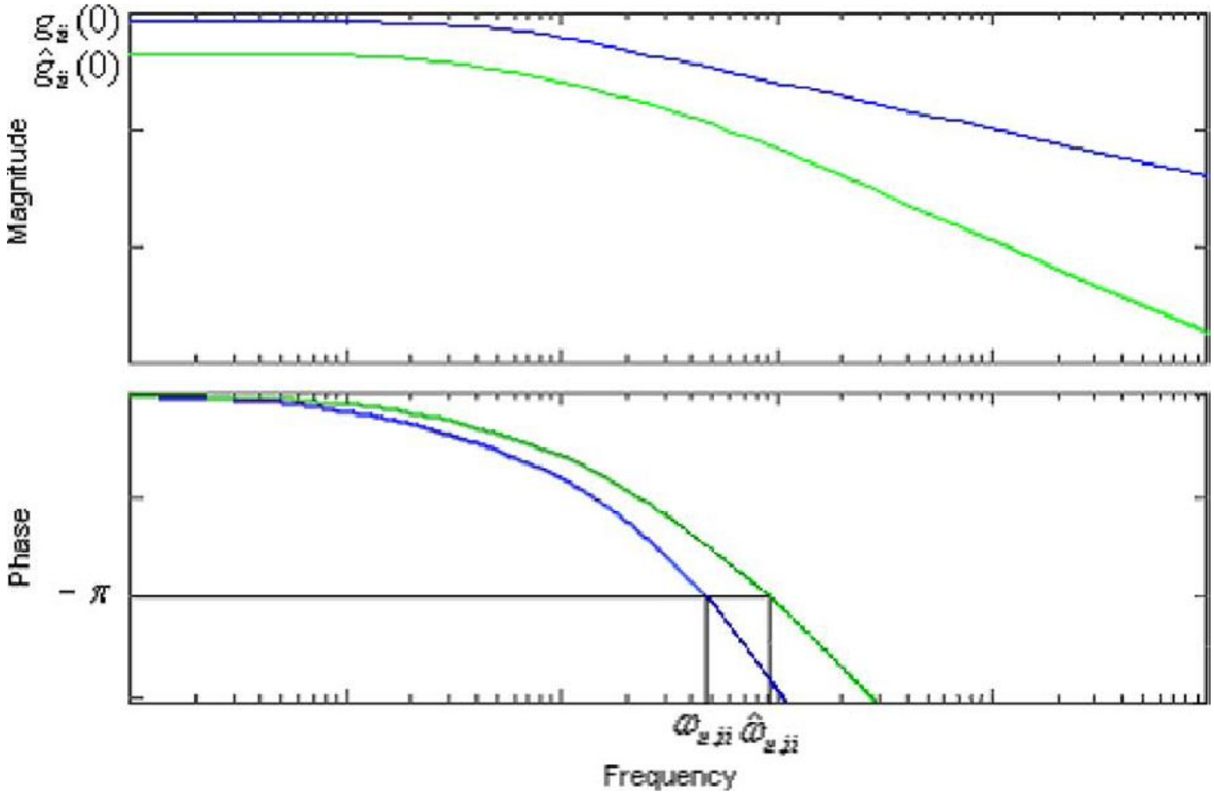


Figure 8: Interaction mode with case 3

Case 4: $\square_{\square\square} > 1, \square_{\square\square} > 1$

In this case, $\left(\frac{1}{\square_{\square\square}} - 1\right) < 0$ and $(\square_{\square\square} - 1) > 0$. According toEqs. (3) and (7), we have $\widehat{\square}_{\square\square}(0) < \square_{\square\square}(0)$ and $\widehat{\square}_{\square\square} > \square_{\square\square}$.

