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Governing the Filtration Process of Contractors to Insolvent Point in Construction Projects with *Nmorcha* (Threshold) Operator

Egwunatum I. Samuel *
Department of Quantity Surveying, Delta State Polytechnic, Ozoro, Nigeria

Opinion Article

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***For Correspondence**

Egwunatum I. Samuel, Department of Quantity Surveying, Delta State Polytechnic, Ozoro, Nigeria.

E-mail: samuelegwunatum@gmail.com

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ABSTRACT

There are recurrent literatures to show that contractors in the construction industry are going insolvent. As a crippling process, there are early signs leading to contractor's insolvency. Conflicting literatures have documented causes of insolvency in construction project in an irregular calibration. This paper argues theoretically that those causes can be stabilized calibrated with a view to establishing a threshold point or the limiting point beyond which a contractor becomes insolvent. A conceptual theory was built on spherical harmonic polynomial for which a sector of the sphere representing an entire economic sphere was taken to represent the construction industry sector. The sectors' investigation precipitated five (5) main causes of insolvency from five (5) significant unethical practices. These variables were subjected to an orthogonal 5 × 5 square matrix to derive a pseudodifferential operator matrix with Fredholm properties narrated in Athanasiadis et al. and Claeys' boundary integral problems. The derived determinant was used to operate as *Nmorcha* operator (∇_N) on the population variable that assumed a chaotic attribute under a normal distribution curve at the critical stressed boundary plane. The *Nmorcha* operator(∇_N) as a determinant operand at the boundary point of the critical region fittered variables to the skew of accepted and reject domains. The filtration operation at the boundary point presented a value of e_2i_4 representing falsification of projects financial status (e_2) arising from poor project cost control (i_4). Beyond this point in the array of causes in the calibrated insolvency scale, the contractor becomes insolvent.

Definition of Terms

Nmorcha operator (∇_N): A theoretical operator derived from the characteristics property of a sectorial polynomial ($m=n$) matrix of spherical harmonic function that operates filtration on functions with indeterminate or chaotic state!

$\nabla_N{:}\rightarrow f(x)\in|\Theta|$

Threshold operation: An operation performed on operand to obtain the value of a threshold function.

Threshold gate: A logic plane where a threshold operation is performed.

Threshold operator: A logic element that performs a threshold operation.

Threshold: The starting point for a new state or experience.

Boundary operator: a certain element that permits transition by gradient profiling at boundary plane of a space to either divides of a domain.

Determinant: The characteristic value of a group of variables that ascribes peculiarity and representative attribute to the variables.

INTRODUCTION

Contractors in the construction business are gliding into insolvency or emphatically becoming insolvent. What remains indeterminate is; at what point in life of the project do they become insolvent? ASHURST sounded a tocsin that contractors' insolvencies will increase from ASHURST [1] and advocated that contractors should devise an organizational metric to indicate the early warning signs of insolvency. Nevertheless, contractors 100% alertness may not be a rescue option to the inevitable because, combating insolvency in the construction industry is almost as a helpless combat of man with climate reclamation! The reason for this is that the world economic recession in the recent past came with global financial warming/depletion that necessarily will and still interfere with any remediation measure to combat insolvency in the real sector of the construction industry [2].

In the last 10 years, research efforts have been directed towards the investigation of the causes of insolvency amongst construction contractors. Typically, Arain and Low proposed measures of minimizing impacts of variations on construction project as a way of avoiding insolvency [3,4]. With the same objective, Bahram offered some legal claim protection methods that can mitigate insolvency on risk transfer basis by insertion of express legal clauses in contract documents. Also, the thought that insolvency may arise from some foundational ethical reasons inspired Wood to prescribe a friendly partnership relational model in construction projects [5]. More the same, the indicators given by ASHURST on early signs of contractor's insolvency are very leading to the expiration of insolvency in construction contract business should contractors adhere. Assuredly, this paper opines that a threshold value from the ladder (hierarchy) of causes of insolvency in construction project contracts based on calibration will certainly give scientific credence to combating insolvency occurrence and friction the gliding into that economic state.

A Survey of Insolvency Literature in the Construction Industry

The concept of insolvency is not peculiar to construction sector of any economy. As a matter of fact, insolvency is an economic state of an organisation that reflects the financial health status of the organisation. The reason for its study is in an attempt to master its forms, its causes, its sources and its prevention. Arain did a classical investigation on the causes of insolvency and unethical practices of contractors in Pakistan construction industry [6]. The investigation was conducted vide a questionnaire survey administered to 90 contractors. Thirty (30) questionnaires were returned and analysed. The findings showed nineteen (19) speculative causes but with five (5) main causes of insolvency. The five significant causes identified in Arain were absence of barriers, cash flow problems poor financial control, knock-on effect and onerous conditions of contract. The study also noted that these five (5) significant causes of insolvency are also intrinsically linked to unethical practices of contractors with the exception of absence of barriers. This outcome collaborates earlier knowledge on the subject by [5-10] Arain, Assaf and Low, Newman, Dalla-Costa, Lowe and Low. A significant outcome of Arain study was that construction industry should pay attention to ethical behaviour and good practices in order not to jeopardize the financial stability of contractors in the supply chain. ASHURST before the Arain study proposed a pre-emptive measure on decimating insolvency, rather than studying the symptoms. The underlying preposition of the ASHURST literature is that contractors are to be vigilant to certain indicators of early warning signs of insolvency that could lead to financial embarrassment. According to the study, the following are early indicators of insolvency; sub-contractors making demands for payments directly, contractors requesting for early/advancement to cover the cost of sub-contractors or materials, scarcity of work on site which is a sign of possible cash flow in the future, slow progress of work and missed date, employees apathy arising from none payment of wages couple with reduced labour force, disappearance of materials, persistent rumours of declining financial status, application for spurious claims, ailing contractor's account, diversion of income, court judgments that conveys orders of mandamus on the contractor and whether if the contractor's parent company is showing any of these symptomatic signs.

