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FREE VIBRATION ANALYSIS OF THIN ISOTROPIC RECTANGULAR PLATE

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Abstract: For free vibration response of rectangular plate, Navier's analytical solution is available only for all edges simply supported (SSSS) plate. But for plate with different boundary conditions such as plate with all edges clamped (CCCC), plate with boundary conditions such as CCCF, CCFF and CFFF solutions are not available. In this paper, an available Modified Discrete Kirchhoff Quadrilateral (MDKQ) element is developed based on Classical Plate Theory (CPT) using discrete Kirchhoff technique. Earlier it is used only for static analysis of plate and no attempt is made to use it for free vibration response. The performance of the developed finite element formulation is assessed for free vibration response of thin isotropic rectangular plate with various boundary conditions and different aspect ratios. A computer program has been developed in FORTRAN using MDKQ interpolation functions for free vibration analysis of thin isotropic rectangular plate. The developed MDKQ element has 3 degrees of freedom at each node. Hamilton's principle is used to formulate the stiffness and consistent mass matrices. It is observed that results of MDKQ element are quite close to the approximate analytical and finite element solutions available in the literature.

Keywords: Modified Kirchhoff quadrilateral element; classical plate theory

I. INTRODUCTION

Now days in modern structures, structural components are many times modeled as plates. Such models are widely used in the field of civil, mechanical, aerospace, marine and automobile engineering. Plates and their differential characteristics enable engineers to design better and lighter structures. Determination of dynamic response of these structures is needed when these are subjected to external complicated dynamic loads such as earthquake, wind, impact and wave forces. Determination of natural frequencies is the major step of dynamic analysis. Analytical solutions for dynamic response of plates are available for very few cases i.e. for plate with simple geometry and boundary conditions. But for plate with complex geometry and boundary conditions, solutions are possible only with the help of numerical methods. The most commonly used numerical method for the analysis of plate is Finite Element Method (FEM). The basic requirement of a finite element method is selection of computationally efficient element based on some suitable theory like classical plate theory. In this paper, a four node available Modified Discrete Kirchhoff Quadrilateral (MDKQ) element is developed based on classical plate theory using smoothed shape functions for free vibration analysis of thin isotropic rectangular plate. The above element is then assessed for its computational efficiency in comparison with approximate analytical and finite element solutions available in the literature.

II. LITERATURE REVIEW

Availability of simple, efficient and reliable elements for thin plate is a need of structural analysis. For that available MDKQ element is developed [1] based on classical plate theory for free vibration analysis of plate using least square smoothing technique of interpolation functions [2,3]. It is earlier used only for static analysis of plate and no attempt is made to use it for free vibration response. Leissa et al. [4] employed the Ritz method to carry out free vibration analysis of thin rectangular plate for different aspect ratios and for different boundary conditions. Liew et al. [5] used Rayleigh-Ritz method to carry out transverse vibration of thick rectangular plates using various comprehensive sets of boundary conditions. Mihir Chandra Manna [6] carried out the vibration analysis of isotropic plates with different thickness ratios, different boundary conditions and different

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aspect ratios using a 24 node high-order triangular element. The element has 18 nodes on the external sides and six internal nodes. It has 51 degrees of freedom, which can be reduced to 39 degrees of freedom by Guyan reduction. Here First Order Shear Deformation Theory (FOSDT) is used to include the effect of transverse shear deformation. Kerboua et al. [7] used a semi-analytical approach for which a mathematical model is developed using a hybrid combination of the finite element method and Sander's shell theory for various boundary conditions. The results obtained in terms of natural frequencies using a four node discrete Kirchhoff quadrilateral element for free vibration analysis of thin isotropic rectangular plate for different boundary conditions and different aspect ratios are compared with the approximate analytical and finite element solutions available in the literature.

