Gain- Function of Two Dissimilar Warm Standby Systems Subject To Failure Due To Voltage Fluctuations and Non-Availability of Water Resulting Failure of Hydroelectric Power with Switch Failure

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ABSTRACT: The energy in the flowing water can be used to produce electricity. Hydro power is one of the best, cheapest, and cleanest sources of energy. New environmental laws affected by the danger of global warming have made energy from small hydropower plants more relevant. The mathematical theory of reliability has grown out of the demands of modern technology and particularly out of experiences in World War II (1939-1945) with complex military systems although the concept of reliability is as old as man himself. In the present paper we have taken two-non-identical warm standby system with failure time distribution as exponential and repair time distribution as general. The Role of voltage fluctuations and non-availability of water resulting failure of hydroelectric power under which the system operates plays significant role on its working. We are considering system under (i) voltage fluctuations and (ii) Non-availability of water resulting failure of hydroelectric power causing different types of failure requiring different types of repair facilities. Using semi Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and gain-function and graphs are drawn.

KEYWORDS: warm standby, voltage fluctuations, non-availability of water resulting failure of hydroelectric power, switches failure.

I. INTRODUCTION

Two-unit standby system subject to environmental conditions such as shocks, change of weather conditions etc. have been discussed in reliability literature by several authors due to significant importance in defenses , industry etc.

II. MATERIALS AND METHODS

A random phenomenon arising through a process which developed in time in a manner controlled by probabilistic laws is called stochastic process. It is called Markovian if the future development is completely determined by the present state and is independent of the way in which state has been achieved. Mathematically, for

$t_0 < t_1 < t_2 < \dots\dots\dots\dots\dots < t_n < t_{n+1} < \dots\dots\dots\dots$

$Pr\{ t_{n+1} = j \mid t_n = i, \dots\dots\dots, \quad t_0 = i_0 \} = Pr\{ t_{n+1} = j \mid t_n = i, \}$

where t_n is the state of the process at the nth transition n= 0,1,2,…………..

A time point at which the system history prior to the time point, is irrelevant to the system conditions is called regenerative point . A process in which each point is a regenerative point is called a regenerative process.

Let $T_0, T_1, T_2, \dots\dots\dots$ denote the epochs at which the system enters any state i belonging to set of regenerative states . Let Z_n denote the state visited at epoch $T_n +$, i.e., just after the transition at T_n. Then { Z_n , T_n } is a Markov renewal process. We have applied the regenerative point technique with renewal process theory

III.ASSUMPTIONS

1.The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.

2.The repair facility is of four types :

Type I, II repair facility - When failure due to voltage fluctuations and non-availability of water resulting failure of hydroelectric power of first unit occurs respectively

Type III, IV repair facility - When failure due to voltage fluctuations and non-availability of water resulting failure of hydroelectric power of the second unit occurs respectively.

3.The repair starts immediately upon failure of units and the repair discipline is FCFS.

4.The repairs are perfect and start immediately as soon as the voltage fluctuations and non-availability of water resulting failure of hydroelectric power of the system becomes normal. The voltage fluctuations and non-availability of water in both the units do not go steep high simultaneously.

5.The failure of a unit is detected immediately and perfectly.

6.The switches are perfect and instantaneous.

7.All random variables are mutually independent.

IV. SYMBOLS FOR STATES OF THE SYSTEM

Superscripts   O, WS, SO, FVF, FNAW, SFO

Operative, Warm Standby, Stops the operation, failure due to voltage fluctuations, failure due to non-availability of water resulting failure of hydroelectric power, Switch failed but operable respectively

Subscripts   nvf, vf, nawf, ur, wr, uR

No voltage fluctuations, voltage fluctuations, non-availability of water resulting failure of hydroelectric power, under repair, waiting for repair, under repair continued respectively

Up states – 0,1,2,9;   Down states – 3, 4, 5, 6, 7, 8, 10, 11;   Regeneration point – 0,1,2,4,7,10

V.STATES OF THE SYSTEM

$0(O_{nvf}, WS_{nvf})$ One unit is operative and the other unit is warm standby and there are no voltage fluctuations and no non-availability of water resulting failure of hydroelectric power in both the units.

$1(SO_{nvf}, O_{nvf})$ The operation of the first unit stops automatically due to voltage fluctuations and warm standby unit's starts operating with no voltage fluctuations.

