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Effects of Non-integer Order Time Fractional Derivative on Coupled Heat and Mass Transfer of MHD Viscous Fluid over an Infinite Inclined Plane with Heat Absorption

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Abstract: Influence of a non-integer order fractional derivative is studied on a MHD generalized viscous fluid model with double convection, caused due to simultaneous effects of heat and mass transfer induced by temperature and concentration gradients. The fluid is considered over an inclined plane moving exponentially accelerated with time dependent of heat and mass at the boundary. Additional effects of heat generation and chemical reaction are also considered. The idea of non-integer order Caputo time fractional derivative is used and exact solutions for temperature, concentration and velocity in dimensionless form are developed. At the end, the graphically illustration is presented and discussed in details for embedded parameters including fractional parameter of order α .

Keywords: Natural convection flow, MHD, Caputo fractional derivative, Heat Absorption, Exact solution, Exponentially accelerated, Inclined plane, Chemical reaction

Nomenclature

B_0 : Uniform Applied Magnetic field
C_w : Concentration of the plate
C_∞ : Concentration of the fluid far away from the plate
c_p : Specific heat at constant pressure
D : Solute mass diffusivity
g : Acceleration due to gravity
Gr : Grashof number for heat transfer
Gm : Grashof number for mass transfer
K_r : Dimensional Chemical reaction parameter
λ : Dimensionless Chemical reaction parameter
M : Magnetic field parameter
$\Pr$: Prandtl number
Q : Dimensional Heat absorption parameter
S : Non-Dimensional Heat absorption parameter

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Sc	: Schmidt number
P	: Pressure
Re	: Reynolds number
T_w	: Temperature of the plate
T_∞	: Temperature of the fluid far away from the plate
k	: Thermal conductivity of the fluid
β_T	: Thermal expansion coefficient
β_C	: Volumetric coefficient of expansion with species concentration
ν	: Kinematic viscosity
ρ	: Density of the fluid
σ	: Electric conductivity
γ	: Slip parameter
Nu	: Nusselt number

I. INTRODUCTION

Although, the concept of fractional calculus was introduced more than 300 years ago, however, only few decades past the fractional calculus particularly fractional differenations (derivatives with non-integer order operators) has embarked many researchers from almost all branches of sciences, technology and engineering due to their capabilities of including more complex natural into mathematical equations. Recently, the fractional calculus has shared much contribution in complex dynamics. In particular, it has been proved to play a valuable role for handling viscoelasticity of materials. The models with fractional derivative provide a better level of adequacy, preserving linearity. Fractional differentiation is a non-local character, appropriate for modeling of complex materials and processes with memory. Some interesting studies on fractional derivatives using different definition and related to this work are cited in [1-10], and the references therein.

The idea of non-local operators of differenation was applied to fluid problems by many researchers [11-19], however, in heat transfer problems, Vieru et al. [20], were the first where they used the idea of fractional derivative and studied the free convection flow of viscous fluid. After that, various researchers reported works for example Imran et al. [21] Shah and Khan [22] studied heat transfer analysis in a second grade fluid. Shakeel et al. [23] discussed time fractional convection flow near a heated vertical plate. Farhad et al. [24] determined solutions for time fractional free convection flow of Brinkman-type fluid. In another investigation, Farhad et al. [25] studied generalized Walters'-B fluid model using Caputo-Fabrizio fractional derivatives. Nadeem et al. [26] applied the modern approach of Caputo-Fabrizio time-fractional derivative to MHD free convection flow of generalized second-grade fluid in a porous medium. Nadeem et al. [27,28], in two other investigations, provided comparative studies for heat transfer and heat and mass transfer heat generation and chemical reaction for convective flow of a generalized Casson fluid respectively. However, all these problems of heat transfer or heat and mass transfer were studied past a vertical flat plate, which is a special case of the inclined plate problem by taking the inclination angel equal to zero along the flow direction. Moreover, the combined phenomenon of heat and mass transfer occurs when buoyancy effects of thermal diffusion and diffusion through chemical species produce together, which plays an important role in geophysics, aeronautics and chemical engineering, with applications in food drying, food processing and polymer production [29]. Motivated by this, Farhad et al. [30], examined the combined effects of heat and mass transfer on MHD free convection flow over an inclined plate embedded in a porous medium.

