



Fractal Based SVPWM Algorithm for Two Legged Three Phase Multilevel Inverters

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Abstract: This paper proposes a Space Vector PWM (SVPWM) algorithm based on fractals for two legged three phase multilevel inverters. The paper presents the observation that the space vector representation of two legged three phase multilevel inverter has an inherent fractal structure. The properties of fractal structure together with the simplicity of fractal arithmetic are utilized to propose a generalized algorithm for SVPWM generation for two legged multilevel inverters and it can be easily extended to n-level inverters. The voltage space vectors of higher level inverter can be generated from the voltage vectors of an equivalent 2-level inverter using simple addition operations. Sector identification and switching vector determination in SVPWM generation are carried out using simple arithmetic operations without any computational complexity. The proposed algorithm doesn't use any look up table for sector identification and switching vector determination. The validation of the algorithm through simulations in MATLAB/SIMULINK has been carried out for two legged 3-level and 5-level inverter configurations for which the results are also presented.

Keywords: Six-switch three phase inverter [SSTPI] topology, Four-switch three phase inverter [FSTPI] topology, Two legged multilevel inverters, Space vector pulse width modulation [SVPWM], Fractals

I. INTRODUCTION

Multilevel inverter technology has emerged as a powerful technique in the area of medium voltage high power applications. Multilevel inverter synthesizes desired output voltage from multiple levels of dc voltages. As the number of voltage level increases the harmonic content of output voltage waveform decreases significantly [1], [2]. Multilevel inverters gained more attention in recent years due to its inherent advantages of high voltage handling and good harmonic rejection capabilities with currently available power devices. The requirement of more number of semiconductor switches is the major disadvantage of multilevel inverters. Although low voltage rated switches can be used in a multilevel inverter, each switch requires a related gate drive protection circuits and the overall system becomes more complex and expensive. In addition increased number of switches increases the switching losses and electromagnetic interference (EMI). The increased switches have limited the usage of multilevel inverters in many critical applications particularly in the field of aerospace applications. The reduction in the number of power semiconductor switches for the inverters is drawing significant research attention. The four switch three phase inverter topology [FSTPI] is one of the techniques to reduce the switches in the inverter [3]. Compared to conventional three phase inverter with six switches [SSTPI], FSTPI has two switches in each two legs and one leg is replaced with capacitors. The two legged 2-level inverter configuration is the four switch topology referred to in literature. Fig.1 shows the two legged 3-level inverter circuit. Space Vector Pulse Width Modulation (SVPWM) is the most popular and best choice for generating PWM for multilevel inverters due to its inherent advantages of better fundamental output voltage, reduced switching losses, lower harmonic distortion and easier implementation [4]-[11]. There has been significant work in the area of Space vector PWM for four switch inverters [12]-[17] but the extension to multilevel inverter configuration is yet to be explored.

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The paper explores the possibility of extending the concept of SVPWM for FSTPI to multilevel configurations. The paper begins with the space vector representation of two legged multilevel inverters and brings out the observation that the voltage space vector representation of a two legged multilevel inverter has an inherent fractal structure with the basic unit of this structure being the triangle made of the vertices of three adjacent inverter voltage space vectors [10]. The inherent fractal structure is utilized in the paper to propose a simple and generalized algorithm to generate SVPWM for any two legged multilevel inverters. The paper presents the simulation results of the algorithm implemented in MATLAB/SIMULINK for 3-level and 5-level inverter configurations.