- $\widehat{\square}_{\square\square}(0) < \square_{\square\square}(0)$ same as in Case 3,
  $\widehat{\square}_{\square\square}(0) = \square_{\square\square}(0)$, same as in Case 2,

- $\widehat{\square}_{\square\square} > \square_{\square\square}$ same as in Case 2,

$$\widehat{\square}_{\square\square} = \square_{\square\square}\square_{\square\square}$$

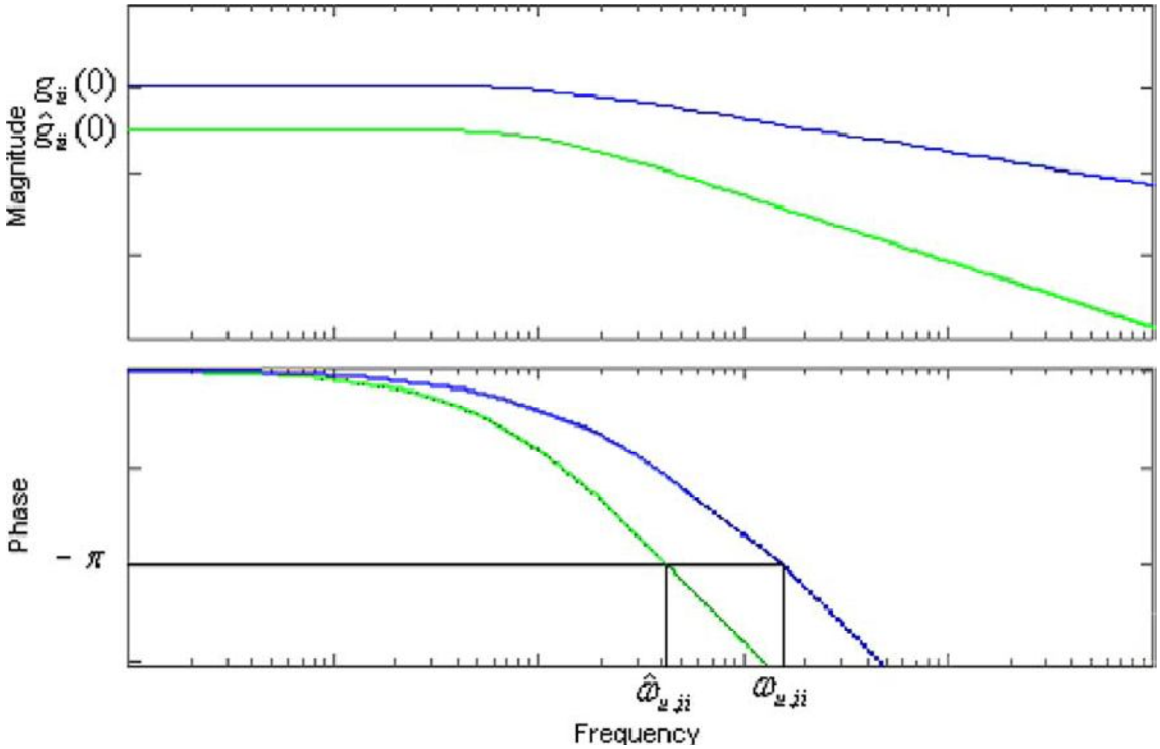


Figure 9: Interaction mode with case 4

A unique problem for decentralized control of MIMO processes is the zero crossing: stable or unstable zeros might be introduced into a particular control loop when other loops are closed. If an unstable zero is introduced, it will result phase shift to the left in the frequency domain. In order to guarantee the entire system stability, the controllers are normally conservatively designed by conventional detuning approaches. By introducing the relative critical frequency, $\square_{\square\square}$, to indicate phase changes after the other loops closed, the effects of unstable zeros can be accurately estimated in each control loop. Consequently, the resultant control systems will be much less conservative.