$2(FVF_{ur}, O_{nvf})$ The first unit fails and undergoes repair after the voltage fluctuations are over and the other unit continues to be operative with no voltage fluctuations.

$3(FVF_{uR}, SO_{uvf})$ The repair of the first unit is continued from state 2 and the operation of second unit stops automatically due to voltage fluctuations.

$4(FVF_{ur}, SO_{nawf})$ The first unit fails and undergoes repair after the voltage fluctuations are over and the other unit also stops automatically due to non-availability of water resulting failure of hydroelectric power.

$5(FVF_{uR}, FNAW_{nawf,wr})$ The repair of the first unit is continued from state 4 and the other unit is failed due to non-availability of water resulting failure of hydroelectric power in it & is waiting for repair.

$6(O_{nvf}, FNAW_{ur})$ the first unit becomes operative with no voltage fluctuations and the second unit is failed due to high non-availability of water is under repair.

$7(SO_{nawf}, SFO_{nvf, ur})$ The operation of the first unit stops automatically due to non-availability of water resulting failure of hydroelectric power and during switchover to the second unit switch fails and undergoes repair and there is no voltage fluctuations.

$8(FNAW_{nawf, wr}, SFO_{nvf, uR})$ The repair of failed switch is continued from state 7 and the first unit is failed after non-availability of water resulting failure of hydroelectric power is waiting for repair.

$9(O_{nvf}, SO_{nawf})$ The first unit is operative with no voltage fluctuations and the operation of warm standby second unit is stopped automatically due to non-availability of water resulting failure of hydroelectric power.

10(SO$_{nawf}$, SF$_{ur}$) The operation of the first unit stops automatically due to non-availability of water resulting failure of hydroelectric power and the second unit switch fails and undergoes repair after the non-availability of water resulting failure of hydroelectric power is over.

11(FNAW$_{nawf, wr}$, SF$_{uR}$) The repair of the second unit is continued from state 10 and the first unit is failed due to non-availability of water resulting failure of hydroelectric power is waiting for repair.

VI.TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions :

$$p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \qquad , \qquad P_{07} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{09} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \qquad , \qquad p_{12} = \frac{\lambda_1}{\lambda_1 + \lambda_3} \qquad , \qquad p_{14} = \frac{\lambda_3}{\lambda_1 + \lambda_3}$$

where

$$P_{20} = G_1^*(\lambda_1) \; , \; P_{22}^{(3)} = G_1^*(\lambda_1) = p_{23} \;\;, \;\; P_{72} = G_2^*(\lambda_4) \; , \; P_{72}^{(8)} = G_2^*(\lambda_4) = P_{78}$$
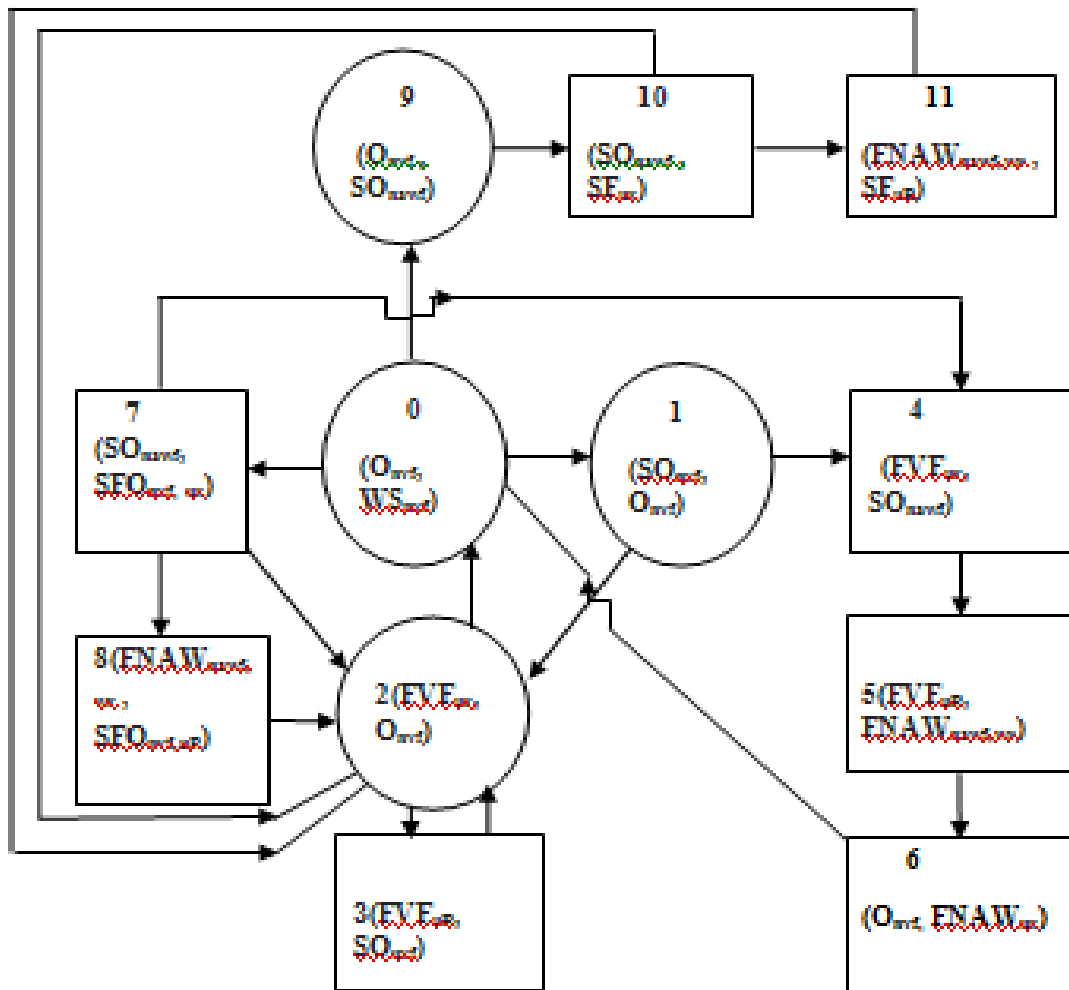