III. FINITE ELEMENT FORMULATION

For the present MDKQ [5] element the finite element formulation is done and is modified for its use in the free vibration analysis of isotropic thin rectangular plate based on the classical plate theory. This element has three degrees of freedom at each node namely transverse displacement , rotations of normals and at the mid surface. The equation of motion and the variationally consistent boundary conditions are derived using the Hamilton's principle. The rotation variables are interpolated over the element using interpolation functions as given in [1]. The consistent mass matrix and the element stiffness matrix are derived from the Hamilton's variational principle.

A. Hamilton's Principle -

Using the notation – $\langle \ldots \rangle =$ Hamilton's principle for the isotropic thin plates can be expressed as-

$$\int_A [\langle \rho \delta u^T \ddot{u} + \rho \delta \omega_0 \ddot{\omega} \rangle] dA + \int_A [\langle \delta \varepsilon^T \sigma \rangle - p_z \delta \omega_0 (x, y, t)] dA$$

$$- \int_{\Gamma_L} \langle \sigma_n \delta u_n + \tau_n \qquad (1)$$

Where, is the mass density of the material, "A" denotes the mid-plane surface area of the plate and " denotes the normal force per unit area applied on the mid-surface of the plate.

To avoid the problem of C^1-continuity the improved discrete Kirchhoff constraint approach is used in which and are replaced by rotation variables and , which then require only C^0-continuity. and are interpolated in-dependently, but the two are subsequently related by imposing the constraints at discrete points on the element boundary. The rotations and are originally interpolated in terms of their values and at the eight nodes $(i = 1, 2, \ldots\ldots$ of a Serendipity element, i.e., nodes 1 to 4 at the corners and nodes 5 to 8 at the mid sides.

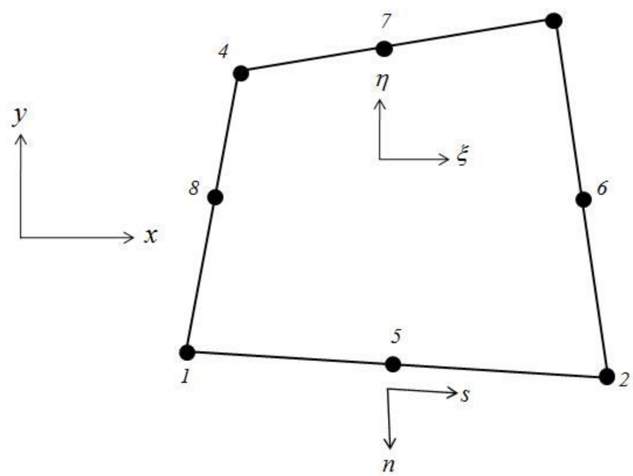

Figure1. Eight nodes serendipity element

and are finally interpolated as as,

$\theta_{0x} = \omega_{0,x} = H^x \omega_e^0$

θ_{0y} (2)

Where, $H_i^x = [H_x^1$

and $H_i^y = [H_y^1$.

For computing the mass matrix, the interpolation function of is required. Hence the deflection is interpolated assuming a bicubic function in terms of parametric co-ordinates and as –

$$\omega_0 = \alpha_1 + \alpha_2\gamma + \alpha_3\eta + \alpha_4\gamma^2 + \alpha_5\gamma\eta + \alpha_6\eta^2 + \alpha_7\gamma^3 + \alpha_8\gamma^2\eta + \alpha_9\gamma\eta^2 + \alpha_{10}\eta^3 + \alpha_{11}\gamma^3\eta + \alpha_1 \qquad (3)$$

The coefficients are evaluated from the system of twelve equations obtained by linking the values of , and at the four corner nodes when the co-ordinates take their appropriate values. Thus, is given by-

(4)

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B. Generalized Strains-

The generalized displacement vector is defined in terms of nodal displacement vector,

$$U^{eT} = [U_1^{eT} \quad (5)$$

(6)

and can be expressed as,

(7)

(8)

Where-

$$N_u = [N_u^1 \quad N_u^2 \quad N_u^3 \quad N_u^4]\, U^e$$

$$N_\omega = [N_\omega^1 \quad N_\omega^2 \quad N_\omega^3 \quad N_\omega^4]\, U^e$$

$$N_u^i = \begin{bmatrix} -H_{i\,2i-2}^{x} & -H_{i\,2i-1}^{x} & -H_{i\,2i}^{x} \\ -H_{i\,2i-2}^{y} & -H_{i\,2i-1}^{y} & -H_{i\,2i}^{y} \end{bmatrix}$$

$$N_\omega^i = [\tilde{N}_{2i-2} \quad \tilde{N}_{2i-1} \quad \tilde{N}_{2i}]$$

The generalized strain is expressed in terms of as-

(9)

Where, is the generalized strain-displacement matrix.