However, symptoms and treatment of insolvency in construction projects need not be a laboratory science. As a cripping process, reminiscence of pathological, metamorphosis, its dynamics requires empirical study before the process converges to a literacy zero (0) i.e. contractor's oblivion. A complementary study by Turner and Townsend proposed [11] a four (4) way test to show that a contractor is gliding to insolvency in consonance with Section 132 of the UK Insolvency Act of 1986. These include failure to pay a statutory demand over a threshold, execution on a judgment is unsatisfied, inability to pay debts arising from cash flow deficit and value of assets are less than liabilities. Bahram attempted a legal prescription to insolvency in construction projects with a review of the Perar BV vs. General Surety & Guarantee Co. Ltd. British Law Report (BLR) 72, Case. The outcome of that study was that insolvency is not a breach of contract as default, unless it is expressly provided in contract documents. Further, the study reported that it is neither to the benefit of the contractor nor the client to resort to termination of contract arising from insolvent situation [12-18]. In the same study, it averred that there is plethora of recovery measures towards diffusing insolvency in contractual arrangements. Specifically, it mentioned administrative receivership; Company Voluntary Arrangement (CVA) and scheme of arrangement that is subject to satisfactory compliance with statutory requirements. Other research efforts in unison offered some legal protection approach but with a caveat that Employer's termination notice must necessarily show compliance with contractual requirements otherwise the situation may turn beneficial by way of compensation to the contractor. As a legal rescue mechanism, contractors and employers should turn to the prescribed procedure of resolving termination of contract due to insolvency. Appreciably, the JCT, under Clause 8 was very benevolent to both parties with Clause 8.2 conditioning that contractor's termination shall not be issued unreasonably or vexatiously [19-22]. Contrary to NEC Clause 90.2 that permits employer to terminate for any reason he deemed fit. On the basis of clustered research reaction, this paper argues that, there are events that culminates to insolvency and seeks to investigate the threshold or the limiting point beyond which a contractor becomes insolvent.

Theoretical Setup

The Arain study, precipitated an outcome of six (6) unethical practices correlating to five (5) significant causes of insolvency on a shifted like scale of 1-6 mean score. Indicating a probability space between most likely and most unlikely as shown below **(Table 1)**.

φ_{i1}represents unethical practice, φ_e represents causes of insolvency such that absence of barriers (φ_{i1}) as a cause of insolvency is likely to be originated by the unethical practice of failure to reconcile with sub-contractor's concern (φ_{e5}) = 2.81. Generally, the Arain source and cause correlation of 6 × 5 matrix is indicated to read **(Table 2)**.

Conceptually, two variables such as source (unethical practices (φ_{ei}) and causes (of insolvency), φ_{ej} are said to be orthogonal if their scalar product vanishes. A family of variables is an orthogonal system on the interval (*a*, *b*) with the weight function ω(x) (or distribution *a(x)*), if for any two distinct members of the family say ($\varphi_e\varphi_i$) = 0. From the Arain results, the correlates can form a system of orthogonal family as;

$$\varnothing_n = \begin{vmatrix} \varphi_1\varphi_1\varphi_1\varphi_2 \cdots\cdots\cdots\cdots \varphi_1\varphi_5 \\ \varphi_2\varphi_1 \cdots\cdots \qquad\qquad \varphi_1\varphi_4 \\ \varphi_2\varphi_1 \cdots\cdots\cdots\cdots\cdots\cdots \\ \varphi_4\varphi_1\varphi_4\varphi_2 \cdots\cdots\cdots\cdots \\ \varphi_5\varphi_1 \cdots\cdots\cdots\cdots\cdots\cdots \\ \varphi_6\varphi_1 \cdots\cdots\cdots\cdots \varphi_6\varphi_5 \end{vmatrix}$$

Consequently, this paper idealizes a threshold estimation function for contractor's insolvency which is a variable that takes a limiting value say "1" if a specified function of the argument exceeds a given threshold and '0' if otherwise. A threshold operation when performed on Equation (1) as an operation performed on operands say $\varphi_1\varphi_1$, $\varphi_1\varphi_2$......... $\varphi_2\varphi_4$, $\varphi_2\varphi_5$......... $\varphi_5\varphi_1$......... $\varphi_6\varphi_5$, in order to obtain the value of a threshold function are therefore consistent with the vanishing properties of an orthogonal function [23-31]. In doing so, the space between $\varphi_1\varphi_1$...... $\varphi_6\varphi_5$,is considered a quadratically integrable space and therefore separable. Since the space consist of finite elements or denumerable vectors been the length of space between each scalar combination, this paper deduces a threshold value as the limiting point of *lim* ($\varphi_{ek,}\varphi_{ik}$) = 0.

The Arain study on causes of insolvency and unethical practices using a cross-assessment table of mean item score, precipitated a 6 × 5 matrix with an overwhelming response determinant of delayed payment and knock-on effect as the resultant combinatorial cause of insolvency in Pakistan from the five (5) main identified causes of insolvency and six (6) unethical practices value **(Table 3)**.

Following this calibration, this paper deduce a semblance of the scalar combination of unethical practices and insolvency causes to an orthogonal system of a finite sequence of $\varphi_{i,}\varphi_e$ *(i = 1, 2, ...n, e = 1, 2, ... n)* or shortly as {φ_n (x)} with orthogonality property of ($\varphi_h\varphi_k$) = 0 , $h^{\Delta r^{-1}}$ *k* noting that the zeros of the orthogonal polynomials are simple and located in the interior of the interval (*a, b*) From the Arain matrix, an orthogonal space of ($\varphi_{i1,}\varphi_{i2},\varphi_{5,}\varphi_{e6}$) wherein the threshold element for insolvency is located in the interior of the interval of (*a, b*) is consequently idealized.

Foundationally, this paper made recourse to Maxwell's theory of poles in acknowledgement of the Bateman's spherical harmonic polynomial description in a reduced form with; λ_1, λ_2, λ_3 'to be independent variables, such that

$$r = \left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2\right)^{1/2} \tag{1}$$

and a differential operator defined as $\forall_k = \frac{\partial}{\partial \lambda_k}$, *k* = 1, 2, 3

Table 1. Outcome of Arain (2013) study on source and cause of insolvency.

