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GENERALIZED DIFFERENCE PARANORMED SEQUENCE SPACE WITH RESPECT TO MODULUS FUNCTION AND ALMOST CONVERGENCE

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Abstract: The aim of the present paper is to introduce some new generalized difference sequence spaces with respect to modulus function involving strongly almost summable sequences. We give some topological properties and inclusion relations on these spaces.

INTRODUCTION

A sequence space is defined to be a linear space of real or complex sequences. Throughout the paper N, R and C denotes the set of non-negative integers, the set of real numbers and the set of complex numbers respectively. Let ω denote the space of all sequences (real or complex). Let l_∞ and c be Banach spaces of bounded and convergent sequences $x=\{x_n\}_{n=0}^{\infty}$ with supremum norm $\|x\|=\sup_n |x_n|$. Let T denote the shift operator on ω, that is, $Tx=\{x_n\}_{n=1}^{\infty}$, $T^2x=\{x_n\}_{n=2}^{\infty}$ and so on. A Banach limit L is defined on l_∞ as a non-negative linear functional such that L is invariant i.e., $L(S_x)=L(x)$ and L(e)=1, e=(1,1,1,…) [1].

Lorentz, called a sequence {xn} almost convergent if all Banach limits of x, L(x), are same and this unique Banach limit is called F-limit of x [1]. In his paper, Lorentz proved the following criterian for almost convergent sequences.

A sequence $x=\{x_n\}\in l_\infty$ is almost convergent with F-limit L(x) if and only if

$$\lim_{m\to\infty} t_{mn}(x)=L(x)$$

where, $t_{mn}(x)=\frac{1}{m}\sum_{j=0}^{m-1} T^j x_n, (T^0=0)$ uniformly in n≥0.

We denote the set of almost convergent sequences by f.

Several authors including Duran [2], Ganie et al. [3-7], King [8], Lorentz [1] and many others have studied almost convergent sequences. Maddox [9,10] has defined x to be strongly almost convergent to a number α if

$$\lim_n \frac{1}{n}\sum_{k=1}^{n}\left|x_{k+m}-\alpha\right|=0, \text{ uniformly in m.}$$

By [f] we denote the space of all strongly almost convergent sequences. It is easy to see that $c\subset f\subset[f]\subset {}_\infty$.

The concept of paranorm is related to linear matric spaces. It is a generalization of that of absolute value. Let X be a linear space. A function P:x→R is called a paranorm, if [11,12].

$(p.1)$ $p(0)\geq 0$

$(p.2)$ $p(x)\geq 0\ \forall\ x\in X$

$(p.3)$ $p(-x)=p(x)\ \forall\ x\in X$

$(p.4)$ $p(x+y)\leq p(x)+p(y)\ \forall\ x,y\in X$ (triangle inequality)

$(p.5)$ $if\ (\lambda_n)$ is a sequence of scalars with $\lambda_n\to\lambda$ (n→∞) and (x_n) is a sequence of vectors with $p(x_n-x)\to 0$ ($n\to\infty$), then $p(x_n\lambda_n-x\lambda)\to 0$ ($n\to\infty$),(continuity of multiplication of vectors).

A paranorm p for which p(x)=0 implies x=0 is called total. It is well known that the metric of any linear metric space is given by some total paranorm [10].

The following inequality will be used throughout this paper. Let p=(pk) be a sequence of positive real numbers with $0<p_k\leq \sup_k p_k=H<\infty$ and let $D=\max(1,2^{H-1})$. For $a_k,b_k\in\mathbb{C}$. We have that (Equation 1) [9,11].

$$|a_k+b_k|^{p_k}\leq D\{|a_k|^{p_k}+|b_k|^{p_k}.\} \quad (1)$$

Nanda defined the following [13,14]:

$$[f,p]=\left\{x:\lim_n\frac{1}{n}\sum_{k=1}^{n}\left|x_{k+m}-\alpha\right|^{p_k}=0 \text{ uniformly in m}\right\},$$

$$[f,p]_0=\left\{x:\lim_n\frac{1}{n}\sum_{k=1}^{n}\left|x_{k+m}\right|^{p_k}=0 \text{ uniformly in m}\right\},$$

$$[f,p]_\infty=\left\{x:\sup_{m,n}\frac{1}{n}\sum_{k=1}^{n}\left|x_{k+m}\right|^{p_k}<\infty\right\}.$$

The difference sequence spaces,

$X(\Delta)=\{x=(x_k):\Delta x\in X\}$,

where X= l_∞, C and C_0, were studied by Kizmaz [15].