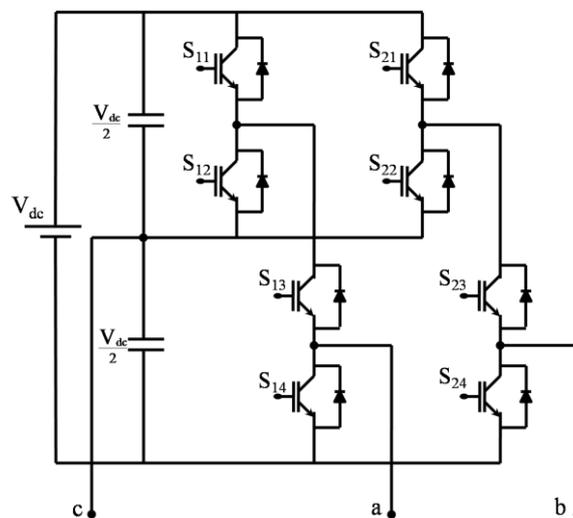


Fig. 1 Circuit of a two legged 3-level inverter

II. INHERENT FRACTAL STRUCTURE IN THE SPACE VECTOR REPRESENTATION OF TWO LEGGED MULTILEVEL INVERTERS

SVPWM utilizes the space vector concept for computing the duty cycle of the switches. Generation of SVPWM involves approximating the instantaneous reference voltage space vector by switching the three nearest inverter voltage vectors [9]. Fig. 2(a) shows the space vector representation of two legged 2-level three phase inverter and it may be noted that the voltage space vector representation of two legged 2-level inverter has four inverter voltage vectors located at the vertices of the parallelogram which be divided into two equilateral triangles (sectors) numbered as S_1 and S_2 .

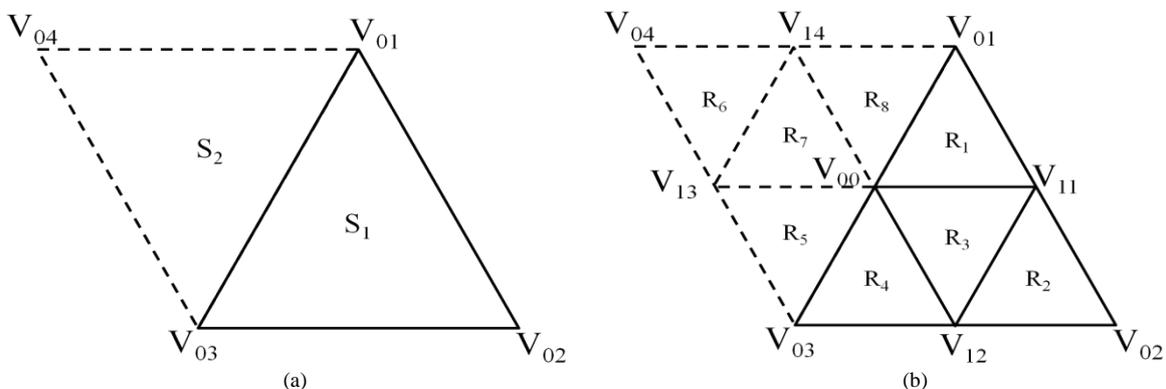


Fig. 2 Space vector representation of two legged (a) 2-level inverter (b) 3-level inverter

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Space vector representation of two legged 3-level inverter is shown in Fig. 2(b), where V_{01} , V_{02} , V_{03} and V_{04} are the same as the locations of the voltage space vectors of 2-level inverter. It may be noted that 3-level inverter has five additional voltage vectors and are located at V_{00} , V_{11} , V_{12} , V_{13} and V_{14} . Consider the region marked as S_1 in the case of 2-level inverter [Fig. 2(a)], formed by the voltage vectors located at V_{01} , V_{02} and V_{03} . In the case of 3-level inverter [Fig. 2(b)], this region has three additional voltage space vectors located at V_{00} , V_{11} and V_{12} and is located at the midpoints of each side of the triangle $[\Delta V_{01}V_{02}V_{03}]$ of equivalent 2-level inverter. Hence the $\Delta V_{01}V_{02}V_{03}$ of 3-level inverter consists of four sectors, formed by the voltage space vectors located at V_{01} , V_{11} , V_{02} , V_{12} , V_{03} , V_{00} . Also the $\Delta V_{01}V_{03}V_{04}$ contains another four sectors. So the total number of sectors in 3-level two legged inverter is 8 $[R_1-R_8]$ as shown in Fig. 2(b).