φ_e / φ_i	φ_{i1}	φ_{i2}	φ_{i4}	φ_{i4}	φ_{i5}
φ_{i1}	4.19	1.93	2.91	1.67	3.24
φ_{i2}	2.94	4.41	4.13	4.59	3.38
φ_{i4}	3.00	4.10	4.17	3.56	4.10
φ_{i4}	4.13	4.03	3.83	3.70	3.38
φ_{i5}	2.81	4.34	4.00	3.67	3.05
φ_{i6}	3.94	2.17	2.17	3.81	3.86

Table 2. Insolvency causes.

S/N	Causes of Insolvency (φ_{ij})	Sources of unethical practices (φ_{ei})
1.	Absence of barriers	Delaying of payments
2.	Cash flow problems	Mishandling of sensitive information
3.	Poor financial control	Improper estimating practices
4.	Knock-on effects	Abuse of resources
5.	Onerous conditions of contract	Failure to reconcile with sub-contractor's concern
6.	-	Misrepresentation of financial status

Table 3. Restructured arain (2013) matrix in tabular form variable of sources and unethical practices

S/N	Combinatorial cause	Scalar value
1	$\varphi_1\varphi_4$	1.67
2	$\varphi_1\varphi_2$	1.93
3	$\varphi_6\varphi_2$	2.17
4	$\varphi_6\varphi_3$	2.17
5	$\varphi_1\varphi_3$	2.17
6	$\varphi_5\varphi_1$	2.81
7	$\varphi_2\varphi_1$	2.94
8	$\varphi_3\varphi_1$	3
9	$\varphi_5\varphi_5$	3.05
10	$\varphi_1\varphi_5$	3.24
11	$\varphi_4\varphi_5$	3.38
12	$\varphi_2\varphi_5$	3.38
13	$\varphi_3\varphi_5$	3.56
14	$\varphi_5\varphi_4$	3.67
15	$\varphi_4\varphi_4$	3.7
16	$\varphi_6\varphi_4$	3.81
17	$\varphi_4\varphi_3$	3.83
18	$\varphi_6\varphi_5$	3.86
19	$\varphi 6\varphi_1$	3.94
20	$\varphi_5\varphi_3$	4
21	$\varphi_4\varphi_2$	4.03
22	$\varphi_3\varphi_2$	4.1
23	$\varphi_3\varphi_5$	4.1
24	$\varphi_4\varphi_1$	4.13
25	$\varphi_2\varphi_3$	4.17
26	$\varphi_3\varphi_3$	4.17
27	$\varphi_5\varphi_2$	4.34
28	$\varphi_1\varphi_1$	4.19
29	$\varphi_2\varphi_2$	4.41
30	$\varphi_2\varphi_4$	4.59

with $\Delta r^{-1} = (\forall_1^2 + \forall_2^2 + \forall_3^2).r^{-1} = 0$ (2)

In consonance with Laplace's equation and homogenous polynomial of degree $n = a + b + c$ multiplied by r^{2n-1} wherein, for every polynomial (H_n) of degree n, the following are equivalently true;

$\Delta H_n = 0$ and = 0 $\Delta H_n.r^{-2n-1} = 0$

Thus, if $n = a + b + c$, then,

$$(\forall_1^2 + \forall_2^2 + \forall_3^2).r^{-1} \; || \; (\forall_1^a + \forall_2^b + \forall_3^c) = H_n\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2\right)^{r^{-2n-1}} \tag{3}$$

From this relationship, it becomes fundamental that to every homogenous polynomial of degree n, with three quantities, $\forall_1$, $\forall_2$, $\forall_3$ for which,

$$\forall_1^2 + \forall_2^2 + \forall_3^2 = 0 \tag{4}$$

There corresponds a harmonic polynomial of λ_1, λ_2, λ_3 of degree n and which in line with Hobson's theory can be shown that,

$$\forall_1^{n-m}(\forall_2 \pm i\forall_3)^m \frac{1}{r} = \frac{(-1)^{n-m}(n-m)!}{r^{n+1}} e^{\pm im\emptyset}.p_n^m(\cos\theta) \;\; m = 0, 1, \ldots, n \tag{5}$$

and λ_1 =r cos θ, λ_2 = r sin θ cos ∅, λ_3 = r sin θ sin ∅

Consequently, for geometric interpretation, of Equatiom (5) surface harmonic function reduction becomes zonal if m =0, sectorial if $m = n$ and tesseral if $1 \leq m \leq n$

From Maxwell's result, this paper, took $\eta = (\alpha_k, \beta_k, \gamma_k)$, $k = 1,2,\ldots,n$ as unit vectors which defines points on the unit-sphere as poles-such that surface harmonic of degree n with poles η_k defined by;

$$S_n(\eta_k) = (-1)^n r^{n+1}[\prod_{k=1}^{n}(\alpha_k \forall_1 + \beta_k \forall_2 + \gamma_k \forall_3)]r^{-1} \quad (6)$$

For the special case of $\rho = 2$, $h(n, \rho) = (n+1)^2$

This paper assumed $\mathfrak{Y}$ to be a vector with four components as $\mathfrak{Y} = (y_1, y_2, y_3, y_4)$ and $\eta = \frac{\mathfrak{Y}}{\rho}, \rho = ||\mathfrak{Y}||$

The following vectors are introduced

U= $(i - its, -its - is, -t+s, 1 = ts)$

Y = $(i\text{-}T^{\sigma}, \text{-}iT - I^{\sigma}, \text{-}T+^{\sigma}, 1 + T^{\sigma})$

So that it becomes;

$$(U, U) = (Y, Y) = 0, (\mathfrak{U}, \bar{\mathfrak{Y}}) = 2(1 + tr)(1+ {}^{s\sigma}) \quad (7)$$

From Equation (7), it shows that all $(n + 1)^2$ polynomials, $H_n^{k,l}(\mathfrak{Y})$ defined by

$$(\mathfrak{U}, \mathfrak{Y})^n = \sum_{k,l=0}^{n} \binom{n}{k} H_n^{k,l}(\mathfrak{Y}) t^k s^l \quad (8)$$

are harmonic polynomial is degree n.