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Keeping in mind the need of fractional derivatives and the generalization of classical fluid problem to generalized fractional fluid problems, this article aims to study the combined phenomenon of heat and mass transfer of viscous fluid over an inclined flat plate moving with exponential acceleration and with variable surface temperature and variable surface concentration. In addition, the effects of heat generation and chemical reaction are also considered. Magnetic field is applied in a transverse direction to the flow making the fluid as electrically conducting. Rest of the paper is arranged in the following fashion. Mathematical formulation of the problem is given in Section 2. Exact solutions using the fractional Laplace transform technique, for velocity, temperature and concentration fields are obtained in Section 3. Section 4 includes the limiting cases. Graphs are plotted in Section 5 and discussed for physical parameters of interest. Concluding remarks are given in Section 6.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

The unsteady magneto-hydrodynamic flow of viscous incompressible fluid past an exponentially accelerated isothermal infinite inclined plan with variable temperature, variable mass diffusion in the presence of heat absorption has been studied. The *x*-axis is along the plane with the inclination θ to the vertical; the *y*-axis is taken normal to it. Initially, the plane and the fluid are at the same temperature T_∞ in the stationary condition with concentration level C_∞ at all the points. At time $t>0$, the plane is exponentially accelerated with a velocity $U_0 H(t)\exp(at)$ and the temperature of the plan is raised linearly with time and species concentration level near the plate is also raised linearly with time t. The temperature and the concentration level on the plane raised or lowered to $T_\infty+(T_w-T_\infty)t$ and $C_\infty+(C_w-C_\infty)t$ respectively. We made the following assumptions: All the fluid physical properties are considered to be constant except the influence of the body force term and applied transverse magnetic field of uniform strength B_0 is normal to the plane. The fluid's conducting property is supposed to be slight and hence the magnetic Reynolds number is lesser than unity and the induced magnetic field is small in comparison with the transverse magnetic field. It is further supposed that there is no applied voltage, as the electric field is absent. Viscous dissipation and Joule heating in energy equation are neglected. According to Boussinesq's approximation, the unsteady flow is governed by the following set of equations:

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta_T\cos\theta\left(T-T_\infty\right)+g\beta_C\cos\theta\left(C-C_\infty\right)-\frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$C_p\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial y^2}+Q(T_\infty-T) \tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}-DK_r(C-C_\infty) \tag{3}$$

with initial and boundary conditions

$$u=0,\ T=0,\ C=0;\ t=0,\ y>0 \tag{4}$$

$$\begin{gathered}u(0,t)=U_oH(t)\exp(at),\ T(y,t)=T_\infty+(T_w-T_\infty)t,\\ C(y,t)=C_\infty+(C_w-C_\infty)t;\ t>0,\ y=0\end{gathered} \tag{5}$$

$$u(y,t)\to 0,\ T(y,t)\to T_\infty,\ C(y,t)\to C_\infty,\ \text{as}\ y\to\infty,\ t>0 \tag{6}$$

Introducing the following dimensionless variables and parameters

$$y^*=\frac{U_0 y}{\nu},\ u^*=\frac{u}{U_0},\ t^*=\frac{tU_0^2}{\nu},\ T^*=\frac{T-T_\infty}{T_w-T_\infty},\ C^*=\frac{C-C_\infty}{C_w-C_\infty},\ Gr=\frac{g\beta_T\,\nu(T_w-T_\infty)}{U_0^3}$$

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$$Gm=\frac{g\beta_C \nu(C_w-C_\infty)}{U_0^3},\ M=\frac{\sigma B_0^2\nu}{\rho U_0^2},\ \Pr=\frac{\nu c_p}{k},\ Sc=\frac{\nu}{D},\ N=\frac{16\sigma' T_\infty^3}{3kk'} \tag{7}$$

into Equations (1) to (6) and dropping the star notation, we have the following initial-boundary problem

$$\frac{\partial u(y,t)}{\partial t}=\frac{\partial^2 u(y,t)}{\partial y^2}+Gr\cos\theta T(y,t)+Gm\cos\theta C(y,t)-Mu(y,t) \tag{8}$$