It was further generalized by Ganie et al. [5], Et and Colak [16], Sengonul [17] and many others.

Further, it was Tripathy et al. [18] generalized the above notions and unified these as follows:

$\Delta_n^m x_k=\{x\in\omega:(\Delta_n^m x_k)\in Z\}$,

Where

$$\Delta_n^m x_k=\sum_{\mu=0}^{n}(-1)^\mu\binom{n}{r}x_{k+m\mu},$$

and
$\Delta_n^0 x_k = x_k \forall k \in \mathbb{N}.$

Recently, M. Et [19] defined the following:

$$[f,p](\Delta^r) = \left\{ x = (x_k) : \lim_n \frac{1}{n}\sum_{k=1}^{n}\left[f\left(\left|\Delta^r x_{k+m} - \alpha\right|\right)\right]^{p_k} = 0, \text{uniformly in m}\right\},$$

$$[f,p]_0(\Delta^r) = \left\{ x = (x_k) : \lim_n \frac{1}{n}\sum_{k=1}^{n}\left[f\left(\left|\Delta^r x_{k+m}\right|\right)\right]^{p_k} = 0, \text{uniformly in m}\right\},$$

$$[f,p]_\infty(\Delta^r) = \left\{ x = (x_k) : \sup_n \frac{1}{n}\sum_{k=1}^{n}\left[f\left(\left|\Delta^r x_{k+m}\right|\right)\right]^{p_k} < \infty, \text{uniformly in m}\right\}.$$

Following Maddox [20]and Ruckle [21], a modulus function g is a function from [0,∞) to [0,∞) such that

(i) g(x)=0 if and only if x=0,

(ii) $g(x+y) \le g(x) + g(y)\ \forall x, y \ge 0$

(iii) g is increasing,

(iv) g if continuous from right at x=0.

Maddox [10] introduced and studied the following sets:

$$f_0 = \{x \in \omega : \lim_n \frac{1}{n}\sum_{k=1}^{n} |x_{k+m}| = 0 \textit{ uniformly in m}\}$$

$$f = \{x \in \omega : x - le \in f_0 \textit{ for some in } l \in \mathbb{C}\}$$

of sequences that are strongly almost convergent to zero and strongly almost convergent.

Let p=(p_k) be a sequence of positive real numbers with $0 < p_k \le \sup_k p_k = M$ and H=max(1, M).

MAIN RESULTS:

In the present paper, we define the spaces $[f,g,p](\Delta_n^r), [f,g,p]_0(\Delta_n^r)$ and $[f,g,p]_\infty(\Delta_n^r)$ as follows:

$$[f,g,p](\Delta_n^r) = \left\{ x = (x_k) : \lim_n \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m} - \alpha\right|\right)\right]^{p_k} = 0, \text{uniformly in m}\right\},$$

$$[f,g,p]_0(\Delta_n^r) = \left\{ x = (x_k) : \lim_n \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} = 0, \text{uniformly in m}\right\},$$

$$[f,g,p]_\infty(\Delta_n^r) = \left\{ x : \sup_n \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} < \infty, \text{uniformly in m}\right\},$$

Where (p_k) is any bounded sequence of positive real numbers.

Theorem 1: Let (p_k) be any bounded sequence and g be any modulus function. Then $[f,g,p](\Delta_n^r), [f,,g,p]_0(\Delta_n^r)$ and $[f,,g,p]_\infty(\Delta_n^r)$ are linear space over the set of complex numbers.