Consequently, this paper note that

$$S_{\Omega(\eta)} S(\mathfrak{U},\eta)^2 (\bar{\mathfrak{Y}}\,\eta)^n \; d\Omega(\eta) = \frac{2^{1-n}\pi^2}{n+1}(\mathfrak{U},\bar{\mathfrak{Y}})^n$$

and therefore the surface harmonics,

$$S_n^{k,l}(\eta) = \rho^{-n} H_n^{k,l}(\mathfrak{Y})$$

Form an orthogonal set of $h(n, 2) = (n+1)^2$ of linearly independent surface harmonics, where,

$$\int_{\Omega} \int_{\Omega} S_n^{k,l}(\eta) \bar{S}_n^{k^1,l^1}(\eta), \delta\Omega = \{ {0 \atop \frac{2\pi^2}{n+1}\binom{n}{l} / \binom{n}{k}} \; {k \neq k^1 \; or \; l \neq l^1 \atop k = k^1 \; or \; l = l^1}$$

Accordingly from Eqn (8), this paper reduces the polynomial conditionalities to; $\bar{S}_n^{k,l}(\eta) = (-1)^{k+l} S_n^{n-k,n-l}(\eta)$

To derive the components of $S_n^{k,l}$ this paper introduced the following units,

$a = y_4 + iy_1$, $b = y_3 - iy_2$, $c = -y_3 - iy_2$, $d = y_4 - iy_1$

Then, $\rho = \|\mathfrak{Y}\| = (ad - bc)^{\frac{1}{2}}$, $(\mathfrak{U}, \mathfrak{Y}) = + bs + (c + ds)t$,

So that, consistently from Eqn (8), this paper derive that,

$$\sum_{l=0}^{n} H_n^{k,l}(\mathfrak{Y}) s^l = (a+bs)^{n-k}(c+ds)^k$$

And

$$H_n^{k,l}(\mathfrak{Y}) = \frac{1}{2\pi i}\int^{(0+)} (a+bs)^{n-k}(c+ds)^k s^{-1-1} ds \quad (9)$$

By substituting,

$\sigma = -s(bc - ad)/bd$,

$$\sigma_0 = ad / c(ad - bc) = (y_1^2 + y_4^2)/(y_1^2 + y_2^2 + y_3^2 + y_4^2)$$

So that expressing a, b, c, d in terms of y_i yields

$$H_n^{k,l}(\mathfrak{Y}) = \frac{(-1)^k}{2\pi i}\rho^n (d/\rho)^{k+l-n}\left(\frac{b}{\rho}\right)^{l-k}\int^{(\sigma_0+1)} \sigma^{n-k}(1-\sigma)^k \frac{d\sigma}{(\sigma-\sigma_0)^{l+1}} \quad (10)$$
$$= \frac{(-1)^k}{l!}\rho^n \left(\frac{y_4 - iy_1}{\rho}\right)^{k+l-n}\left(\frac{y_3 - iy_2}{\rho}\right)^{l-k}\frac{d^l}{d\sigma_0 1}\sigma_0^{n-k}(1-\sigma_0)^k$$

Arising therefrom, the following precipitates;

$$S_n^{k,l}(\eta)=\rho^{-n}H_n^{k,l}(\mathfrak{Y}) \;=(-1)^k\binom{n-k}{l}(\eta_4-i\eta_1)^{n-k-l}(\eta_3-i\eta_2)^{k-l} \tag{11}$$

$$\chi_2F_1(-1,n-1+1;n-k-l+1;\eta_4^2-\eta_1^2)=(-1)^k(\eta_4-i\eta_1)^{n-k-l}(\eta_3-i\eta_2)^{k-l}=(-1)^k(\eta_4-i\eta_1)^{n-k-l}(\eta_3-i\eta_2)^{k-l}$$

$\chi\rho_1^{(n-k-l,k-l)}(\ \eta_3^2+\eta_2^2-\eta_4^2-\eta_1^2)$ provided $n < k + l$

Similarly,

$$S_n^{k,l}(\eta)=\ \rho^{-n}\ H_n^{k,l}(\mathfrak{Y})=(-1)^{n-1}k^{(n-1)}(\eta_4-i\eta_1)^{k+i-n}(\eta_3-i\eta_2)^{i-k} \tag{12}$$

$$\chi_2F_1^{(1-n,l+1,1+k-n+1;\eta_4^2+\eta_1^2)}$$

Ditto;

$$S_n^{k,l}(\eta)=\ \rho^{-n}\ H_n^{k,l}(\mathfrak{Y})=(-1)^{n-1}(\eta_4-i\eta_1)^{k+i-n}(\eta_3-i\eta_2)^{i-k} \tag{13}$$

$X\rho_{n-l}^{(1+k-n,i-k)}(\eta_3^2+\eta_2^2+\eta_4^2+\eta_1^2)$ where $\rho_m^{(\alpha,\beta)}$ denotes a purely Jacobi polynomial

Derivatively, if this paper associate cause of insolvency (φ_e) and unethical practices (φ_i) to two vectors with four components each and letting $\mathfrak{Y}$ be a vector with the components;

$$\left.\begin{aligned} w_1 &= y_1z_4+y_4z_1-y_2z_3+y_3z_2\\ w_2 &= y_2z_4+y_4z_2-y_3z_1-y_3z_1\\ w_3 &= y_3z_4+y_4z_3-y_1z_2+y_2z_1\\ w_4 &= y_4z_4-y_1z_1y_2z_2-y_3z_3 \end{aligned}\right\} \tag{14}$$

where $Y_iz_i = \varphi_e\varphi_i$

With the introduction of quaternions, into Eqn (14) it reduces to $w_4 + iw_2 + iw_3 + kw_1 = (z_4 + iz_2 + iz_3 + kz_1)(y_4 + iy_2 + iy_3 + ky_1)$ where 1, *i, j, k* are fundamental coefficients. Then from the addition theorem;

$$H_{2n}^{k,l}(\mathfrak{Y})=\sum_{n=0}^{2n}H_{2n}^{k,m}(\varphi_i)H_{2n}^{m,l}(\varphi_e) \tag{15}$$