Fig.1 The State Transition Diagram

We can easily verify that

$p_{01} + p_{07} + p_{09} = 1$, $p_{12} + p_{14} = 1$, $p_{20} + p_{23}(=p_{22}^{(3)}) = 1$, $p_{46}^{(6)} = 1$ $p_{60} = 1$,
$p_{72} + P_{72}^{(5)} + p_{74} = 1$, $p_{9,10} = 1$, $p_{10,2} + p_{10,2}^{(11)} = 1$ 　　　　　　　　　　　　　(1)

And mean sojourn time is

$$\mu_0 = E(T) = \int_0^\infty P[T > t]\,dt \qquad (2)$$

VII. MEAN TIME TO SYSTEM FAILURE

We can regard the failed state as absorbing

$$\theta_0(t) = Q_{01}(t)[s]\theta_1(t) + Q_{09}(t)[s]\theta_9(t) + Q_{07}(t)$$
$$\theta_1(t) = Q_{12}(t)[s]\theta_2(t) + Q_{14}(t), \ \theta_2(t) = Q_{20}(t)[s]\theta_0(t) + Q_{22}^{(3)}(t)$$
$$\theta_4(t) = Q_{9,10}(t) \qquad (3\text{-}5)$$

Taking Laplace-Stiltjes transform of eq. (3-5) and solving for

$$Q_0^*(s) \quad = \quad N_1(s) \ / \ D_1(s) \qquad (6)$$

where

$N_1(s) = Q_{01}^*(s) \ \{ \ Q_{12}^*(s) \ Q_{22}^{(3)*}(s) + Q_{14}^*(s) \} + Q_{09}^*(s) \ Q_{9,10}^*(s) + Q_{07}^*(s)$
$D_1(s) = 1 - Q_{01}^*(s) \quad Q_{12}^*(s) \ Q_{20}^*(s)$

Making use of relations (1) & (2) it can be shown that $\theta_0^*(0) = 1$, which implies that $\theta_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = d/ds \ \theta_0^*(s) \Big|_{s=0} = \ (D_1'(0) - N_1'(0)) \ / \ D_1(0)$$

$$= \ (\mu_0 + p_{01} \ \mu_1 + p_{01} \ p_{12} \ \mu_2 + p_{09} \ \mu_9) \ / \ (1 - p_{01} \ p_{12} \ p_{20})$$

where

$$\mu_0 = \mu_{01} + \mu_{07} + \mu_{09}, \quad \mu_1 = \mu_{12} + \mu_{14}, \ \mu_2 = \mu_{20} + \mu_{22}^{(3)}, \quad \mu_9 = \mu_{9,10}$$

VIII. AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state I is up at time t without making any other regenerative state . By probabilistic arguments, we have

The value of $M_0(t)$, $M_1(t)$, $M_2(t)$, $M_4(t)$ can be found easily.