$$\frac{\partial T(y,t)}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 T(y,t)}{\partial y^2}-\frac{S}{\Pr}T(y,t) \tag{9}$$

$$\frac{\partial C(y,t)}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C(y,t)}{\partial y^2}-\frac{\lambda}{Sc}C(y,t) \tag{10}$$

with dimensionless initial and boundary conditions

$$u(y,t)=0,\ T(y,0)=0,\ C(y,0)=0,\ y>0 \tag{11}$$

$$u(0,t)=H(t)\exp(at),\ T(0,t)=t,\ C(0,t)=t,\ t>0 \tag{12}$$

$$u(y,t)\to 0,\ T(y,t)\to 0,\ C(y,t)\to 0 \text{ as } y\to\infty \tag{13}$$

Here, we have to developed fractional model, replacing the time derivative in Equations (8), (9) and (10), with time-fractional derivatives, we obtain the following fractional differential equations

$$D_t^\alpha u(y,t)=\frac{\partial^2 u(y,t)}{\partial y^2}+Gr\cos\theta T(y,t)+Gm\cos\theta C(y,t)-Mu(y,t) \tag{14}$$

$$D_t^\alpha T(y,t)=\frac{1}{\Pr}\frac{\partial^2 T(y,t)}{\partial y^2}-\frac{S}{\Pr}T(y,t) \tag{15}$$

$$D_t^\alpha C(y,t)=\frac{1}{Sc}\frac{\partial^2 C(y,t)}{\partial y^2}-\frac{\lambda}{Sc}C(y,t) \tag{16}$$

where $D_t^\alpha u(y,t)$ represent the Caputo time-fractional derivative $u(y,t)$ defined as

$$D_t^\alpha u(y,t)=\begin{cases}\dfrac{1}{\Gamma(1-\alpha)}\displaystyle\int_0^t \frac{1}{(t-\tau)^\alpha}\frac{\partial u(y,\tau)}{\partial \tau}d\tau; & 0\le\alpha<1\\ \dfrac{\partial u(y,t)}{\partial t}; & \alpha=1\end{cases} \tag{17}$$

III. Solution Of The Problem

Temperature Distribution

Applying Laplace transform to Equations (15), $(12)_2$, $(13)_2$ and using initial condition, we obtain

$$\left(q^\alpha+\frac{S}{\Pr}\right)\bar{T}(y,q)=\frac{1}{\Pr}\frac{\partial^2\bar{T}(y,q)}{\partial y^2} \tag{18}$$

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$$\bar{T}(0,q)=\frac{1}{q^2},\quad \bar{T}(y,q)\to 0 \text{ as } y\to 0 \tag{19}$$

The solution of the partial differential equation (18) by using conditions in Equation (19) is

$$\bar{T}(y,q)=\frac{1}{q^2}\exp\left(-y\sqrt{\Pr}\sqrt{\left(q^{\alpha}+\frac{S}{\Pr}\right)}\right) \tag{20}$$

Taking inverse Laplace transform of Equation (20), we obtain

$$T(y,t)=\int_0^{\infty}\int_0^{t} erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{x}}\right)\frac{\exp\left(-\frac{S}{\Pr}x\right)}{\tau}\Phi\left(0,\alpha;-x\tau^{-\alpha}\right)\left[\frac{(t-\tau)^{1-\alpha}}{\Gamma(2-\alpha)}+\frac{S}{\Pr}(t-\tau)\right]d\tau dx \tag{21}$$

where $\phi(\beta,-\sigma;z)$, is a Wright function defined in Appendix.