The matrix of the output of Eqn (14) becomes

$$[H_{2n}^{k,l}(\varphi_{ei})] \tag{16}$$

where *k* denotes the rows and *l* denotes the columns for which *k = m, l = n* and *m = n* in this study and recursively gives the determinant;

$$(y_1^2+y_2^2+y_3^2+y_4^2)^{n(2n+1)} \tag{17}$$

which is in tandem with Eqn (2) ditto *Zi*'s with characteristic roots $\lambda_1^m,\lambda_2^{2n-m},m=0,1,\ldots,z_n$ where λ_1 and λ_2 the roots of the association given by

$$\begin{vmatrix} a-\lambda, & b\\ c, & d-\lambda \end{vmatrix}=0$$

Study Objective

To identify a threshold point on a calibrated insolvency scale by threshold operation been an operation performed on operands in order to obtain the value of a threshold point through a threshold gate. This point on the calibration scale is the point beyond which a contractor turn's insolvent.

METHODS OF RESEARCH

This paper reviewed the existing literatures on contractors' insolvency in construction projects and zeroed the investigation to the causes and sources of insolvency. This study mirrored the Arain investigation on causes of insolvency arising from some unethical practices amongst contractors in the Pakistan construction industry as a control "experiment". The Arain study identified 5-causes of insolvency originating from 6-unethical practices. As replication logic to test the validity of that outcome, a similar investigation was conducted on the Nigerian construction industry sector in this paper. The investigation was done using a survey design method with 108 questionnaire soliciting information on causes of insolvency arising from some certain sharp or unethical practices of contractors. The respondents (sample size) was derived from a population of 150, made up of professional Civil

Engineers, M &E Engineers, Builders, Quantity Surveyors and Architects in public, private and contracting organizations using a sample size formula of;

$$S = \chi^2 N\rho\left(a-\rho\right) \div \partial^2\left(N-1\right) + \chi^2\rho(1-\rho)$$

S = sample size being sought.

χ^2 = Table value for chi-square at 1 degree of freedom at the desired alpha level 0.05 = 3.841; 0.01 = 6.64

N = Population size.

$\rho =$ The population proportion usually 0.05 for maximum sample.

∂ = Degree of accuracy designed, expressed as a proportion (usually 0.05).

81 questionnaires were returned with 67 suitable for analysis indicating a response rate of 75%. The questionnaire was structured in sections in relation to the objective of the study. An open ended system of questioning was used in achieving this aim reflecting a 5-point likert scale with the following ordinate value

5: Very high, 4: High, 3: Moderate, 2: Low, 1 : Very low

The mean score method of analysis which usually represents the average value of all factors associated to a specific inquiry was employed. The justification for the use of mean item score instrument for analysis was in consonance with the opinion of Egwunatum, citing Atsar and Ogunsemi that the instrument is prevalent amongst construction management researchers and reported its use by Kulugara, Ling and Akintoye. Usually arithmetic mean is employed unless otherwise a condition for application is giving. A pool of several types of mean item score exists viz; Arithmetic mean, Geometric mean, Harmonic mean and weighted mean. This study applied the weighted mean score, which involves assigning numerical weight (i.e. ordinal value) to respondents' ratings of factors with respect to severity. The weighted mean was computed from Rogers as;

$$\text{Weighted Mean} = \frac{w_1x_1 + w_2x_2 \ldots + w_nx_n}{w_1 + w_2 + \ldots + w_n}$$ $x_1, x_2 \ldots x_n$ represents factors under evaluation.

$w_1, w_2, \ldots w_n$ represents the weightings of the factors that translate to

w_1 = number of respondents who answered Very Low.

w_2 = number of respondents who answered Low.

w_3 = number of respondents who answered Moderate.

w_4 = number of respondents who answered High.

w_5 = number of respondents who answered Very High.

The mean score values of five (5) main causes of insolvency originating from five (5) main unethical practices that might have been the source of the cause of insolvency in Nigerian construction industry was precipitated in matrix form to show a multinomial scalar combination in **Table 2**, and a calibrated scale by ranking shown in **Table 3**. This was against the Arain study of the Pakistan construction industry that gave six (6) unethical practices from five (5) significant causes. This paper resorted to transition operator (see Equations 1 and 2 for recourse) that skews distribution variables to regions permissible only to them by theorizing a characteristic value for the distribution, been the distribution determinant from the $\varphi_e\varphi_i$ distribution matrix as the skew Eigen vector. The transition from a critical point (i.e. a threshold to a rejection region) requires that an operand operates at the boundary domain to govern the filtration process. Krall had demonstrated that such process exist with stieltjes differential – boundary operators using the uniform limit of Picard-like approximations [$\mu_\lambda(s, t)$] that can give the limit of an integration of say an *m* = *n* or 5 × 5 matrix illustrated in:

This context as; $$\mu_\lambda\left(\varphi_e\varphi_i\right) = \begin{pmatrix} e_1i_1 \ldots\ldots\ldots e_1i_5 \\ \ldots\ldots\ldots e_2i_2 \ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots e_3i_5 \\ e_4i_1 \ldots\ldots\ldots\ldots \\ e_5i_1 \ldots\ldots e_4i_3 \ldots\ldots \end{pmatrix}.$$

Arising from **Table 2**, a matrix determinant value was computed from Eqn 16, that corresponds to an operator been the characteristic property of the matrix responsible for the skew distributing gradient of $\varphi_e\varphi_i$ distribution. This property shows symmetric relation to the coercivity properties of combined boundary integral investigation carried out by Spence on high-frequency scattering of variables in distributive fields. This paper considered the $\varphi_e\varphi_i$ values as normally distributed having been subjected to a normality test and a chaotic state occasioned by their transition attempts to accept or reject domain. An investigation as to why certain values are restricted at the critical plane of a normal distribution made a logical appeal that bears semblance to a threshold gate. On this basis, the adaptation of the boundary point/critical plane of a geometrically concave normal distribution as threshold gate shows congruency to the Feischl, Feischl, Werndland, and Kita and Fausmann conditions of Adaptive Boundary Element Methods which *ab initio* restrains variables transition to regions. Most values of $\varphi_e\varphi_i$ will be impeded by operator filtration from entering the rejection region, except exceeding the transient threshold of the transition gradient operator that performs the threshold operation of filtration at the threshold gate as a replica of mixed impedance transmission boundary. Congruency of this preposition has been demonstrated in Costabel on Boundary integral operators on Lipschitz Domains and Bernardi, Costabel, Dauge and Girault on continuity properties of the *Inf-Sup* Constants for divergence in conservative spaces **(Figure 1)**.

