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# Fitting of Exponential Distribution in Application with the Rainfall Data of All the Districts of Tamilnadu, India

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Research Article

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## ABSTRACT

Nature is the mother of all the wonders, which never fails to admire us in all the time. Rainfall is a gift which is the backbone of the wealth of all biotic and abiotic factors. The present study analyses the rainfall of the districts of Tamilnadu, which considers the rainfall of all the 32 districts of Tamilnadu from 2015 to 2017. In this study an endeavor is made to fit univariate continuous exponential distribution and tested the goodness of fit by using chi-square test. It is observed that the exponential distribution is an acceptable fit for the year 2016 of rainfall data of the all the districts of Tamilnadu and it is not preferred for the rest of years.

## INTRODUCTION

India occupies the greater part of South Asia with the capital of New Delhi. The Indian government is a constitutional republic which represents a highly assorted population consisting of thousands of ethnic groups and likely hundreds of languages. With around one-sixth of the world's total population, India is the second most popular country, after China. So, our nation has a greatest weapon called the youngsters [1]. India is a federal union comprising 28 states and 8 union territories [2-9].

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Tamil Nadu state has a geographical area of 1, 30,058 sq.km and is placed between North Latitudes 08 ° 00’ and 13 ° 30’ and East Longitudes 76° 15’ and 80 ° 18’. This State is bounded by the Bay of Bengal in the eastern side, the Indian Ocean in the southern side, the Western Ghats in the western side and the States of Karnataka and Andhra Pradesh in the northern sides.

The Tamilnadu State is divided into 32 Districts, 209 Taluks, and 1139 Firkas. It has 10 Corporations, 150 Municipalities, 559 Town Panchayats, 12,620 Panchayat Villages and 93,699 Habitations.

### Objectives

Our main objective is to obtain continuous frequency distributions for 2015, 2016 and 2017, and fitting the exponential distribution to the frequency distribution which we obtained for the years 2015, 2016 and 2017.

Estimating the expected frequencies to the fitted exponential frequency distribution and testing its goodness of fit using $\chi^2$ test, whether the Exponential distribution is a good fit or not.

### Study area

Salem is the fifth largest city in Tamil Nadu by population and covers 124 km². The district lies between 11° 14' and 12° 53' North of latitude and between 77° 44' and 78° 50' East of longitude, covering an area of 5245 sq. km. Salem is a Geologist’s paradise, surrounded by hills and the landscape dotted with hillocks. It is located about 160 km northeast of Coimbatore, 186 km southeast of Karnataka state capital Bangalore and about 340 km southwest and with the state’s capital Chennai.

### Rainfall data

The average annual rainfall data is used for this study and they are the secondary source and are collected from the online web source namely, World Weather Online for the study period 2015, 2016 and 2017 of all the districts of Tamilnadu, India, which has complete weather details of all countries and its states as well as districts wise. The average annual rainfall of all the districts of Tamilnadu was analyzed.

## MATERIAL AND METHODS

### Exponential distribution

Let the variable x be an average rainfall of the district of Tamilnadu in a year with the parameter λ and if it is distributed exponentially, then the probability density function is

$$\mathbf{f(x,\lambda)} = \begin{cases} \boldsymbol{\lambda e^{-\lambda x}, x \geq 0} \\ \mathbf{0, Otherwise} \end{cases}$$

The cumulative distribution function of this exponential distribution is

$$\mathbf{F(X<}a\mathbf{)} = \int_0^a \boldsymbol{\lambda} \mathbf{e^{-\lambda x} dx} = \begin{cases} \mathbf{1 - e^{-\lambda x}, x \geq 0} \\ \mathbf{0, Otherwise} \end{cases}$$

In short, $\boldsymbol{F(x) = 1 - e^{-\lambda x}}$

### Estimation of the parameter λ

The probability density function of the exponential distribution is

$$\boldsymbol{f(x,\lambda)} = \begin{cases} \boldsymbol{\lambda e^{-\lambda x}, x \geq 0} \\ \boldsymbol{0, Otherwise} \end{cases}$$

$$\boldsymbol{i.e., f(x) = \lambda e^{-\lambda x}}$$

The likelihood function of the exponential distribution is

$$\boldsymbol{L(x,\lambda) = \prod_{i=1}^{n} f(x,\lambda)}$$

$$L(x,\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x}$$
$$L(x,\lambda) = \lambda^{n} e^{-\lambda \sum_{i=1}^{n} x_i}$$