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{07}(t)[c]A_7(t) + q_{09}(t)[c]A_9(t)$
$A_1(t) = M_1(t) + q_{12}(t)[c]A_2(t) + q_{14}(t)[c]A_4(t)$, $A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + q_{22}^{(3)}(t)[c]A_2(t)$
$A_4(t) = q_{46}^{(3)}(t)[c]A_6(t)$, $A_6(t) = q_{60}(t)[c]A_0(t)$
$A_7(t) = (q_{72}(t) + q_{72}^{(8)}(t)) \ [c]A_2(t) + q_{74}(t)[c]A_4(t)$
$A_9(t) = M_9(t) + q_{9,10}(t)[c]A_{10}(t)$, $A_{10}(t) = q_{10,2}(t)[c]A_2(t) + q_{10,2}^{(11)}(t)[c]A_2(t)$
(7-14)

Taking Laplace Transform of eq. (7-14) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) \quad = \quad N_2(s) \ / \ D_2(s) \qquad (15)$$

where

$N_2(s) = (1 - \hat{q}_{22}^{(3)}(s)) \ \{ \hat{M}_0(s) + \hat{q}_{01}(s) \ \hat{M}_1(s) + \hat{q}_{09}(s) \ \hat{M}_9(s)\} + \hat{M}_2(s)\{ \hat{q}_{01}(s) \ \hat{q}_{42}(s) + \hat{q}_{07}(s)(\hat{q}_{72}(s) + \hat{q}_{73}^{(8)}(s)) + \hat{q}_{09}(s) \ \hat{q}_{9,10}(s)(\hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s))\}$

$D_2(s) = (1 - \hat{q}_{22}^{(3)}(s)) \ \{ 1 - \hat{q}_{46}^{(5)}(s) \ \hat{q}_{60}(s) \ (\hat{q}_{01}(s) \ \hat{q}_{44}(s) + \hat{q}_{07}(s) \ \hat{q}_{74}(s))$

$\quad - \hat{q}_{20}(s)\{ \hat{q}_{01}(s) \ \hat{q}_{12}(s) + \hat{q}_{07}(s)(\hat{q}_{72}(s)) + \hat{q}_{72}^{(8)}(s) + \hat{q}_{09}(s) \ \hat{q}_{9,10}(s) \ (\hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s))\}$

The steady state availability

$A_0 = \lim_{t \to \infty}[A_0(t)] \quad = \lim_{s \to 0}[s\,\hat{A}_0(s)] \quad = \lim_{s \to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s \to 0}\frac{N_2(s) + s\,N_2{'}(s)}{D_2{'}(s)} = \frac{N_2(0)}{D_2{'}(0)}$ 　　　　　　　　　　　　　　(16)

Where

$N_2(0) = p_{20}(\hat{M}_0(0) + p_{01}\hat{M}_1(0) + p_{09}\,\hat{M}_9(0)) + \hat{M}_2(0)\ (p_{01}p_{12} + p_{07}(p_{72} + p_{72}^{(8)} + p_{09}))$

$D_2{'}(0) = p_{20}\{\ \mu_0 + p_{01}\,\mu_1 + (p_{01}\,p_{14} + p_{07}\,p_{74})\,\mu_4 + p_{07}\,\mu_7 + p_{07}\,\mu_7 + p_{09}(\mu_9 + \ \mu_{10}) +$

$\qquad \mu_2\ \{\ 1 - ((p_{01}p_{14} + p_{07}\,p_{74})\}$

$\mu_4 = \mu_{46}^{(5)},\ \mu_7 = \mu_{72} + \mu_{72}^{(8)} + \mu_{74},\quad \mu_{10} = \mu_{10,2} + \mu_{10,2}^{(11)}$

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^\infty A_0(z)dz$ 　　So that $\overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ 　　　　　　(17)

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$ 　　So that $\overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s)$ 　　　　　　　　(18)

IX. THE EXPECTED BUSY PERIOD OF THE SERVER FOR REPAIRING THE FAILED UNIT UNDER NON-AVAILABILITY OF WATER RESULTING FAILURE OF HYDROELECTRIC POWER IN (0,T]