Temperature Field for the Ordinary Case ($\alpha=1$)

By taking $\alpha\to 1$, into Equation (20), we obtain the

$$\bar{T}(y,q)=\frac{1}{q^2}\exp\left(-y\sqrt{\Pr}\sqrt{\left(q+\frac{S}{\Pr}\right)}\right) \tag{22}$$

with inverse Laplace transform

$$T(y,t)=\frac{1}{2}\left[\begin{array}{l}\left(t-\frac{y}{2\sqrt{S}}\right)\exp\left(-\frac{y\sqrt{S}}{\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{S}{\Pr}t}\right)+\\ +\left(t+\frac{y}{2\sqrt{S}}\right)\exp\left(\frac{y\sqrt{S}}{\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{S}{\Pr}t}\right)\end{array}\right] \tag{23}$$

The Nusselt number Nu, which measures the heat transfer rateat the plate, can be obtained using any of the above expressions for temperature, namely

$$Nu=-\left.\frac{\partial T(y,t)}{\partial y}\right|_{y=0}=-\frac{\partial}{\partial y}L^{-1}\left\{\bar{T}(y,q)\right\}\Big|_{y=0}=-L^{-1}\left\{\left.\frac{\partial \bar{T}(y,q)}{\partial y}\right|_{y=0}\right\} \tag{24}$$

$$Nu=-\sqrt{\Pr_{eff}}\left[\frac{1}{\Gamma\left(1-\frac{\alpha}{2}\right)}t^{-\frac{\alpha}{2}}-aG_{1,\frac{\alpha}{2},1}(-b,t)\right],\quad \alpha\in(0,1) \tag{25}$$

and

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$$Nu = -\sqrt{\Pr_{eff}}\left[\frac{1}{\sqrt{\pi t}} - aG_{1,\frac{1}{2},1}(-bt)\right], \quad \alpha = 1 \tag{26}$$

Where $G_{a,b,c}(d,t)$ is the $G-$function of Lorenzo-Hartley (27)

Concentration Distribution

It is observed that, the initial-boundary value problem for concentration $C(y,t)$ is of the same form with the problem for temperature field. Therefore, using results from the previous section, we have

$$\overline{C}(y,q) = \frac{1}{q^2}\exp\left(-y\sqrt{Sc}\sqrt{q^\alpha + \frac{\lambda}{SC}}\right) \tag{28}$$

with inverse Laplace transform for

$$C(y,t) = \int_0^\infty\int_0^t erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{x}}\right)\frac{\exp\left(-\frac{\lambda}{Sc}x\right)}{\tau}\Phi\left(0,\alpha;-x\tau^{-\alpha}\right)\left[\frac{(t-\tau)^{1-\alpha}}{\Gamma(2-\alpha)} + \frac{\lambda}{Sc}(t-\tau)\right]d\tau dx,\ \alpha\in(0,1) \tag{29}$$

respectively,

$$C(y,t) = \frac{1}{2}\left[\begin{array}{l}\left(t - \frac{y}{2\sqrt{\lambda}}\right)\exp\left(-\frac{y\sqrt{\lambda}}{Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{\frac{\lambda}{Sc}t}\right) + \\ + \left(t + \frac{y}{2\sqrt{\lambda}}\right)\exp\left(\frac{y\sqrt{\lambda}}{Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{\frac{\lambda}{Sc}t}\right)\end{array}\right], \quad \alpha = 1 \tag{30}$$

Calculation for Velocity

Applying Laplace transform to Equations (14), (12), (13) and using initial condition andusing expressions for $\overline{T}(y,q)$ and $\overline{C}(y,q)$ from Equations (22) and (28) respectively

$$\left(q^\alpha + M\right)\overline{u}(y,q) = \frac{\partial^2\overline{u}(y,q)}{\partial y^2} + \frac{Gr\cos\theta}{q^2}\exp\left(-y\sqrt{\Pr}\sqrt{\left(q^\alpha + \frac{S}{\Pr}\right)}\right) + \frac{Gm\cos\theta}{q^2}\exp\left(-y\sqrt{Sc}\sqrt{\left(q^\alpha + \frac{\lambda}{Sc}\right)}\right) \tag{31}$$

$$\overline{u}(0,q) = \frac{1}{q-a},\ \overline{u}(y,q)\to 0 \text{ as } y\to\infty \tag{32}$$

where $\overline{u}(y,q)$ is the Laplace transform of the function $u(y,t)$ and q is the transform variable.