Taking logarithms on both sides,

$$\log L(x,\lambda) = n \log \lambda - \lambda \sum_{i=1}^{n} x_i$$

Now, differentiating this equation with respect to the parameter λ

$$\frac{d}{d\lambda} \log L(x,\lambda) = \frac{d}{d\lambda}\left\{ n \log \lambda - \lambda \sum_{i=1}^{n} x_i \right\}$$
$$\frac{d}{d\lambda} \log L(x,\lambda) = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i$$

For estimating the parameter, according to the principle of maximum likelihood estimation, equate the equation to zero and the estimator for the parameter λ.

$$\frac{d}{d\lambda} \log L(x,\lambda) = 0$$
$$\frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0$$
$$\frac{1}{\lambda} = \frac{1}{n} \sum_{i=1}^{n} x_i$$
$$\frac{1}{\lambda} = \bar{x}, i.e., \lambda = \frac{1}{\bar{x}}$$

Where, $\bar{x}$ is the sample mean and it can be obtained by using $\bar{x} = \frac{1}{n}\sum x_i$.

We can also check whether this estimator maximizes the Likelihood function or not by integrating the first order derivative with respect the parameter λ.

If the value of the derivative is negative, then it maximizes the Likelihood function, otherwise, not.

$$\frac{d}{d\lambda}\left\{ \frac{d}{d\lambda} \log L(x,\lambda) \right\} = \frac{d}{d\lambda}\left\{ \frac{n}{\lambda} - \sum_{i=1}^{n} x_i \right\}$$
$$\frac{d^2}{d\lambda^2} \log L(x,\lambda) = -\frac{n}{\lambda^2} < 0$$

It is clear, that the estimator of the parameter is maximizing the Likelihood function,

**Obtaining expected frequencies**

Let us consider E(x) be the expected frequency and is obtained by multiplying the total frequencies the probability values of the Exponential Distribution.

$$E(x) = N.P(x)$$

Here, $P(x)$ is the probability density function of the exponential distribution.

## RESULTS AND DISCUSSION

This part is consisting of four sub topics. Namely, Descriptive Statistics, obtaining frequency distribution, fitting exponential distribution for the frequency distribution and finally testing its goodness of fit by using Chi-square distribution for the years 2015, 2016 and 2017 of average annual rainfall of the 32 districts of Tamilnadu.

**Descriptive statistics**

The descriptive statistics in terms of mean, standard deviation, coefficient of variation, skewness; the maximum amount of rainfall and the minimum amount of rainfall for the years 2015, 2016 and 2017 are summarized in the Table 1.

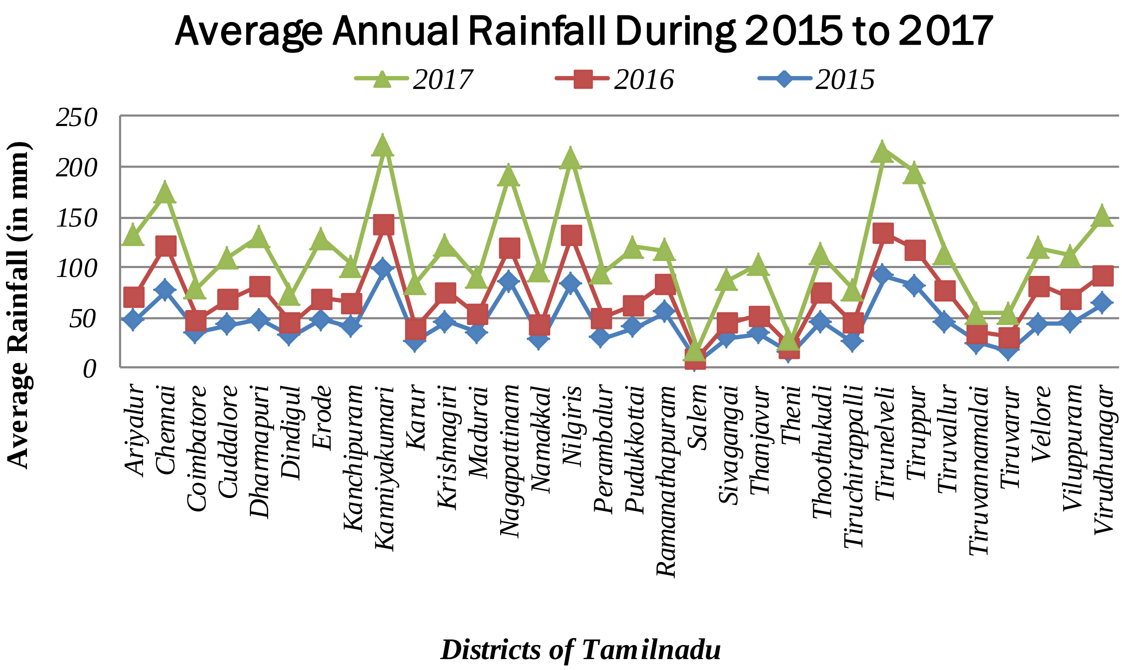
**Table 1:** Statistic of annual average rainfall from 2015 to 2017