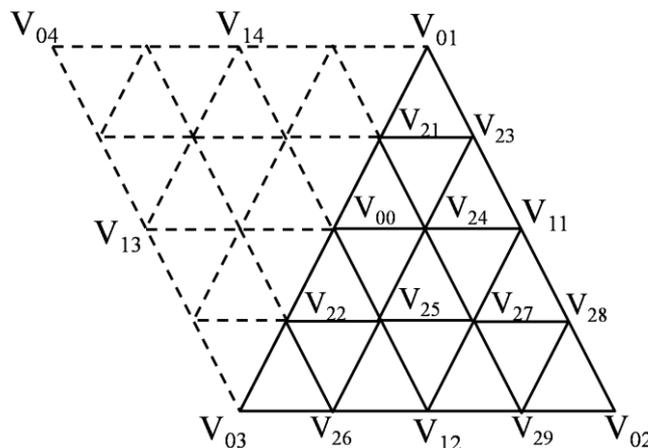


Fig. 3 Voltage space vector locations for two legged 5-level inverter

Fig. 3 shows the voltage space vector locations for two legged 5-level inverter. In Fig. 3, in the case of the triangular region formed by the voltage space vectors located at V_{01} , V_{02} and V_{03} , besides the voltage space vectors of 3-level inverter, nine additional voltage space vectors are present and are located at V_{21} , V_{22} , V_{23} , V_{24} , V_{25} , V_{26} , V_{27} , V_{28} and V_{29} . The nine additional voltage vectors are located at the midpoints of the sides of sectors of 3-level inverter and these nine voltage vectors together with the voltage vectors of 3-level inverter results 16 sectors within $\Delta V_{01}V_{02}V_{03}$. By considering the sectors in the $\Delta V_{01}V_{03}V_{04}$, total sectors formed for 5-level inverter becomes 32. In this manner, each sectors formed by the voltage space vectors of 2-level inverter divided into four smaller sectors results the voltage space vector representation of 3-level inverter with 8 sectors. All these 8 sectors of the 3-level inverter further divided in to four smaller sectors and this results the voltage space vector representation of 5-level inverter with 32 sectors. The process gets repeated as the number of level increases and the voltage space vector locations of any level can be generated. Generalizing, the number of triangles for the two legged n level inverter can be obtained as,

$$s = 2 \times (n - 1)^2 \tag{1}$$

Whereas the number of triangles for the conventional n level inverter is, $6 \times (n - 1)^2$. Thus the number of triangles for the two legged inverter configuration is only one-third of the number for conventional inverters. The advantage of lesser triangles further simplifies the generation of SVPWM for two legged higher level inverters.

The space vector representation of 2-level inverter can be grows to that of higher level inverters by repeated division of each sectors. At every stage due to the presence of additional voltage space vectors, each triangular sector is divided into four smaller triangular sectors. On moving from 2-level vectors to 3-level each triangle in the space vector location gets subdivided into four smaller triangles, which further subdivides for the 5-level inverter. The division of each triangle into four smaller triangles by joining the midpoints of the sides of the triangle is the basis for Sierpinski triangle generation in fractal theory. According to fractal theory, it may be noted that the switching voltage space vector representation of two legged multilevel inverter has a fractal structure with the basic structure is a triangle. In the

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present work this observation is used to propose a generalized algorithm for the SVPWM generation for any two legged n -level configuration. The switching voltage vectors and switching states of higher level inverters can be generated from an equivalent 2-level inverter using repeated triangularization algorithm [10].

III. GENERALIZED SVPWM ALGORITHM FOR TWO LEGGED MULTILEVEL INVERTERS THROUGH FRACTALS

This section explains the steps for the implementation of fractal based SVPWM generation for two legged multilevel inverters.

A. Sector Identification and switching vector determination

Sector identification determines the triangle that encloses the tip of the reference space vector and the vertices of the identified sector represent the locations of switching voltage space vectors which are used to synthesize the reference space vector. Repeated triangularization algorithm is used for the sector identification. Consider three instantaneous reference phase voltages V_a , V_b and V_c . Using coordinate transformation, three phase voltages are transformed in to (α, β) plane. This can be obtained as,

$$\begin{aligned} V_\alpha &= \frac{3}{2}V_a \\ V_\beta &= \frac{\sqrt{3}}{2}(V_b - V_c) \end{aligned} \quad (2)$$