$R_0(t) = S_0(t) + q_{01}(t)[c]R_1(t) + q_{07}(t)[c]R_7(t) + q_{09}(t)[c]R_9(t)$

$R_1(t) = S_1(t) + q_{12}(t)[c]R_2(t) + q_{14}(t)[c]R_4(t)$,

$R_2(t) = q_{20}(t)[c]R_0(t) + q_{22}^{(3)}(t)[c]R_2(t)$

$R_4(t) = q_{46}^{(3)}(t)[c]R_6(t)$, $R_6(t) = q_{60}(t)[c]R_0(t)$

$R_7(t) = (q_{72}(t) + q_{72}^{(8)}(t))\ [c]R_2(t) + q_{74}(t)[c]R_4(t)$

$R_9(t) = S_9(t) + q_{9,10}(t)[c]R_{10}(t)$, $R_{10}(t) = q_{10,2}(t) + q_{10,2}^{(11)}(t)[c]R_2(t)$

(19-26)

Taking Laplace Transform of eq. (19-26) and solving for $\overline{R_0}(s)$

$\overline{R_0}(s) = N_3(s)\ /\ D_2(s)$ 　　　　　　　　　　　　　　　　　　　　　(27)

where

$N_2(s) = (1 - \hat{q}_{22}^{(3)}(s))\ \{\ \hat{S}_0(s) + \hat{q}_{01}(s)\,\hat{S}_1(s) + \hat{q}_{09}(s)\,\hat{S}_9(s)\}$ and $D_2(s)$ is already defined.

In the long run, 　　　　　$R_0 = \frac{N_3(0)}{D_2{'}(0)}$ 　　　　　　　　　　　　　　　　(28)

where $N_3(0) = p_{20}(\hat{S}_0(0) + p_{01}\hat{S}_1(0) + p_{09}\,\hat{S}_9(0))$ and $D_2{'}(0)$ is already defined.

The expected period of the system under non-availability of water resulting failure of hydroelectric power in (0,t] is

$\lambda_{rv}(t) = \int_0^\infty R_0(z)dz$ 　　So that $\overline{\lambda_{rv}}(s) = \frac{\hat{R}_0(s)}{s}$

X. THE EXPECTED BUSY PERIOD OF THE SERVER FOR REPAIRING THE FAILED UNITS UNDER VOLTAGE FLUCTUATIONS BY THE REPAIRMAN IN (0,T]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{07}(t)[c]B_7(t) + q_{09}(t)[c]B_9(t)$

$B_1(t) = q_{12}(t)[c]B_2(t) + q_{14}(t)[c]B_4(t)$, $B_2(t) = q_{20}(t)[c]\,B_0(t) + q_{22}^{(3)}(t)[c]B_2(t)$

$B_4(t) = T_4(t) + q_{46}^{(3)}(t)[c]B_6(t)$, $B_6(t) = T_6(t) + q_{60}(t)[c]B_0(t)$

$B_7(t) = (q_{72}(t) + q_{72}^{(8)}(t))\ [c]B_2(t) + q_{74}(t)[c]B_4(t)$

$B_9(t) = q_{9,10}(t)[c]B_{10}(t)$, $B_{10}(t) = T_{10}(t) + (q_{10,2}(t) + q_{10,2}^{(11)}(t)[c]B_2(t)$

(29- 36)

Taking Laplace Transform of eq. (29-36) and solving for $\overline{B_0}(s)$

$$\overline{\overline{B_0}}(s) = N_4(s) / D_2(s) \tag{37}$$

where

$N_4(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ \ \hat{q}_{01}(s) \ \hat{q}_{14}(s)(\ \overline{\hat{T}}_4(s) + \hat{q}_{46}^{(5)}(s) \ \overline{\hat{T}}_6(s)) + \hat{q}_{07}^{(3)}(s) \ \ \hat{q}_{74}(s)(\ \overline{\hat{T}}_4(s)$

$\qquad + \ \hat{q}_{46}^{(5)}(s) \ \overline{\hat{T}}_6(s)) + \hat{q}_{09}(s) \ \hat{q}_{09,10}(s) \ \ \overline{\hat{T}}_{10}(s) \ )$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \dfrac{N_4(0)}{D_2{}'(0)}$ (38)

where $N_4(0) = p_{20} \{ (\ p_{01} \ p_{14} + \ p_{07} \ p_{74}) \ (\hat{T}_4(0) + \hat{T}_6(0)) + p_{09} \ \hat{T}_{10}(0) \ \}$ and $D_2{}'(0)$ is already defined.