Accordingly, this paper deduce a threshold operation performed on $\varphi_e\varphi_i$ distribution in order to obtain a threshold value at the boundary of a critical plane using the distribution characteristic value (determinant) of $\varphi_e\varphi_i$ computed from Equatio (16) as an operand that permits the transition of variables by gradient filtration at the critical plane of the normal curve when operated on at point $\bar{x}+2\sigma$ been the threshold gate/boundary. This process shows congruency with typical mixed impedance transmission problems which has been given exhaustive narration and applicability in Athanasiadis and obtained an adaptive logic license for it with recourse to Claeys "Quasi-local multitrace boundary integral formulation" process. This filtration process is affirmed to be consistent with the replication logic of the "Maxwell's demon" which is an imaginary creature to whom Maxwell assigned the task of operating a door in a partition dividing a volume containing gas at uniform temperature. The door is usually opened by the demon to enable fast molecules to move (say) from left to right through a partition. In this way, without expenditure of external work, the gas on the right could be made hotter than before and that on the left made cooler. The probable value of the threshold, det (∇_*), operator on $[\bar{x}+2\sigma]$ as a boundary point correlated to the introduction of quaternion in Equation (14, 15) and fitted to the calibration on **Table 4** to deduce the threshold value of insolvency as an insolvent point **(Figure 2)**.

Paper Structure

Section 1, gave an insight and exposed the issue and problems of insolvency in construction projects.

Section 2, identified the trend of research efforts towards resolving or mitigating insolvency in construction projects and the attendant gaps created by the research.

For instance;

1. ASHURT proposed organizational sensitivity to early warning signs and alarms.
2. Turner and Townsend proposed a 4-way test of identifying insolvent situation.
3. Bahram proposed legal recovery measures and claim protection methods towards diffusing insolvency to mitigate insolvency on risk transfer basis.
4. Arain and Low proposed minimizing variation orders to stern the tide of insolvency
5. Wood prescribed friendly relational model in the form of mutual commitment to offset insolvency.
6. Arain identified several causes of insolvency amounting from some unethical practices of contractors.

In line with the objective of this paper, no effort has been made in terms of research to identify a contractor's insolvent point amid the series of causes.

Section 3, attempts a logical description of the method employed to achieve the objective of identifying the point at which a contractor becomes insolvent. This was done by treating the economic sphere of Nigeria as a spherical harmonic polynomial and took a sector of the sphere as construction industry sector. Did a field investigation on the sector thereafter subjected the result to a rank order and performed a threshold operation on the variable to identify the threshold point of becoming insolvent.

Section 4, presented the results of the field survey and showed contrast or correlation with existing knowledge.

Section 5, presented the findings from the analysis of the field survey with respect to the theoretical framework.

Basic Assumption of Variables Chaotic State

This paper typifies its construct of a chaotic state to the Vieru general definitions of chaos for continuous and discrete-time processes. Vieru posited that let $(\varnothing_a)_{a\in\grave{U}}$ be a family of continuous functions, mapping a metric space (Θ, d_1) into a metric space (Λ,d_2) and depending on a parameter a, whose values are called *initial conditions.* Let, (Ω, d_0) be the metric space of initial

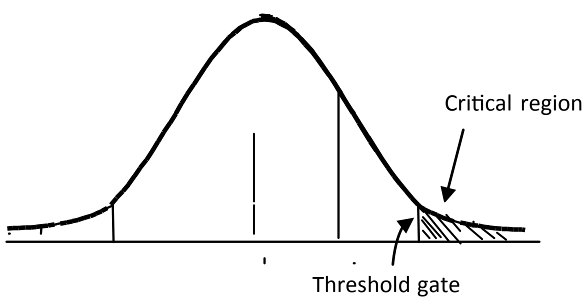


Figure 1. Constants for divergence in conservative spaces.

Table 4. Summary of demographic information of respondents.

Categories	Classification	No.	%
Academic qualification of respondents	Asso. Bachelor/ND	21	31.34
	HND	19	28.36
	B.Sc.	17	25.37
	M.Sc.	9	13.43
	Ph.D	1	1.49
Professional qualification of respondents	NSE	25	37.31
	NIQS	21	31.34
	NIOB	12	17.91
	NIA	9	13.43
Type of Organization	Contracting	23	34.32
	Consulting	19	28.36
	Government office	17	25.37
	Corporate client	7	10.44
Number of projects executed over the last 10 years	0-5	18	26.87
	6-10	16	23.88
	11-15	12	17.91
	16-20	11	16.42
	Above 20	10	14.92
Construction experience of respondents in years	0-5	17	25.37
	6-10	16	23.88
	11-15	12	17.91
	16-20	12	17.91
	Above 20	10	14.92
Age of establishment	5-15	6	8.96
	16-20	54	79.10
	21-25	3	4.48
	26-30	2	2.99
	Above 30	2	2.99
Source: Field Survey 2015			