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The solution of the partial differential equation (31), with conditions (32), in suitable form

$$\bar{u}(y,q)=\frac{Gr\cos\theta}{(\mathrm{Pr}-1)(q^{\alpha}+c)q^{2}}\left[\exp\left(-y\sqrt{q^{\alpha}+M}\right)-\exp\left(-y\sqrt{\mathrm{Pr}\,q^{\alpha}+S}\right)\right]+$$
$$+\frac{Gm\cos\theta}{(\mathrm{Sc}-1)(q^{\alpha}+d)q^{2}}\left[\exp\left(-y\sqrt{q^{\alpha}+M}\right)-\exp\left(-y\sqrt{Scq^{\alpha}+\lambda}\right)\right]+$$
$$+\frac{1}{q-a}.\exp\left(-y\sqrt{q^{\alpha}+M}\right),\quad c=\frac{S-M}{\mathrm{Pr}-1}\qquad d=\frac{\lambda-M}{Sc-1}\qquad \mathrm{Pr}\neq 1\,,Sc\neq 1 \tag{33}$$

By applying inverse Laplace transform to Equation (33), using Appendix as well as convolution theorem

$$u(y,t)=\frac{Gr\cos\theta}{(\mathrm{Pr}-1)}\int_0^t(t-\tau)^{\alpha+1}E_{\alpha,\alpha+2}\left(-c(t-\tau)^{\alpha}\right)\int_0^{\infty}\frac{y}{2u\sqrt{\pi u}}\exp\left(-\frac{y^2}{4u}\right)\times$$
$$\times\left[e^{-Mu}\frac{1}{\tau}\Phi\left(0,-\alpha,-u\tau^{-\alpha}\right)-e^{-Su}\frac{1}{\tau}\Phi\left(0,-\alpha,-u\,\mathrm{Pr}\,\tau^{-\alpha}\right)\right]dud\tau+$$
$$+\frac{Gm\cos\theta}{(\mathrm{Sc}-1)}\int_0^t(t-\tau)^{\alpha+1}E_{\alpha,\alpha+2}\left(-d(t-\tau)^{\alpha}\right)\int_0^{\infty}\frac{y}{2u\sqrt{\pi u}}\exp\left(-\frac{y^2}{4u}\right)\times$$
$$\times\left[e^{-Mu}\frac{1}{\tau}\Phi\left(0,-\alpha,-u\tau^{-\alpha}\right)-e^{-\lambda u}\frac{1}{\tau}\Phi\left(0,-\alpha,-uSc\tau^{-\alpha}\right)\right]dud\tau+$$
$$+\int_0^t e^{a(t-\tau)}\int_0^{\infty}\frac{y}{2u\sqrt{\pi u}}\exp\left(-\frac{y^2}{4u}\right)e^{-Mu}\frac{1}{\tau}\Phi\left(0,-\alpha,-u\tau^{-\alpha}\right)dud\tau \tag{34}$$

Velocity Field for Ordinary Case

By taking $\alpha\to 1$, into Equation (33), we obtain

$$\begin{aligned}u(y,t)=&\int_0^t\exp(a(t-\tau))\Phi\left(y,\tau;a+M\right)d\tau\\
&-\frac{Gr\cos\theta\,\mathrm{Pr}}{(\mathrm{Pr}-1)}\int_0^t\left[(t-\tau)\exp(m_1\tau)\Phi\left(y,\tau;m_1+M\right)\right]d\tau\\
&-\frac{Gm\cos\theta Sc}{(Sc-1)}\int_0^t\left[(t-\tau)\exp(m_2\tau)\Phi\left(y,\tau;m_2+M\right)\right]d\tau\\
&+\frac{Gr\cos\theta\,\mathrm{Pr}}{(\mathrm{Pr}-1)}\int_0^t\left[(t-\tau)\exp(m_1\tau)\Phi\left(y\sqrt{\mathrm{Pr}},\tau;m_1+\frac{S}{\mathrm{Pr}}\right)\right]d\tau\\
&+\frac{Gm\cos\theta Sc}{(Sc-1)}\int_0^t\left[(t-\tau)\exp(m_2\tau)\Phi\left(y\sqrt{Sc},\tau;m_2+\frac{\lambda}{Sc}\right)\right]d\tau,\\
&m_1=\frac{S-M}{\mathrm{Pr}-1},\ \mathrm{m}_2=\frac{\lambda-M}{Sc-1},\ \mathrm{Pr}\neq 1,\ Sc\neq 1\end{aligned} \tag{35}$$