The expected busy period of the server for repair in (0,t] is

$$\lambda_{ru}(t) = \int_0^\infty B_0(z)\,dz \quad \text{So that} \quad \overline{\lambda_{ru}}(s) = \frac{\hat{B}_0(s)}{s} \tag{39}$$

XI. THE EXPECTED BUSY PERIOD OF THE SERVER FOR REPAIR OF SWITCH IN (O,T]

$P_0(t) = q_{01}(t)[c]P_1(t) + q_{07}(t)[c]P_7(t) + q_{09}(t)[c]P_9(t)$

$P_1(t) = q_{12}(t)[c]P_2(t) + q_{14}(t)[c]P_4(t) , \ P_2(t) = q_{20}(t)[c]P_0(t) + q_{22}^{(3)}(t)[c]P_2(t)$

$P_4(t) = q_{46}^{(3)}(t)[c]P_6(t) , \quad P_6(t) = q_{60}(t)[c]P_0(t)$

$P_7(t) = L_7(t) + (q_{72}(t) + q_{72}^{(8)}(t)) \ [c]P_2(t) + q_{74}(t)[c]P_4(t)$

$P_9(t) = q_{9,10}(t)[c]P_{10}(t) , \quad P_{10}(t) = (q_{10,2}(t) + q_{10,2}^{(11)}(t))[c]P_2(t) \quad$ (40-47)

Taking Laplace Transform of eq. (40-47) and solving for

$$\overline{\overline{P_0}}(s) = N_5(s) / D_2(s) \tag{48}$$

where $N_2(s) = \hat{q}_{07}(s) \ \hat{L}_7(s) \ (1 - \hat{q}_{22}^{(3)}(s))$ and $D_2(s)$ is defined earlier.

In the long run, $P_0 = \dfrac{N_5(0)}{D_2{}'(0)}$ (49)

where

$N_5(0) = p_{20} \ p_{07} \ \hat{L}_4(0)$ and $D_2{}'(0)$ is already defined.

The expected busy period of the server for repair of the switch in (0,t] is

$$\lambda_{rs}(t) = \int_0^\infty P_0(z)\,dz \quad \text{So that} \quad \overline{\lambda_{rs}}(s) = \frac{\hat{P}_0(s)}{s} \tag{50}$$

XII. THE EXPECTED NUMBER OF VISITS BY THE REPAIRMAN FOR REPAIRING THE DISSIMILAR UNITS IN (0,T]

$H_0(t) = Q_{01}(t)[c]H_1(t) + Q_{07}(t)[c]H_7(t) + Q_{09}(t)[c]H_9(t)$

$H_1(t) = Q_{12}(t)[c][1+H_2(t)] + Q_{14}(t)[c][1+H_4(t)] , \ H_2(t) = Q_{20}(t)[c]H_0(t) + Q_{22}^{(3)}(t)[c]H_2(t)$

$H_4(t) = Q_{46}^{(3)}(t)[c]H_6(t) , \ H_6(t) = Q_{60}(t)[c]H_0(t)$

$H_7(t) = (Q_{72}(t) + Q_{72}^{(8)}(t)) \ [c]H_2(t) + Q_{74}(t)[c]H_4(t)$

$H_9(t) = Q_{9,10}(t)[c][1+H_{10}(t)] , \quad H_{10}(t) = (Q_{10,2}(t)[c] + Q_{10,2}^{(11)}(t))[c]H_2(t)$

$$(51\text{-}58)$$

Taking Laplace Transform of eq. (51-58) and solving for $H_0^*(s)$

$$H_0^*(s) = N_6(s) / D_3(s) \tag{59}$$

where

$N_6(s) = (1 - Q_{22}^{(3)*}(s)) \{ \ Q^*_{01}(s)(\ Q^*_{12}(s) + Q^*_{14}(s)) + \ Q^*_{09}(s) \ Q^*_{9,10}(s) \}$