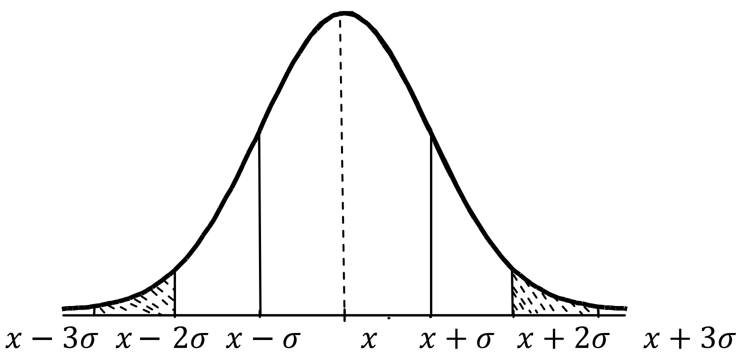


Figure 2. Distribution mean and standard deviation (Where $\bar{x} - 3\sigma =$ distribution mean; σ = standard deviation of the distribution).

conditions. . This paper will more often assume Θ and Λ are either connected sets or everywhere dense parts of connected sets. In discrete processes, this paper will only assume Θ has at least one cluster point (usually ∞). This paper will consider continuous processes and more often assume (Θ, d_1) is a *path-connected metric space*. However, our definitions still hold for metric spaces that are not path-connected. The restrictions contained in our definitions drop by themselves if the set of paths in the metric space (or in its considered subsets) is empty.

This paper will call the elements of Ω points, while the elements of the domain È of the functions ψ_a will be called moments, even if Θ may not necessarily be a subset of R.

In this article, this paper will not consider cases in which Θ and Ωare sets of fractal dimension. Since there is no possible confusion, this paper will designate d_0 (*x*, *y*), d_1(*z*, *t*) and d_2 (*u*, *v*) by $|x - y|$, $|z - t|$ and $|u - v|$. Regarding the normal distribution continuum, this paper subjected the entities of the distribution been $\varphi_e\varphi_i$ variables to the Vieru chaotic state for continuous field by idealizing a mapping metric space by an *Nmorcha* operator as:

∇_N:⟶$f(x) \in |\Theta|$

RESULTS AND DISCUSSION

The outcome of the result showing a multinomial combination values of φ_i unethical practices (φ_i) and causes of insolvency (φ_e) as $\varphi_e\varphi_i$

i_5 = cash flow problem — e_i = payment delays

i_4 = Poor cost control — e_2 = falsification of financial status

i_3 = Onerous conditions of contract — e_3 = Abuse of resources

i_2 = Knock-on effects — e_4 = Inaccurate estimating practices

i_1 = Absence of barriers — e_5 = Irreconcilable contractor's concern

$$\mu_{\nabla_*}(\varphi_e\varphi_i) = \begin{pmatrix} 3.67 & 2.59 & 3.29 & 3.18 & 3.45 \\ 3.58 & 3.10 & 3.52 & 3.40 & 2.59 \\ 3.09 & 4.50 & 4.09 & 3.13 & 2.33 \\ 3.65 & 3.20 & 4.12 & 3.85 & 3.41 \\ 3.68 & 3.62 & 3.89 & 3.73 & 2.80 \end{pmatrix}$$

On the basis of Eqn (16) our $\left[H_{2n}^{k,l}(\varphi_{ei})\right]$ value read as det (∇_*) yielded 0.5252.

Recursive applications

In this study, this paper associated more appropriately the Nigerian economy as a closed sphere to a spherical harmonic polynomial and from Maxwell's theory of poles on spherical polynomial, a geometric interpolation showed that spherical sections from Equation (5) reduces the sphere to a sector when $m = n$, zonal when $m = 0$ and tesseral when $1 \leq m \leq n$ (see Equation 5). The Nigerian construction industry sector became a consequence of Equation (5) from the spherical economy. The sector (construction industry) is burdened with having to determine the cause(s) of contractor's insolvency emanating from some unethical practices. Field survey result from **Table 5**, yielded five (5) significant causes of insolvency arising from five (5) unethical practices i.e.($\varphi_e\varphi_i$)= (5, 5). The sector's result by status of Equation (5) conditionalities yield a scalar combinatorial value of $\varphi_e\varphi_i$ in matrix form of order 5 × 5 that precipitated 25 combinatorial variables **(Table 6)**.

From Equation (16), the matrix determinant value of **Table 5**, was computed by MATLAB ver. 6.0 to be det (∇_*)= 0.5252. The methodology specified the subjection of values of **Table 5** to normality test for which they were associated to normal distribution estimation. The estimation behaviour of variables under a normal distribution treatment is shown to be at interval of; $\bar{x}+3\sigma$ to $\bar{x}+3\sigma$ i. e.,

A value derived by the operation of det (∇_*) on $\bar{x}+2\sigma$ of a normal distribution will naturally represents a threshold which depicts a critical point of transition to rejection region. This paper's methodology have posited that such point have been correlated to boundary recourse problems highlighted in Athanasiadis *et. al.* mixed impedance transmission problems. A recall of Athanasiadis invoked $\Gamma^{(j)}(x,\omega)$ and $\Gamma^{(j)}(x)$ as matrices of fundamental solutions of the differential operator of elasto-oscillations and its principal homogenous part by $A^{(j)}(\partial,\omega)$. The explicit expression of these matrices are given formulae (A1) and (A2) where

Table 5. Mean score value on causes of insolvency arising from unethical practices of contractors) with m = n rank (m).

ö$_e$ / ö$_i$	ö$_{e5}$	φ_{e4}	φ_{e3}	φ_{e2}	φ_{e1}
ö$_{e1}$	3.67	2.59	3.29	3.18	3.45
ö$_{e2}$	3.58	3.10	3.52	3.40	2.59
ö$_{e3}$	3.09	4.50	4.09	3.13	2.33
ö$_{e4}$	3.65	3.20	4.12	3.85	3.41
ö$_{e5}$	3.68	3.62	3.89	3.73	2.80

Table 6. Prepotency of variable in ranked calibration.