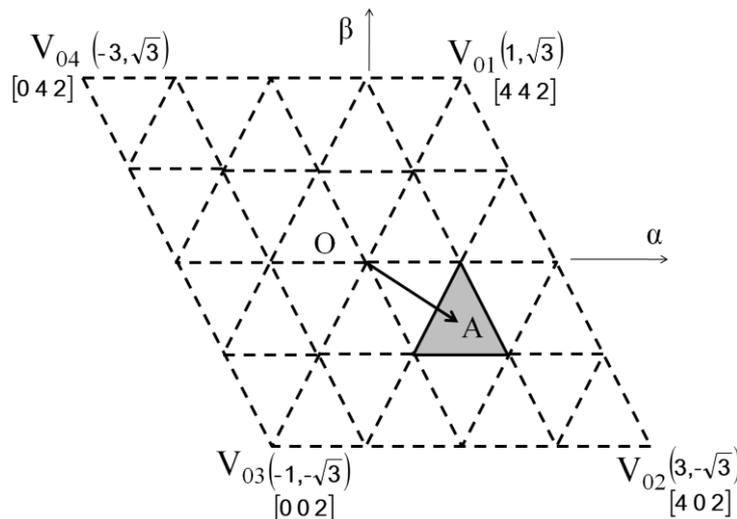


Fig. 4 The position of instantaneous reference space vector OA in two legged 5-level inverter

The method of sector identification in the proposed work can be explained by considering a position of reference space vector OA for a two legged 5-level inverter as in Fig. 4. In order to identify the sector, first determine the location of the tip of the reference space vector OA from among the two regions of the equivalent 2-level inverter. In the previous sections, it is mentioned that the switching voltage representation of 2-level inverter contains two sectors (equilateral triangles) as a results of four voltage space vectors located at V_{01} , V_{02} , V_{03} and V_{04} . α and β coordinates of V_{01} , V_{02} , V_{03} and V_{04} for equivalent 2-level inverter are $(1, \sqrt{3})$, $(3, -\sqrt{3})$, $(-1, -\sqrt{3})$ and $(-3, \sqrt{3})$ as shown in Fig. 5(a). The switching states of V_{01} , V_{02} , V_{03} and V_{04} are also shown in Fig. 5(a) [in square brackets]. To identify the sector which encloses the tip of the reference space vector, compute the coordinates of the centroid of the both two equilateral triangles of the equivalent 2-level inverter and found the triangle with centroid closest to tip of reference voltage space vector. The coordinates of the centroid of the triangles can be determined by taking the average of the coordinates of

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the three vertices. In this case, it is found that $\Delta V_{01}V_{02}V_{03}$ is the triangular sector in which tip of the reference space vector lies as shown in Fig. 5(a). The number of applications of triangularization algorithm for a 5-level inverter is two [10]. In the first application of triangularization algorithm to the region $[\Delta V_{01}V_{02}V_{03}]$ which encloses the tip of the reference space vector, it gets divided into four equilateral triangles. Sector I of the equivalent 2-level inverter in which tip of the reference space vector undergoes first triangularization algorithm and generate three additional vectors V_{00} , V_{11} , V_{12} and gets divided into four smaller triangular regions as in Fig. 5(b).