| Statistic | 2015 | 2016 | 2017 |
|---|---|---|---|
| Mean | 41.63 | 22.74 | 43.19 |
| Standard Deviation | 21.77 | 11.08 | 18.03 |
| Coefficient of Variation | 52.29 | 48.73 | 41.75 |
| Maximum Average Rainfall | 98.96 | 48.17 | 82.00 |
| Minimum Average Rainfall | 5.93 | 3.39 | 7.38 |
| Skewness | 0.78 | 0.28 | 0.13 |

The average annual rainfall is high for the year 2017 and is 43.19 mm which is also less skewed. The standard deviation is lowest for the year 2016 and is 11.08 mm.

The average rainfall of all the districts of Tamilnadu is shown in the Figure 1. From the Figure 1, it is observed that the average rainfall of the district Kanya kumari is high and Salem is low.

**Figure 1:** Average Annual Rainfall during 2015 to 2017




**Frequency distribution**

Using the Sturges method the frequency distribution is formed the years 2015, 2016 and 2017. They are given in Tables 2-4.

**Table 2:** Frequency distribution of average annual rainfall for the year 2015 is

| Average Rainfall (in mm) | 5-21 | 21-37 | 37-53 | 53-69 | 69-85 | 85-101 | Total |
|---|---|---|---|---|---|---|---|
| No. of Districts | 3 | 10 | 11 | 2 | 3 | 3 | 32 |

**Table 3:** Frequency distribution of average annual rainfall for the year 2016 is

| Average Rainfall (in mm) | 3-11 | 11-19 | 19-27 | 27-35 | 35-43 | 43-51 | Total |
|---|---|---|---|---|---|---|---|
| No. of Districts | 3 | 9 | 7 | 7 | 3 | 3 | 32 |

**Table 4:** Frequency distribution of average annual rainfall for the year 2017 is

| Average Rainfall (in mm) | 7-20 | 20-33 | 33-46 | 46-59 | 59-72 | 72-85 | Total |
|---|---|---|---|---|---|---|---|
| No. of Districts | 3 | 4 | 11 | 7 | 2 | 5 | 32 |

Expected frequency based the exponential distribution for the years 2015, 2016 and 2017 are shown in Tables 5-7.

**Table 5:** Expected Frequency based the exponential distribution for the year 2015 is given below

| Class Interval | $m$ | $f$ | $fm$ | Lower Limit (x) | $\lambda x$ | $e^{-\lambda x}$ | $F(x)$ | $P(x) = F(x+1) - F(x)$ | Expected Frequency |
|---|---|---|---|---|---|---|---|---|---|
| Below 5 | | | | 0 | 0 | 1.0000 | 0.0000 | 0.1041 | 3.330 ≅ 3 |
| 5-21 | 13 | 3 | 39 | 5 | 0.1099 | 0.8959 | 0.1041 | 0.2656 | 8.500 ≅ 9 |
| 21-37 | 29 | 10 | 290 | 21 | 0.4616 | 0.6303 | 0.3697 | 0.1869 | 5.980 ≅ 6 |
| 37-53 | 45 | 11 | 495 | 37 | 0.8133 | 0.4434 | 0.5566 | 0.1315 | 4.207 ≅ 4 |
| 53-69 | 61 | 2 | 122 | 53 | 1.1649 | 0.3119 | 0.6881 | 0.0925 | 2.960 ≅ 3 |
| 69-85 | 77 | 3 | 231 | 69 | 1.5166 | 0.2195 | 0.7805 | 0.0651 | 2.082 ≅ 2 |
| 85-101 | 93 | 3 | 279 | 85 | 1.8683 | 0.1544 | 0.8456 | 0.1544 | 4.940 ≅ 5 |
| 101 and above | | | | 101 | 2.2200 | 0.1086 | 1.0000 | | |
| Total | | N=32 | $\Sigma fm = 1456$ | | | | | | 32 |
| Mean=45.5 and λ=0.021978 | | | | | | | | | |