Mathematically, the equivalent transfer function should incorporate the controllers of all other loops.
To solve such a complex problem, recursive solution is required by first assigning initial controllers, then finding the equivalent loop transfer functions and designing controllers again. This process is continuous until a stable solution is obtained. To simplify the problem, both detuning and independent methods proposed so far assume that all other closed loops are under perfect control when designing the controller for a particular loop and consider only the gain change. In the proposed method, the changes are considered for both gain and frequency. Especially, Eq. (3) focuses on the gain impact while Eq. (7) contributes to time delay portion, i.e., frequency impact. As will be shown later, it is far more accurate than those existing methods

3.3 DECENTRALISED CONTROL SYSTEM DESIGN

Without loss of generality, we assume that each main loop, i.e., diagonal element in the transfer function matrix is represented by a second order plus dead time (SOPDT) model, which can be used to describe most of the industrial processes:

$$\square_{\square\square}(\square) = \frac{\square_{0,\square\square}}{\square_{2,\square\square}\square^{2} + \square_{1,\square\square}\square + 1}\,\square^{-\square_{\square\square}\square}$$

Similarly, ETF is represented as,

ISSN 2278 - 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 2, August 2012

$$\widehat{\square}_{\square\square}(\square) = \frac{\widehat{\square}_{\square\square}(0)}{\square_{2,\square\square}\square^{2} + \square_{1,\square\square}\square + 1}\square^{-\widehat{\square}_{\square\square}\square}$$

The decentralized controllers can then be independently designed by single loop approaches based on the corresponding ETFs. Here we employ the gain and phase margins approach. This is primary because the frequency response method provides good performance in the face of uncertainty in both plant model and disturbances.
The PID controller of each loop is supposed of the following standard form:

$$\square_{\square,\square}(\square) = \square_{\square,\square} + \frac{\square_{\square,\square}}{\square} + \square_{\square,\square}\square$$

The controller can be rewritten as

$$\square_{\square,\square}(\square) = \frac{\square_{\square,\square} + \square_{\square,\square}\square + \square_{\square,\square}\square^{2}}{\square} = \frac{\square\square^{2} + \square\square + \square}{\square},$$

Where A=$\frac{\square_{\square,\square}}{\square}$, B=$\frac{\square_{\square,\square}}{\square}$, c=$\frac{\square_{\square,,\square}}{\square}$ By selecting A=$\square_2$, B=$\square_1$, C=1, the open loop transfer function becomes

$$\square_{\square,\square}(\square)\widehat{\square}_{\square\square}(\square) = \square\frac{\widehat{\square}_{\square\square}(0)}{\square}\square^{-\widehat{\square}_{\square\square}\square}$$

Denoting the gain and phase margin specifications as $\square_{\square,\square}$and $\square_{\square,\square}$ and their crossover frequencies as and $\square_{\square,\square}$, respectively, we have

$$\square\square\square[\square_{\square,\square}(\square\square_{\square,\square,})\widehat{\square}_{\square\square}(\square\square_{\square,\square,})] = -\square,$$
$$\square_{\square,\square}|\square_{\square,\square}(\square\square_{\square,\square,})\widehat{\square}_{\square\square}(\square\square_{\square,\square,})| = 1,$$
$$|\square_{\square,\square}(\square\square_{\square,\square\square,\square\square,})\widehat{\square}_{\square\square}(\square\square_{\square,\square,})| = 1,$$
$$\square_{\square,\square} = \square + \square\square\square[\square_{\square,\square}(\square\square_{\square,\square\square,})\widehat{\square}_{\square\square}(\square\square_{\square,\square\square,})].$$

By substitution and simplification to above equations, we obtain

$$\square_{\square,\square,}\widehat{\square}_{\square\square} = \frac{\square}{2} \qquad \square_{\square,\square} = \frac{\square_{\square,\square,}}{\widehat{\square}_{\square\square}(0)\square},$$
$$\widehat{\square}_{\square\square}(0)\square = \square_{\square,\square,} \qquad \square_{\square,\square} = \frac{\square}{2} - \square_{\square,\square,}\widehat{\square}_{\square\square},$$

Which results

$$\square_{\square,\square} = \frac{\square}{2}\left(1 - \frac{1}{\square_{\square,\square}}\right), \qquad \square = \frac{\square}{2\square_{\square,\square}\widehat{\square}_{\square\square}\widehat{\square}_{\square\square}(0)}$$

By this formulation, the gain and phase margins are interrelated to each other, some possible gain and phase margin selections are given in Table 1.
The PID parameters are given by

$$\begin{bmatrix}\square_{\square,\square}\\ \square_{\square,\square}\\ \square_{\square,\square}\end{bmatrix} = \frac{\square}{2\square_{\square,\square}\widehat{\square}_{\square\square}\widehat{\square}_{\square\square}(0)}\begin{bmatrix}\square_{1,\square\square}\\ 0\\ \square_{2,\square\square}\end{bmatrix} \qquad (8)$$

Applying Eq. (8) for each case discussed in Section 3, we can easily obtain both ETFs and the PID parameters which are summarized in Table 2.