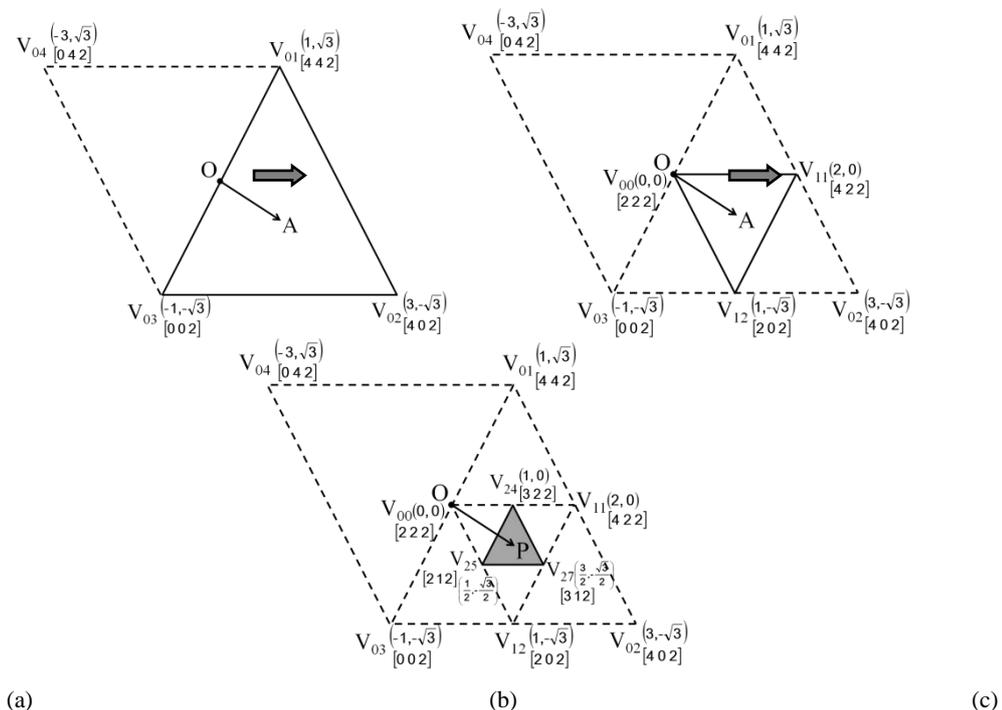


Fig. 5 Illustration of the proposed sector identification for a 5-level two legged inverter

According to Triangularization algorithm, the average of α and β coordinates of the voltage vectors at V_{01} and V_{02} will result in the coordinates of V_{11} as (2, 0). The averaging applied to the switching states at V_{01} and V_{02} will result in the switching states corresponding to the switching vector located at V_{11} as [4 2 2]. The voltage vectors and switching states of V_{12} and V_{00} are also computed similarly and are also shown in Fig. 5(b). Then again compute the centroid of the new four triangular sectors and from among these sectors find the region in which the tip of the reference space vector lies. It is found that the triangle which encloses the tip of the reference space vector OA is $\Delta V_{00}V_{11}V_{12}$.

For a 5-level inverter, the triangularization algorithm has to be applied once more. The application of triangularization algorithm within the $\Delta V_{00}V_{11}V_{12}$ generate three additional vectors V_{24} , V_{25} , V_{27} and the corresponding switching vectors and inverter states can be determined and are shown in Fig. 5(c). These three new vectors divide the $\Delta V_{00}V_{11}V_{12}$ into four smaller triangles. From among these four triangles, determine the triangle in which the tip of the reference space vector lies by computing the centroids. It is found that the $\Delta V_{24}V_{25}V_{27}$ encloses the tip of the reference space vector and V_{24} , V_{25} and V_{27} are the location of the switching vectors which are used to synthesize the reference voltage space vector OA. This is the procedure for the sector identification and the switching vector determination. For higher level inverters the procedure repeated to identify the sector and determine corresponding switching vector and switching state of voltage space vectors.

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B. Switching vector time calculations

The procedure for identification of sector and determination of switching voltage vectors are explained in the above section. The next stage involves determining the duration for which the voltage space vectors located at the vertices of the identified sector are to be switched. Since one phase is clamped to a constant voltage, space vector representation of two legged multilevel inverter has unique zero vector. In the above case, the tip of the reference voltage space vector OA lies in the triangular sector formed by the voltage space vectors located at V_{24} , V_{25} and V_{27} . The coordinates of the switching voltage space vectors located at V_{24} , V_{25} and V_{27} are $(1, 0)$, $(1/2, -\sqrt{3}/2)$ and $(3/2, -\sqrt{3}/2)$ as shown in Fig. 5(c). The switching time duration of the voltage vector is determined by using the volt-sec balancing principle [8]. There is no mapping is used in the proposed work. The equations for switching time duration (T_1 and T_2) of the voltage vectors for two legged 3-level inverter is given in Table I. The sampling time period for the space vector modulation is denoted as T_s and it is equal to the sum of T_1 , T_2 and T_0 . Hence the equation for T_0 is obtained by,

$$T_0 = T_s - T_1 - T_2 \quad (3)$$

From the computed T_0 , T_1 and T_2 , the inverter leg switching time for phase A and B (denoted as T_{ga} and T_{gb}) are computed as given in Table I.