Proof: We shall prove the result for $[f,g,p]_0(\Delta_n^r)$ and the others follows on similar lines. Let $x, y \in [f,g,p]_0(\Delta_n^r)$. Now for $\alpha, \beta \in \mathbb{C}$, we can find positive numbers Aα,B^β such that $|\alpha| \le A_\alpha$ and $|\beta| \le B_\beta$. Since f is subadditive and Δ_n^r is linear

$$\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r(\alpha x_{k+m} + \beta y_{k+m})\right|\right)\right]^{p_k}$$

$$\le \frac{1}{n}\sum_{k=1}^{n}\left[g\left(|\alpha|\left|\Delta_n^r x_{k+m}\right|\right) + g\left(|\beta|\left|\Delta_n^r \beta y_{k+m}\right|\right)\right]^{p_k}$$

$$\le D(A_\alpha)^H \frac{1}{n}\sum_{k=1}^{n}\left[g\left(|\alpha|\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k}$$

$$+ D(B_\beta)^H \frac{1}{n}\sum_{k=1}^{n}\left[g\left(|\alpha|\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} \to 0$$

As n→∞, uniformly in m. This proves that $[f,p]_0(\Delta_n^r)$ is linear and the result follows. □

Theorem 2: Let g be any modulus function. Then $[f,g,p](\Delta_n^r) \subset [f,g,p]_\infty(\Delta_n^r)$ and $[f,g,p]_0(\Delta_n^r) \subset [f,g,p]_\infty(\Delta_n^r)$.

Proof: We shall prove the result for $[f,g,p](\Delta_n^r) \subset [f,g,p]_\infty(\Delta_n^r)$ and the second shall be proved on similar lines. Let $x \in [f,g,p](\Delta_n^r)$. Now, by definition of g, we have

$$\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} = \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m} - L + L\right|\right)\right]^{p_k}$$

$$\le \frac{D}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m} - L\right|\right)\right]^{p_k} + \frac{D}{n}\sum_{k=1}^{n}\left[g\left(|L|\right)\right]^{p_k}.$$

Thus, for any number L, there exists a positive integer K_L such that $|L| \le K_L$, we have

$$\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} = \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m} - L + L\right|\right)\right]^{p_k}$$

$$\le \frac{D}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m} - L\right|\right)\right]^{p_k} + \frac{D}{n}[K_L g(1)]^{p_k}\sum_{k=1}^{n} 1.$$

Since, $x \in [f,g,p](\Delta_n^r)$, we have and the proof of the result follows.

Theorem 3: $[f,,g,p]_0(\Delta_n^r)$ is a paranormed space with

$$h_\Delta(x) = \sup_{m,n}\left(\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k}\right)^{\frac{1}{H}}.$$

Proof: From Theorem 2, for each $x \in [f,g,p]_0(\Delta_n^r)$, $h\ (x)$ exists. Also, it is trivial that $h_\Delta(x) = h_\Delta(-x)$ and $\Delta_n^r x_{k+m} = 0$ for x=0. Since, h(0)=0, we have $h_\Delta(x) = 0$ for x=0. Since, $\frac{p_k}{M} \le 1$ for M≥1, therefore, by Minkowski's inequality and by definition of g for each n that

$$\left(\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m} + \Delta_n^r y_{k+m}\right|\right)\right]^{p_k}\right)^{\frac{1}{H}}$$

$$\le \left(\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right) + g\left(\left|\Delta_n^r y_{k+m}\right|\right)\right]^{p_k}\right)^{\frac{1}{H}}$$

$$\le \left(\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k}\right)^{\frac{1}{H}} + \left(\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r y_{k+m}\right|\right)\right]^{p_k}\right)^{\frac{1}{H}},$$

which shows that $h_\Delta(x)$ is sub-additive. Further, let α be any complex number. Therefore, we have by definition of g, we have

$$h_\Delta(\alpha x) = \sup_{m,n}\left(\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r \alpha x_{k+m}\right|\right)\right]^{p_k}\right)^{\frac{1}{H}} \le S_\alpha^{\frac{H}{M}} h_\Delta(x),$$

where, Sα is an integer such that α<Sα. Now, let α→0 for any fixed x with $h_\Delta(x) \ne 0$. By definition of g for $|\alpha| < 1$, we have for $n > N(\varepsilon)$ that (Equation 2)

$$\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} < \varepsilon. \tag{2}$$