$D_3(s) = (1 - Q_{22}^{(3)*}(s)) \{ \ 1 - \ (Q^*_{01}(s) \ Q^*_{14}(s) + \ Q^*_{07}(s) \ Q^*_{74}(s)) \ Q_{46}^{(5)*}(s) \ Q^*_{60}(s)\} \ - Q^*_{20}(s)\{ \ Q^*_{01}(s) \ Q^*_{12}(s) + Q^*_{07}(s)(\ Q^*_{72}(s)) \ + Q^*_{72}^{(8)}(s) \ + $

$\qquad Q^*_{09}(s) \ Q^*_{9,10}(s) \ (\ Q^*_{10,2}(s) + Q_{10,2}^{(11)*}(s))\}$

In the long run , $H_0 = \dfrac{N_6(0)}{D_3{}'(0)}$ (60)

where $N_6(0) = p_{20} (p_{01} + p_{09})$ and $D'_3(0)$ is already defined.

XIII. THE EXPECTED NUMBER OF VISITS BY THE REPAIRMAN FOR REPAIRING THE SWITCH IN (0,T]

$V_0(t) = Q_{01}(t)[c]V_1(t) + Q_{07}(t)[c]V_7(t) + Q_{09}(t)[c]V_9(t)$

$V_1(t) = Q_{12}(t)[c]V_2(t) + Q_{14}(t)[c]V_4(t)$, $V_2(t) = Q_{20}(t)[c]V_0(t) + Q_{22}^{(3)}(t)[c]V_2(t)$

$V_4(t) = Q_{46}^{(3)}(t)[c]V_6(t)$, $V_6(t) = Q_{60}(t)[c]V_0(t)$

$V_7(t) = (Q_{72}(t)[1+V_2(t)]+ Q_{72}^{(8)}(t))$ $[c]V_2(t) + Q_{74}(t)[c]V_4(t)$

$V_9(t) = Q_{9,10}(t)[c]V_{10}(t)$, $V_{10}(t) = (Q_{10,2}(t) + Q_{10,2}^{(11)}(t))[c]V_2(t)$ $\hspace{2cm}$ (61-68)

Taking Laplace-Stieltjes transform of eq. (61-68) and solving for $V_0^*(s)$

$\hspace{1.5cm} V_0^*(s) \hspace{1cm} = N_7(s) \ / \ D_4(s)$ $\hspace{4cm}$ (69)

where

$N_7(s) = Q^*_{07}(s) \ Q^*_{72}(s) \ (1 - Q_{22}^{(3)*}(s))$ and $D_4(s)$ is the same as $D_3(s)$

In the long run , $V_0 = \dfrac{N_7(0)}{D_4'(0)}$ $\hspace{4cm}$ (70)

where

$N_7(0) = p_{20} \ p_{07} \ p_{72}$ and $D'_3(0)$ is already defined.

XIV. GAIN- FUNCTION ANALYSIS

The gain- function of the system considering mean up-time, expected busy period of the system under voltage fluctuations when the units stops automatically, expected busy period of the server for repair of unit due to non-availability of water resulting failure of hydroelectric power and expected busy period of the server to repair the switch, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for switch failure.

The expected total cost-benefit incurred in (0,t] is

C(t) = Expected total revenue in (0,t] - expected total repair cost for switch in (0,t]

$\hspace{1.5cm}$ - expected total repair cost for repairing the units due to voltage fluctuations when the units automatically stop in (0,t]

$\hspace{1.5cm}$ - expected busy period of the system under non-availability of water resulting failure of hydroelectric power

$\hspace{1.5cm}$ - expected number of visits by the repairman for repairing the switch in (0,t]

$\hspace{1.5cm}$ - expected number of visits by the repairman for repairing of the dissimilar units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t \to \infty} (C(t)/t) \hspace{1cm} = \lim_{s \to 0} (s^2 C(s))$

$= K_1 A_0 \ - \ K_2 P_0 \ - \ K_3 B_0 \ - \ K_4 R_0 \ - \ K_5 V_0 \ - \ K_6 H_0$

Where

K_1 - revenue per unit up-time,

K_2 - cost per unit time for which the system is under switch repair,

K_3 - cost per unit time for which the system is under repair due to voltage fluctuations when units automatically stop,

K_4 - cost per unit time for which the system is under repair due to non-availability of water resulting failure of hydroelectric power,

K_5 - cost per visit by the repairman for which switch repair,

K_6 - cost per visit by the repairman for units repair.

XV. CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to voltage fluctuations, failure rate due to non-availability of water resulting failure of hydroelectric power increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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