TABLE I
SWITCHING TIME EQUATIONS FOR 3-LEVEL INVERTER

Sector	T_1	T_2	T_{ga}	T_{gb}
R ₁	$\left[2V_\alpha - \frac{2}{\sqrt{3}}V_\beta\right] T_s$	$\frac{4}{\sqrt{3}}V_\beta T_s$	T_1+T_2	T_2
R ₂	$\left[2V_\alpha - \frac{2}{\sqrt{3}}V_\beta - 1\right] T_s$	$\left[\frac{4}{\sqrt{3}}V_\beta + 1\right] T_s$	T_1+T_2	T_2
R ₃	$\left[-2V_\alpha + \frac{2}{\sqrt{3}}V_\beta + 1\right] T_s$	$\left[2V_\alpha + \frac{2}{\sqrt{3}}V_\beta\right] T_s$	T_2	T_1+T_2
R ₄	$\left[2V_\alpha - \frac{2}{\sqrt{3}}V_\beta\right] T_s$	$\left[\frac{4}{\sqrt{3}}V_\beta T_s + 1\right]$	T_1+T_2	T_2
R ₅	$\left[-2V_\alpha + \frac{2}{\sqrt{3}}V_\beta\right] T_s$	$\left[2V_\alpha + \frac{2}{\sqrt{3}}V_\beta + 1\right] T_s$	T_2	T_1+T_2
R ₆	$\left[-2V_\alpha + \frac{2}{\sqrt{3}}V_\beta - 1\right] T_s$	$\left[2V_\alpha + \frac{2}{\sqrt{3}}V_\beta + 1\right] T_s$	T_2	T_1+T_2
R ₇	$\left[2V_\alpha - \frac{2}{\sqrt{3}}V_\beta + 1\right] T_s$	$\frac{4}{\sqrt{3}}V_\beta T_s$	T_1+T_2	T_2
R ₈	$\left[-2V_\alpha + \frac{2}{\sqrt{3}}V_\beta\right] T_s$	$\left[2V_\alpha + \frac{2}{\sqrt{3}}V_\beta\right] T_s$	T_2	T_1+T_2

IV. SIMULATION RESULTS

To verify the validity of the proposed algorithm for generation of SVPWM, simulation is carried out in MATLAB/SIMULINK for two legged three phase inverter fed Permanent Magnet Synchronous Motor (PMSM) drive with a DC link voltage of 400V. Fig.6 shows the simulink model of SVPWM generation for two legged multilevel inverter fed PMSM. Fig. 7 presents the simulation results of two legged inverter for 3-level and 5-level operations with phase C clamped to 200V. T_{ga} represents the inverter leg switching time waveform and are shown in Fig. 7(a). Fig. 7(b) shows the inverter pole voltage corresponding to 400V operation. For the 3-level operation (V_{a0}) the inverter switches between the levels of 150V, 200V and 250V. The levels of pole voltage for 5-level operations are 0V, 100V, 200V, 300V and 400V. The motor phase voltage and current are also shown in Fig. 7(c) and Fig. 7(d) respectively.

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Simulation parameters are:

Fundamental frequency	50 Hz
Switching frequency	10 kHz
PMSM parameters:	
Stator resistance, R_s	0.7 Ω
Stator inductance, L_s	2.72mH
Moment of inertia, J	0.0002 kg.m ²
Damping coefficient, B	0.002 N.m.s.rad ⁻¹
Back-emf constant, K_b	0.5128 V/rad/sec
Rated speed, ω_r	3000 rpm