C. Element Inertia and Stiffness Matrices

Substituting equation (7), (8) and (9) into Hamilton's variational equation (1) we get the consistent mass matrix ( ), element stiffness matrix ( ) and the load vector ( ) as follows –

$$M^e = \int_{A^e} [N_u^T\, I\, N_u + N_\omega^T\, \bar{I}\, N_\omega]\, dxdy$$

$$K^e = \int_{A^e} [\, B_{\kappa 0}^T\, \bar{D}\, B_{\kappa 0}]\, dxdy$$

$$P^e = \int_{A^e} N_\omega^T\, p_z\, dx\, dy$$

Substituting the contribution of all the elements in equation (1), since the virtual displacements are arbitrary, Equation (1) yields to- , where M, K and P are the assembles counterparts of matrices . As we are considering un-damped free vibration case, load vector is zero and thus the above equation reduces to- , which is the required equation from which the generalised eigen value problem is solved to obtain the natural frequencies

IV NUMERICAL RESULTS-

A. Material properties-

The properties of the material used =1, thickness (h) = 1 and = 0.3. The results for the natural circular frequency are non-dimensionalised as-

Where flexural rigidity D of the plate is given by,

B. Results –

Here results for first two fundamental frequencies for free vibration of thin isotropic plate for plate with boundary conditions CCCC, CCCF, CCFF and CFFF and aspect ratios (b/a) 1 and 2.5 are shown. The element results are obtained with 8×8 and 16×16 mesh sizes and compared with the results available in the literature. It is observed that as the mesh is made finer results are converged.

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TABLE 1. Natural frequencies for rectangular plates

B.C.	Aspect Ratio	Mode			MDKQ ELEMENT	
			Manna[6]	Kerboua et al[7].	8×8	16×16
CCCC	1	1	3.6460	3.5450	3.5685	3.6259
		2	7.4360	7.2030	7.2337	7.3816
			Manna[6]	Liew et al.[5]	8×8	16×16
	2.5	1	2.3960	2.3960	2.3560	2.3853
		2	2.8180	2.8180	2.6751	2.7796
CCCF			Kerboua et al.[7]	Leissa. et al[4]	8×8	16×16
	1	1	2.3940	2.4020	2.4184	2.4219
		2	3.9490	4.0030	3.9626	4.0290
	2.5	1	2.2530	2.2570	2.2500	2.2548
		2	2.4120	2.4620	2.4316	2.4765
CCFF	1	1	0.6920	0.6940	0.6997	0.7007
		2	2.3960	2.4030	2.4186	2.4208
	2.5	1	0.3960	0.3980	0.4008	0.4021
		2	0.7100	0.7150	0.7150	0.7204
CFFF	1	1	0.3470	0.3490	0.3514	0.3516
		2	0.8510	0.8524	0.8598	0.8613
	2.5	1	0.3490	0.3510	0.3538	0.3542
		2	0.4750	0.4780	0.4803	0.4823

IV. CONCLUSIONS

1. The MDKQ element for thin isotropic plate is suitable for the general purpose finite element programming.
2. For different boundary conditions the performance appears to be excellent in comparison with the approximate analytical and finite element results available in the literature.
3. It is observed that as the number of free edges increases i.e. from CCCC, CCCF, CCFF and CFFF the natural frequencies decreases for all the cases studied.

It is also seen that as the aspect ratio increases the natural frequency decreases for all cases studied in the work.

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