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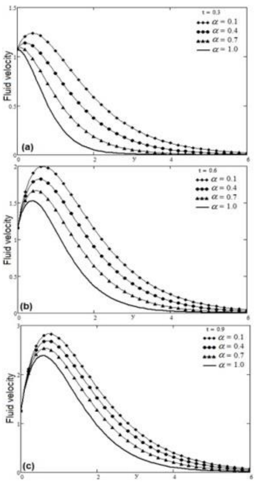

Fig. 1. Profiles of dimensionless velocity for α variation versus y for $\Pr = 0.3,\ Gr = 10,\ \ Gm = 8,\ Sc = 0.1,$ $\theta = \dfrac{\pi}{6},\ S = 0.6,\ \lambda = 0.7,\ M = 0.4,\ a = 0.25$ and different small values of time.

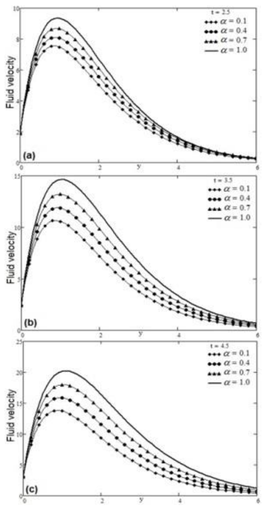

Fig. 2. Profiles of dimensionless velocity for α variation versus y for $\Pr = 0.3,\ Gr = 10,\ \ Gm = 8,\ Sc = 0.1,$ $\theta = \dfrac{\pi}{6},\ S = 0.6,\ \lambda = 0.7,\ M = 0.4,\ a = 0.25$ and different large values of time.

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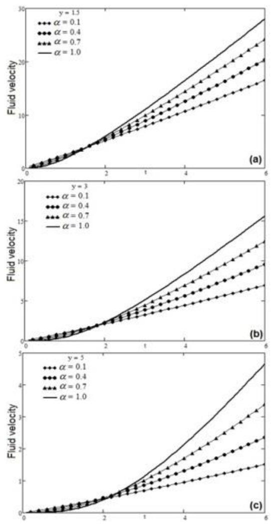

Fig. 3. Profiles of dimensionless velocity for α variation versus t for $\Pr = 0.3,\ Gr = 10,\ Gm = 8,\ Sc = 0.1,$ $\theta = \dfrac{\pi}{6},\ S = 0.6,\ \lambda = 0.7,\ M = 0.4,\ a = 0.25$ and different values of y.

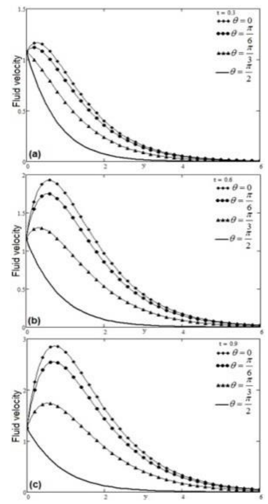

Fig. 4. Profiles of dimensionless velocity for θ variation versus y for $\Pr = 0.3,\ Gr = 10,\ Gm = 8,\ Sc = 0.1,$ $\alpha = 0.4,\ S = 0.6,\ \lambda = 0.7,\ M = 0.4,\ a = 0.25$ and different values of time.

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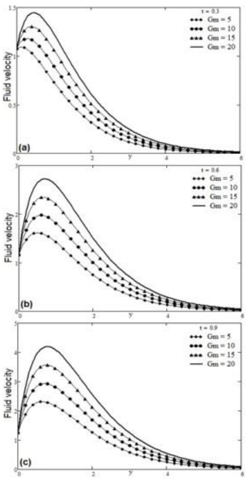

Fig.5. Profiles of dimensionless velocityfor Gm variation versus y for $\Pr = 0.3,\ Gr = 10,\ \theta = \dfrac{\pi}{6},\ Sc = 0.1,$ $\alpha = 0.4,\ S = 0.6,\ \lambda = 0.7,\ M = 0.4,\ a = 0.25$ and different values of time.