As g is continuous, we have, for 1≤n≤N and by choosing α so small that (Equation 3)

$$\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} < \varepsilon. \tag{3}$$

Consequently, (2) and (3) gives that $h_\Delta(\alpha x) \to 0$ as α→0. □

Theorem 4: Let X be any of the spaces [f,g], $[f,g]_0$ and $[f,g]_\infty$. Then, $X(\Delta_n^{r-1}) \subset X(\Delta_n^r)$ is strict. In general, $X(\Delta_n^j) \subset X(\Delta_n^r)$ for all j=1,2,…,r-1 and the inclusion is strict.

Proof: We give the proof for the space $[f, g]_\infty$ and others can be proved similarly. So, let $x \in x \in [f,g,p]_\infty(\Delta_n^{r-1})$. Then, we have

$$\sup_{m,n} \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^{r-1}x_{k+m}\right|\right)\right] < \infty.$$

Since, g is increasing function, we have

$$\frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^{r}x_{k+m}\right|\right)\right] = \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^{r-1}x_{k+m} - \Delta_n^{r-1}x_{k+m+1}\right|\right)\right]$$

$$\le \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^{r}x_{k+m}\right|\right)\right] + \frac{1}{n}\sum_{k=1}^{n}\left[g\left(\left|\Delta_n^{r-1}x_{k+m+1}\right|\right)\right]$$

$$< \infty.$$

Thus, $[f,g]_\infty(\Delta_n^{r-1}) \subset [f,g]_\infty(\Delta_n^r)$. Continuing in this way, we shall get $[f,g]_\infty(\Delta_n^j) \subset [f,g]_\infty(\Delta_n^r)$ for j=1,2,…,r-1. The inclusion is strict. For this, we consider x=(k^r) and is in $[f,g]_\infty(\Delta_n^r)$ but does not belong to $[f,g]_\infty(\Delta_n^{r-1})$ for f(x)=x and n=1. (if x=(k^r), then $\Delta_n^r x_k = (-1)^r r!$ and $\Delta_n^r x_k = (-1)^{r+1} r!\left(k + \frac{r-1}{2}\right)$ for all $k \in \mathbb{N}$). □

Theorem 5: $[f,g,p](\Delta_n^{r-1}) \subset [f,g,p]_0(\Delta_n^r)$

Proof: The proof is obvious from Theorem 4 above.

Theorem 6: Let g, g_1 and g_2 be any modulus functions. Then,

(i) $[f,g_1,p]_0(\Delta_n^r) \subset [f,gog_1,p]_0(\Delta_n^r)$.

(ii) $[f,g_1,p]_0(\Delta_n^r) \cap [f,g_2,p]_0(\Delta_n^r) \subset [f,g_1+g_2,p]_0(\Delta_n^r)$.

Proof: Let ε be given small positive number and choose δ with 0< δ<1 such that g(t)< ε for 0<t≤ δ. We put $y_{k+m} = f_1\left(\left|\Delta_n^r x_{k+m}\right|\right)$ and consider

$$\sum_{k=1}^{n}\left[g\left(y_{k+m}\right)\right]^{p_k} = \sum_{I}\left[g\left(y_{k+m}\right)\right]^{p_k} + \sum_{II}\left[g\left(y_{k+m}\right)\right]^{p_k}$$

where the first summation is over $y_{k+m} \le \delta$ and second summation is over yk+m> δ. As g is continuous, we have (Equation 4)

$$\sum_{I}\left[g\left(y_{k+m}\right)\right]^{p_k} < n\varepsilon^H \quad (4)$$

and for y_{k+m}> δ, we use the fact that

$$\frac{1}{n} < \frac{y_{k+m}}{\delta} \le 1 + \frac{y_{k+m}}{\delta}.$$

Now, by definition of g, we have for y_{k+m}> δ that

$$g\frac{y_{k+m}}{\delta} < 2g(1)\frac{y_{k+m}}{\delta}.$$

Thus (Equation 5),

$$\frac{1}{n}\sum_{II}\left[g\left(y_{k+m}\right)\right]^{p_k} \le \max\left(1,\left(2g(1)\delta^{-1}\right)^H\right)\frac{1}{n}\sum_{k=1}^{n} y_{k+m}^{p_k}. \quad (5)$$