**Table 6:** Expected Frequency based the exponential distribution for the year 2016 is given below:

| Class Interval | m | f | fm | Lower Limit (x) | λx | e^(-λx) | F(x) | P(x)= F(x+1)-F(x) | Expected Frequency |
|---|---|---|---|---|---|---|---|---|---|
| Below 3 | | | | 0 | 0 | 1.0000 | 0.0000 | 0.1141 | 3.653 ≅ 4 |
| 3-11 | 7 | 3 | 21 | 3 | 0.1212 | 0.8859 | 0.1141 | 0.2446 | 7.829 ≅ 8 |
| 11-19 | 15 | 9 | 135 | 11 | 0.4444 | 0.6412 | 0.3588 | 0.1771 | 5.667 ≅ 6 |
| 19-27 | 23 | 7 | 161 | 19 | 0.7676 | 0.4641 | 0.5359 | 0.1282 | 4.102 ≅ 4 |
| 27-35 | 31 | 7 | 217 | 27 | 1.0908 | 0.3359 | 0.6641 | 0.0928 | 2.969 ≅ 3 |
| 35-43 | 39 | 3 | 117 | 35 | 1.414 | 0.2432 | 0.7568 | 0.0672 | 2.149≅ 2 |
| 43-51 | 47 | 3 | 141 | 43 | 1.7372 | 0.1760 | 0.8240 | 0.1760 | 5.432 ≅ 5 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 51 and above | | | | 51 | 2.0604 | 0.1274 | 1.0000 | | |
| Total | | N=32 | Σfm=792 | | | | | | 32 |
| Mean=24.7 and λ=0.040404 | | | | | | | | | |

**Table 7:** Expected Frequency based the exponential distribution for the year 2017 is given below

| Class Interval | m | f | fm | Lower limit (x) | λx | e^(-λx) | F(x) | P(x)= F(x+1)-F(x) | Expected Frequency |
|---|---|---|---|---|---|---|---|---|---|
| Below to 7 | | | | 0 | 0 | 1.0000 | 0.0000 | 0.1412 | 4.517 ≅ 5 |
| 7-20 | 13.5 | 3 | 40.5 | 7 | 0.1522 | 0.8588 | 0.1412 | 0.2114 | 6.766 ≅ 7 |
| 20-33 | 26.5 | 4 | 106 | 20 | 0.4348 | 0.6474 | 0.3526 | 0.1594 | 5.100 ≅ 5 |
| 33-46 | 39.5 | 11 | 434.5 | 33 | 0.7174 | 0.4880 | 0.5120 | 0.1201 | 3.845 ≅ 4 |
| 46-59 | 52.5 | 7 | 367.5 | 46 | 1.0000 | 0.3679 | 0.6321 | 0.0906 | 2.898 ≅ 3 |
| 59-72 | 65.5 | 2 | 131 | 59 | 1.2827 | 0.2773 | 0.7227 | 0.0683 | 2.185 ≅ 2 |
| 72-85 | 78.5 | 5 | 392.5 | 72 | 1.5653 | 0.2090 | 0.7910 | 0.2090 | 6.389 ≅ 6 |
| 85 and above | | | | 85 | 1.8479 | 0.1576 | 1.0000 | | |
| Total | | N=32 | Σfm=1472 | | | | | | 32.000 |
| Mean=46 and λ=0.0217391 | | | | | | | | | |

**Testing goodness of fit: $\chi^2$ test**

This method is one of the famous methods used widely to test whether the distribution which was taken is perq-fectly fits the data or not. That is, the distribution function which is tried to fit the data is a right distribution function or not.

The chi-square statistic is given by

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2 \; with \; (n-1) \; d.f.$$

Where, $O_i$-Observed frequency
$E_i$-Expected frequency
n-Number of frequencies and
*d. f.*-degrees of freedom

**$\chi^2$ test for the year 2015**

**Null Hypothesis [$H_0$]:** The Exponential distribution is a good fit for the rainfall data of all the districts of Tamilnadu for the year 2015.