IV. NUMERICAL RESULTS AND DISCUSSION

This section is related to the impact of embedded parameters on the fluid velocity. Several graphs have been drawn to see some physical aspects of the studied problem. We have studied the influence of non-integer order fractional parameter α for small and large values of time t, spatial variable y, inclination θ and mass Grashof number on the velocity of the fluid. Fig. 1 has shown the influence of non-integer parameter α on the velocity of the fluid and observed that velocity increases for decreasing the value of fractional parameter for small values of time. Increasing time the amplitude of the fluid velocity becomes significant. On the other hand in Fig. 2 for large values of time an opposite behavior can be seen. This is due to the physical advantage of fractional of fractional model that fluid flow can be enhanced by adjusting the value of non-integer order fractional parameter for small and large time t. Fig. 3 is plotted between the fluid velocity and different value of spatial variable y. It can be seen that velocity decreases by increasing the value of spatial variable. Fig. 4 presented for the inclination of the inclined plane and found that in the absence of inclination that is limiting case when the fluid flow over an infinite vertical plate, fluid velocity increases and flow becomes faster and as we increased the value of inclination velocity decreases and approaches asymptotically zero. Also, by increasing time highest peak can be significant. Finally, in Fig. 5 velocity is an increasing function of Gm for different values of time t.

V. CONCLUSION

Fractional model for viscous fluid flow over an infinite inclined plane with heat absorption and mass transfer has been studied. Exact solutions have been obtained by means of Laplace transform. Effect of various parameters on fluid velocity has been studied especially the non-integer fractional parameter. Some concluding remarks of the present analysis are:

- For small time velocity field increases for decreasing value of α.
- For large time an opposite behaviour can be seen.
- Velocity is a decreasing function of spatial variable y.
- Fluid velocity was increased by increasing the mass Grashof number Gm.