Consequently, we see from (4) and (5) that $[f,g_1,p]_0(\Delta_n^r) \subset [f,gog_1,p]_0(\Delta_n^r)$.

To prove (ii), we have from (1) that

$$\left[(g_1+g_2)\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} \le D\left[g_1\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k} + D\left[g_2\left(\left|\Delta_n^r x_{k+m}\right|\right)\right]^{p_k}.$$

Let $x \in [f,g_1,p]_0(\Delta_n^r) \cap [f,g_2,p]_0(\Delta_n^r)$. Consequently, by adding above inequality form k=1 to k=n, we have and the result follows. □

Theorem 7: Let g, g_1 and g_2 be any modulus functions. Then,

$[f,g_1,p](\Delta_n^r) \subset [f,gog_1,p](\Delta_n^r)$

$[f,g_1,p](\Delta_n^r) \cap [f,g_2,p](\Delta_n^r) \subset [f,g_1+g_2,p](\Delta_n^r)$

$[f,g_1,p]_\infty(\Delta_n^r) \subset [f,gog_1,p]_\infty(\Delta_n^r)$

$[f,g_1,p]_\infty(\Delta_n^r) \cap [f,g_2,p]_\infty(\Delta_n^r) \subset [f,g_1+g_2,p]_\infty(\Delta_n^r)$

Proof: The follows as a routine verification as of the Theorem 6.

Theorem 8: The spaces $[f,g,p](\Delta_n^r), [f,,g,p]_0(\Delta_n^r)$ and $[f,,g,p]_\infty(\Delta_n^r)$ are not solid in general.

Proof: To show that the spaces $[f,g,p](\Delta_n^r), [f,,g,p]_0(\Delta_n^r)$ and $[f,,g,p]_\infty(\Delta_n^r)$ are not solid in general, we consider the following example.

Let p_k=1 for all k and g(x)=x with r=1=n. Then, $(x_k) = (k) \in [f,,g,p]_\infty(\Delta_n^r)$ but $(\alpha_k x_k) \notin [f,,g,p]_\infty(\Delta_n^r)$ when $\alpha_k=(-1)^k$ for all $k \in \mathbb{N}$. Hence is result follows. □

From above Theorem, we have the following corollary.

Corollary 9: The spaces $[f,g,p](\Delta_n^r), [f,,g,p]_0(\Delta_n^r)$ and $[f,,g,p]_\infty(\Delta_n^r)$ are not perfect.

Theorem 10: The spaces $[f,g,p](\Delta_n^r), [f,,g,p]_0(\Delta_n^r)$ and $[f,,g,p]_\infty(\Delta_n^r)$ are not symmetric in general.

Proof : To show that the spaces $[f,g,p](\Delta_n^r), [f,,g,p]_0(\Delta_n^r)$ and $[f,,g,p]_\infty(\Delta_n^r)$ are not perfect in general, to show this, let us consider p_k=1 for all k and g(x)=x with n=1. Then, $(x_k) = (k) \in [f,,g,p]_\infty(\Delta_n^r)$ Let the re-arrangement of (x_k) be (y_k) where (y_k) is defined as follows,

$(y_k) = \{x_1, x_2, x_4, x_3, x_9, x_5, x_16, x_6, x_25, x_7, x_36, x_8, x_49, x_10, ...\}$.

Then, $(y_k) \notin [f,,g,p]_\infty(\Delta_n^r)$ and this proves the result. □

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