Table 1: Typical gain and phase margin values:

$\square_{\square,\square}$	□/□	□/□	□□/□	□□/□
$\square_{\square,\square}$	2	3	4	5

Table 2: Decentralized PID controller design

Mode	$\widehat{\square}_{\square\square}(\square)$	$\square_{\square,\square\square}$	$\square_{\square,\square\square}$	$\square_{\square,\square\square}$
Case 1	$\frac{\square_{\square\square}(0)/\ \square_{\square\square}}{\square_{2,\square\square}\square^2+\square_{1,\square\square}\square+1}\square^{-\square}$	$\frac{\square\square_{1,\square\square}\square_{\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}(0)}$	$\frac{\square\square_{\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}(0)}$	$\frac{\square\square_{\square\square}\square_{2,\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}(0)}$
Case 2	$\frac{\square_{\square\square}(0)/\ \square_{\square\square}}{\square_{2,\square\square}\square^2+\square_{1,\square\square}\square+1}\square^{-\square}$	$\frac{\square\square_{1,\square\square}\square_{\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}\square_{\square}}$	$\frac{\square\square_{\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}\square_{\square\square}(}$	$\frac{\square\square_{\square\square}\square_{2,\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}\square_{\square\square}(0)}$
Case 3	$\frac{\square_{\square\square}(0)}{\square_{2,\square\square}\square^2+\square_{1,\square\square}\square+1}\square^{-\square}$	$\frac{\square\square_{1,\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}(0)}$	$\frac{\square}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}(0)}$	$\frac{\square\square_{2,\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}(0)}$
Case 4	$\frac{\square_{\square\square}(0)}{\square_{2,\square\square}\square^2+\square_{1,\square\square}\square+1}\square^{-\square}$	$\frac{\square\square_{1,\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}\square_{\square}}$	$\frac{\square}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}\square_{\square\square}(}$	$\frac{\square\square_{2,\square\square}}{2\square_{\square,\square}\square_{\square\square}\square_{\square\square}\square_{\square\square}(0)}$

V.CONCLUSION

Effective transfer function approach is a novel method for decentralized control system design of multivariable interactive processes. An extension of the effective transfer function approach by taking into consideration all the interactions was proposed and implemented successfully with improved responses. The simplicity and effectiveness of the method is based on the incorporation of the interaction frequency directly in the controller design. This approach ensures that all the necessary information of the gain and interaction frequency changes are provided. The decentralized controllers are obtained by simply using the single loop design approaches. Simulation results for the four 2x2 processes and a 3x3 process show that the proposed method provides a better overall performance compared to the other design approaches even after taking into account the interactions. The advantage of this method is more significant when applied to higher dimensional processes with complicated interaction modes. Since this is an extension of the ETF approach, it can also be easily integrated into an auto-tuning control structure. This method can also be successfully tested for the other MIMO processes. Also employment of BLT tuning after obtaining the effective transfer function can also be performed for better results.

REFERENCES

1. W.L.Luyben, M.L.Luyben, Essentials of process control, McGraw-Hill Chemical Engineering Series, 1997.
2. Qiang Xiong, Wen-Jian Cai, Effective transfer function method for decentralized control system of MIMO processes, JPC 16(2006).
3. Deshpande, P.B. (1989), *Multivariable Process Control*, ISA, North Carolina.
4. W.L.Luyben, Simple Method for Tuning SISO Controllers in Multivariable Systems, I&EC Process Design and Development ,Vol. 25, No.3,1986
5. George Stephanopoulos, Chemical Process Control – An Introduction to theory and practice, Prentice Hall Of India,2008