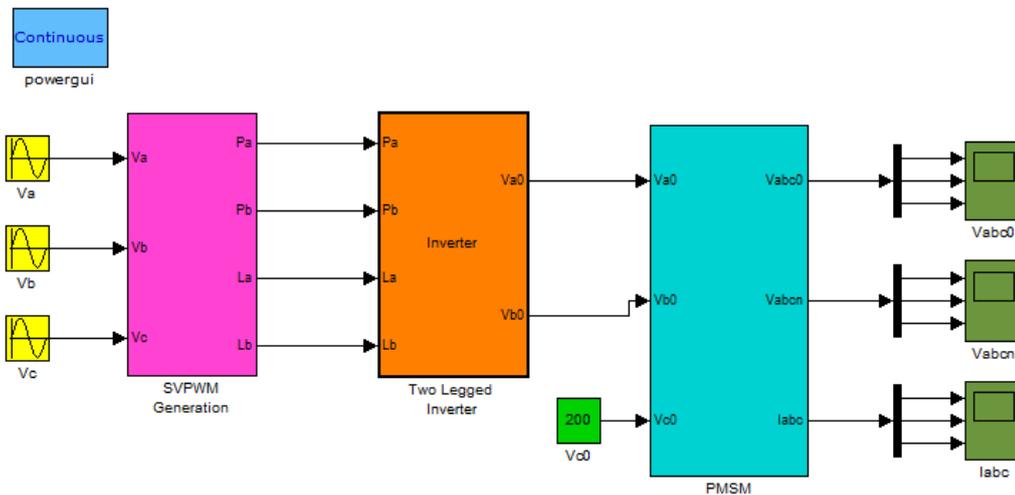
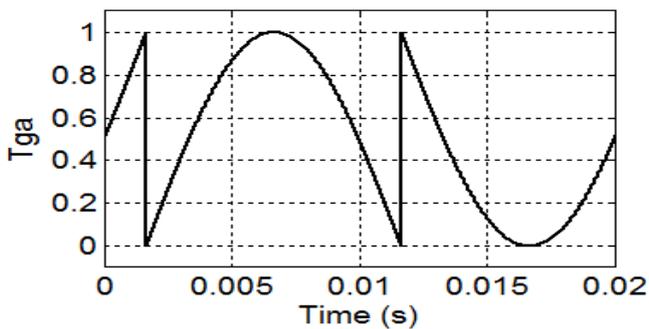
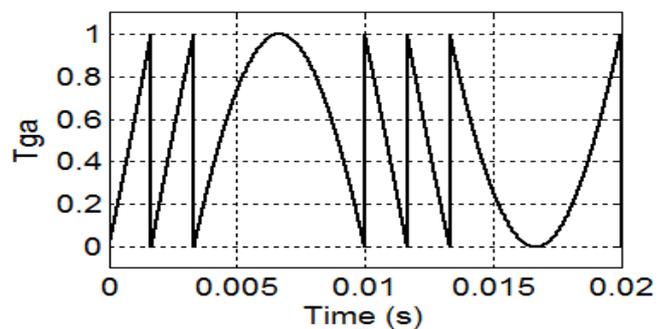


Fig. 6 Simulink model of SVPWM generation for two legged three phase inverter fed PMSM