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REFERENCES

[1] Y. Hatano, "Determination of order in fractional diffusion equation", Journal of Math-for- Industry, vol. 5, pp. 51-57, 2013.
[2] CF. Lorenzo, TT. Hartley, "Generalized functions for the fractional calculus", Critical Reviews™ in Biomedical Engineering, vol. 36 no. 1, pp. 39-55, 2008.
[3] UK. Saha, LK. Arora, BK. Dutta, "On the fractional differ integration of some special functions of fractional calculus and related functions", Int J of Math and Computer Sci, vol. 6 no. 2, pp. 65-68,2010.
[4] Y. Hatano, "Determination of order in fractional diffusion equation", Journal of Math-for- Industry, vol. 5, pp.51-57, 2013.
[5] A. Atangana, D. Baleanu, "New fractional derivatives with non-local and non-singular kernel: theory and application to heat transfer model", Therm Sci, vol. 20 no. 2, pp. 763–769, 2016.
[6] I. Atangana, Koca, "Chaos in a simple nonlinear system with Atangana–Baleanu derivatives with fractional order", Chaos Solitons Fract, vol. 89, pp. 447-454, 2016.
[7] M. Caputo, M. Fabrizio, "A new definition of fractional derivative without singular kernel", Progr Fract Differ Appl, vol. 1, pp. 73-85, 2015.
[8] Losada, JJ. Nieto, "Properties of the new fractional derivative without singular kernel", Progr Fract Differ Appl, vol. 1, pp. 87-92, 2015.
[9] A. Atanganaa, I. Kocab, "On the new fractional derivative and application to nonlinear Baggs and Freedman model", J Nonlinear Sci Appl, vol.9, pp. 2467-2480, 2016.
[10] BST. Alkahtani, A. Atangana,"Controlling the wave movement on the surface of shallow water with the Caputo-Fabrizio derivative with fractional order", Chaos Solitons & Fractals, vol. 89, pp. 539-546, 2016.
[11] BST. Alkahtani, A. Atangana, "Analysis of non-homogeneous heat model with new trend of derivative with fractional order", Chaos, Solitons & Fractals, vol. 89, pp. 566-571, 2016.12.
[12] J. Hristov, "Transient heat diffusion with a non-singular fading memory", Thermal Science, vol. 20 no. 2, pp. 757-762, 2016.
[13] J.Hristov, "Steady-state heat conduction in a medium with spatial non-singular fading memory: Derivation of Caputo-Fabrizio space-fractional derivative with Jeffrey's kernel and analytical solutions".
[14] A. Atangana, D. Baleanu, "New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model", 2016.
[15] A. Atangana, D. Baleanu, "Caputo-Fabrizio derivative applied to groundwater flow within confined aquifer", Journal of Engineering Mechanics, 2016.
[16] A. Atangana, I. Koca, "Chaos in a simple nonlinear system with Atangana–Baleanu derivatives with fractional order", Chaos, Solitons and Fractals, pp.1-8, 2016.
[17] OJJ.Algahtani,"Comparing the Atangana–Baleanu and Caputo–Fabrizio derivative with fractional order: Allen Cahn model", Chaos, Solitons & Fractals, vol. 89, pp. 552-559, 2016.
[18] T. Wenchang, XU Mingyu, "Unsteady flows of a generalized second grade fluid with the fractional derivative model between two parallel plates", Acta Mechanica Sinica, Vol. 20 no. 5, pp. 471-476, 2004.
[19] C. Fetecau, M. Athar, C. Fetecau, "Unsteady flow of a generalized Maxwell fluid with fractional derivative due to a constantly accelerating plate", Computers and Mathematics with Applications, vol. 57, pp. 596-603, 2009.
[20] NA. Shah, I. Khan, "Heat transfer analysis in a second grade fluid over and oscillating vertical plate using fractional Caputo-Fabrizio derivatives", Eur Phys JC Vol. 76, pp. 362, 2016.
[21] D.Vieru, C. Fetecau, C. Fetecau, "Time fractional free convection flow near a vertical plate with Newtonian heating and mass diffusion", Thermal Science, vol. 19, pp. S85-S98, 2015.
[22] MA. Imran, I.Khan, M. Ahmad, NA.Shah, M. Nazar, "Heat and mass transport of differential type fluid with non-integer order time-fractional Caputo derivatives", Journal of Molecular Liquids, 2017.
[23] NA.Shah, I. Khan, "Heat transfer analysis in a second grade fluid over and oscillating vertical plate using fractional Caputo–Fabrizio derivatives", The European Physical Journal C, vol. 76 no.7, pp. 1-11, 2016.
[24] A.Shakeel, A.Sohail, H.Khan, D. Vieru,"Solutions with Wright functions for time fractional convection flow near a heated vertical plate", Advances in Difference Equations, Vol. 2016 no.5, 2016.
[25] F. Ali, SAA.Jan, I. Khan, M. Gohar, NA. Sheikh, "Solutions with special functions for time fractional free convection flow of Brinkman-type fluid", The European Physical Journal Plus, vol. 131 no.9, pp.310, 2016.
[26] F Ali, M. Saqib, I. Khan, NA. Sheikh, "Application of Caputo-Fabrizio derivatives to MHD free convection flow of generalized Walters'-B fluid model", The European Physical Journal Plus, vol. 131 no. 10, pp. 377, 2016.
[27] SA. Nadeem, A. Farhad, I. Khan, M. Saqib, "A modern approach of Caputo-Fabrizio time-fractional derivative to MHD free convection flow of generalized second-grade fluid in a porous medium", Neural Computing and Applications, pp. 1-11.
[28] SA. Nadeem, "A comparative study of Atangana-Baleanu and Caputo-Fabrizio fractional derivatives to the convective flow of a generalized Casson fluid." The European Physical Journal Plus vol. 132 no. 1, pp. 54, 2017.
[29] SA. Nadeem, F. Ali, M. Saqib, I. Khan, SAA. Jan, AS. Alshomrani, MS. Alghamdi, "Comparison and analysis of the atangana-baleanu and caputo-fabrizio fractional derivatives for generalized casson fluid model with heat generation and chemical reaction". Results in Physics, 2017.
[30] Z. Guo, HJ. Sung, "Conjugate heat and mass transfer in metal hydride beds in the hydriding process", Int J Heat Mass Transf, vol.42, pp. 379-382, 1999.
[31] F. Ali, I. Khan , Samiulhaq, S. Shafie, "Conjugate effects of heat and mass transfer on MHD free convection flow over an inclined plate embedded in a porous medium", PLoS ONE vol. 8 no.6, pp. e65223, 2013.