**Alternative Hypothesis [$H_1$]:** The Exponential distribution is not a good fit for the rainfall data of all the districts of Tamilnadu for the year 2015.

Level of significance: 5%

Degrees of Freedom: n-1

$\chi^2$ test statistic is

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2 \text{ with } (n-1)\ d.f.$$

**Table 8:** $\chi^2$ test for the year 2015

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|
| 13 | 12 | 1 | 1 | 0.0833 |
| 11 | 6 | 5 | 25 | 4.1667 |
| 8 | 14 | -6 | 36 | 2.5714 |
| | | | | $\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = 6.8214$ |

$\chi^2$ Calculated value is 6.8214 obtained from Table 8.
$\chi^2$ Tabulated value for (n-1-3*=6-1-3=) 2 degrees of freedom for 5% level of significance is 5.991.
Since, calculated $\chi^2$ value is greater than the tabulated $\chi^2$ value. So, the null hypothesis is rejected.
The Exponential distribution is not a good fit for the rainfall data of all the districts of Tamilnadu for the year 2015.

### $\chi^2$ test for the year 2016

**Null Hypothesis** [$H_0$]: The Exponential distribution is a good fit for the rainfall data of all the districts of Tamilnadu for the year 2016.
**Alternative Hypothesis** [$H_1$]: The Exponential distribution is not a good fit for the rainfall data of all the districts of Tamilnadu for the year 2016.
Level of significance: 5%
Degrees of Freedom: n-1

$\chi^2$ test statistic is

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2 \text{ with } (n-1)\ d.f.$$

**Table 9:** $\chi^2$ test for the year 2016

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|
| 12 | 12 | 0 | 0 | 0 |
| 7 | 6 | 1 | 1 | 0.1667 |
| 13 | 14 | -1 | 1 | 0.0714 |
| | | | | $\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = 0.2381$ |

$\chi^2$ Calculated value is 0.2381 obtained from Table 9.
$\chi^2$ Tabulated value for (n-1-3*=6-1-3=) 2 degrees of freedom for 5% level of significance is 5.991.
Since, calculated $\chi^2$ value is lesser than the tabulated $\chi^2$ value. So, the null hypothesis is accepted.
The Exponential distribution is a good fit for the rainfall data of all the districts of Tamilnadu for the year 2016.

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*When applying $\chi^2$ test, all the frequencies are should be equal to 5 or more than 5. If any of the observed or expected frequencies are less than 5, it should be pooled to the immediate frequencies to make it equal to or more than 5. When frequencies are pooled, then the number of pooled value minus 1 should be subtracted from the degrees of freedom.

### $\chi^2$ test for the year 2017

**Null Hypothesis [$H_0$]:** The Exponential distribution is a good fit for the rainfall data of all the districts of Tamilnadu for the year 2017.

**Alternative Hypothesis [$H_1$]:** The Exponential distribution is not a good fit for the rainfall data of all the districts of Tamilnadu for the year 2017.

Level of significance: 5%

Degrees of Freedom: n-1

$\chi^2$ test statistic is

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2 \text{ with } (n-1)\ d.f.$$

**Table 10:** $\chi^2$ test for the year 2017

| $O_i$ | $E_i$ | $O_i - E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|
| 7 | 12 | -5 | 25 | 2.0833 |
| 11 | 5 | 6 | 36 | 7.2000 |
| 14 | 15 | -1 | 1 | 0.0667 |
| | | | $\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = 9.3500$ | |

$\chi^2$ Calculated value is 9.3500 obtained from Table 10.

$\chi^2$ Tabulated value for (n-1-3*=6-1-3=) 2 degrees of freedom for 5% level of significance is 5.991.

Since, calculated $\chi^2$ value is greater than the tabulated $\chi^2$ value. So, the null hypothesis is rejected.

The Exponential distribution is not a good fit for the rainfall data of all the districts of Tamilnadu for the year 2017.

## CONCLUSION

This study shows that the detailed evaluation of fitting and testing of Exponential distribution for the Rainfall data with the known chi-square test. From the Analysis and Results part, it is found that the Exponential distribution is not a good fit for the rainfall data of all the districts of Tamilnadu for the years 2015 and 2017. The Exponential distribution is a good fit for the rainfall data of all the districts of Tamilnadu for the year 2016.

The results shows that the exponential distribution is partially fit the rainfall data of all the districts of Tamilnadu.

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