For 3-Level Operation



For 5-Level Operation

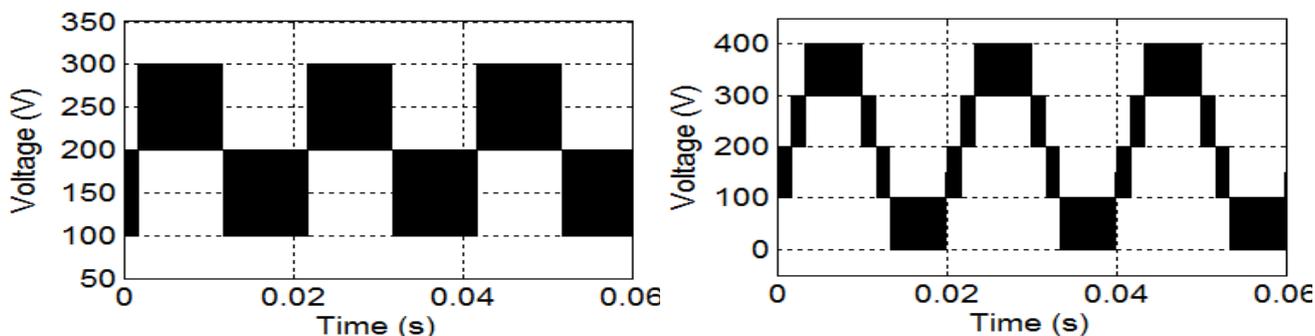


(a) Gating signal waveform

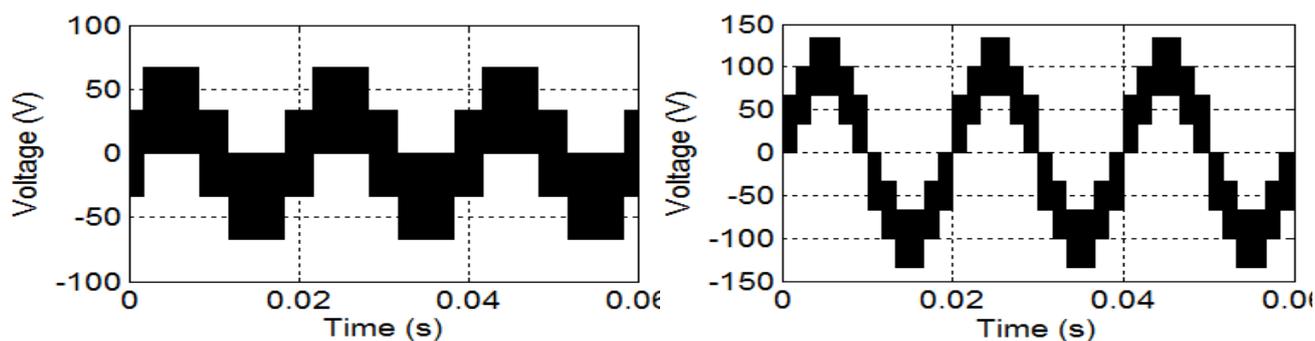
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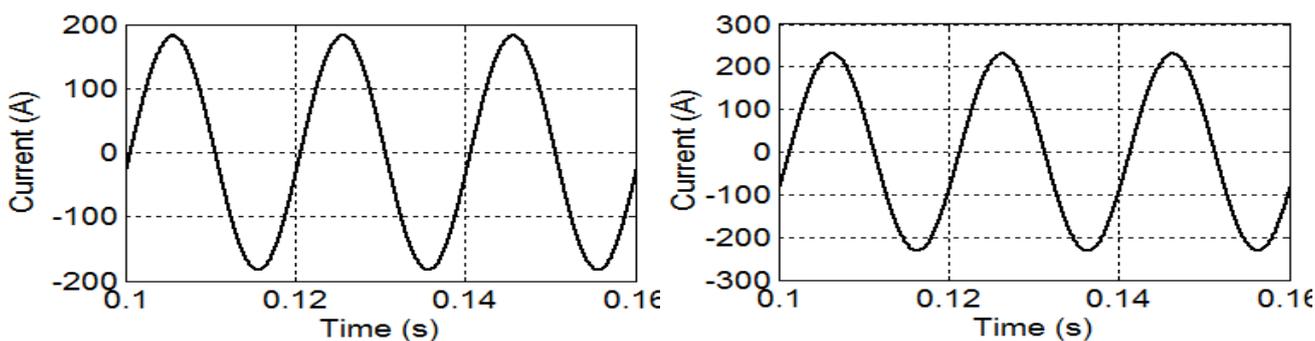
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(b) Pole Voltage waveform corresponding to 400V operation



(c) Phase Voltage waveform for the PMSM motor



(d) Current waveform for the PMSM motor

Fig. 7 Simulation results of two legged inverters (3-level and 5-level configurations)

V. CONCLUSION

This paper presents the fractal structure in the voltage representation of the two legged multilevel inverter and utilizing the fractal structure the paper proposes a method for the SVPWM generation for the n-level two legged inverter. The proposed work involves triangularization algorithm for the generation of the SVPWM and thus it simplify the implementation process. No need of look up table for the sector identification. The basic unit of the fractal structure of the two legged multilevel inverter is the triangle and each triangle is undergoes repeated division for higher



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level inverters. The implementation algorithm is explained for 5-level inverter and simulation results are presented for 3-level and 5-level inverter configurations. The proposed work can be easily extended for higher level two legged inverters.

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