Unranked calibration		Prepotency of Variable in ranked calibration	
$\mu(\varphi_{ei}i_j)$	X	$\mu(\varphi_{ei}i_j)$	X
$e_1 i_5$	3.69	$e_3 i_4$	4.50
$e_1 i_4$	2.59	$e_4 i_3$	4.12
$e_1 i_3$	3.29	$e_3 i_3$	4.09
$e_1 i_2$	3.18	$e_5 i_3$	3.89
$e_1 i_1$	3.45	$e_4 i_2$	3.85
$e_2 i_5$	3.58	$e_5 i_2$	3.73
$e_2 i_4$	3.10	$e_1 i_5$	3.69
$e_2 i_3$	3.52	$e_5 i_5$	3.68
$e_2 i_2$	3.40	$e_4 i_5$	3.65
$e_1 i_1$	2.59	$e_5 i_4$	3.62
$e_3 i_5$	3.09	$e_2 i_5$	3.58
$e_3 i_4$	4.50	$e_2 i_3$	3.52
$e_3 i_3$	4.09	$e_1 i_1$	3.45
$e_3 i_2$	3.13	$e_4 i_1$	3.41
$e_3 i_1$	2.33	$e_2 i_2$	3.40
$e_4 i_5$	3.65	$e_1 i_3$	3.29
$e_4 i_4$	3.20	$e_4 i_4$	3.20
$e_4 i_3$	4.12	$e_1 i_2$	3.18
$e_4 i_2$	3.85	$e_3 i_2$	3.13
$e_4 i_1$	3.41	$e_2 i_4$	3.10
$e_5 i_5$	3.68	$e_3 i_4$	3.09
$e_5 i_4$	3.62	$e_5 i_4$	2.80
$e_5 i_3$	3..59	$e_1 i_4$	2.59
$e_5 i_2$	3.73	$e_2 i_1$	2.59
$e_5 i_1$	2.80	$e_3 i_1$	2.33
		$\bar{x}$= 3.4184	σ = 1.244

the material parameters, the density ϱ and the Lame constant λ and μ are to be replaced by $\varrho_j\lambda_j$ and μ_j, respectively. Evidently, the columns of the matrix $\Gamma^{(j)}(x,\omega)$ satisfy the Sommerfeld-Kupradze radiation conditions, and we have the following standard general integral representation formulae for solution vectors.

$$u^{(1)} \in \left[W_p^1(\Omega_1)\right]^3 \text{ and } u^{(0)} \in \left[W_{p,\,loc}^1(\Omega_0)\right]^3 \cap SK(\Omega_0)$$

$$W_0^{(1)}\left(\left\{u^{(1)}\right\}+\right)(x) - V_0^{(1)}\left(\left\{T^{(1)}u^{(1)}\right\}+\right)(x) - W_1^{(1)}\left(\left\{u^{(1)}\right\}-\right)(x) + V_1^{(1)}\left(\left\{T^{(1)}u^{(1)}\right\}-\right)(x) = \begin{cases} u^{(1)}(x) in\Omega_1, \\ 0 \quad in\mathbb{R}^3 \setminus \Omega_1 \end{cases}$$

$$-W_0^{(0)}\left(\left\{u^{(0)}\right\}-\right)(x) + V_0^{(0)}\left(\left\{T^{(0)}u^{(0)}\right\}-\right)(x) = \begin{cases} 0 \quad in\mathbb{R}^3 \setminus \Omega_0 \\ u^{(0)}(x) in\Omega_0, \end{cases}$$

where $V_k^{(j)}$ and $W_k^{(j)}$ are, respectively, the single and double layer potentials over the surface S_k associated with the fundamental matrix $\Gamma^{(j)}(x-y,\omega)$ of the differential operator $A^{(j)}(\partial,\omega)$

$$V_k^{(j)}(g)(x) : \int_{S_k} \Gamma^{(j)}(x-y,\omega)\, g(y)\, dS_{y,}\ \ x \in \mathbb{R}^3 \setminus S_k \ \ k, j = 0,1$$

$$W_k^{(j)}(h)(x) := \int_{S_k} [\Gamma^{(j)}(\partial_y, n(y))\Gamma^{(j)}(x-y,\omega)]^T h(y)\, dS_{y,}\ \ x \in \mathbb{R}^3 \setminus S_k \ \ k, j = 0,1$$

The aforementioned representation formulae remain true in the case of Lipschitz domains as well. We will employ the following notation for the boundary integral operators generated by the layer potentials

$$\left(H_k^{(j)} g\right)(x) := \int_{S_k} \Gamma^{(j)}(x-y,\omega)\, g(y)\, dS_{y,}\ \ x \in S_k,$$

$$\left(K_k^{(j)} g\right)(x) := \int_{S_k} \{T^{(j)}(\partial_x, n(x))\Gamma^{(j)}(x-y,\omega)]g(y)\, dS_{y,}\ \ x \in S_k,$$

$$\left(\bar{K}_k^{(j)} h\right)(x) := \int_{S_k} \{T^{(j)}(\partial_x, n(x))\Gamma^{(j)}(x-y,\omega)]^T h(y)\, dS_{y,}\ \mathrm{x}\ S_k,$$

$$\left(L_k^{(j)} h\right)(x) := \{T^{(j)}(\partial_x, n(x)) W_k^{(j)}(h)(x)\}^{\pm},\ \ x \in S_k$$

Mapping properties and jump relations of these operators are described. Recall that the operators $K_k^{(j)}$ and $H_k^{(j)}$ are mutually adjoint singular integral operators, while $H_k^{(j)}$ and $L_k^{(j)}$ weakly singular and singular integro-differential operators, respectively.

In that paper, they showed that direct scattering (either for chaotic or indeterminate states) problems for homogenous and non-homogenous layered obstacles in a linearly elastic conditions often results to mixed impedance transmission in as steady state oscillation. In a typical homogenous isotropic field synonymous to the normal distribution space, potential conscription method was employed or drafted to reduce the mixed impedance transmission problem to an idealized boundary pseudo-differential operator that lives on the interface between layers which in this case is the layer separating the accept domain from the reject domain in the normal distribution space as a proper sub-manifold of an homogenous (causes of insolvency) space. Their results confirmed the unique existence of original mixed impedance transmission problems oscillatory states. With the establishment of such obstructive transmission problem, a chaotic interpolation by collision curves with different boundary conditions colliding at a confluence resulted to as smoother (operator) diffuser. The smoother (operator) diffuser portends the conquest of stress vectors underlying the asymptotic behaviour of collision curves at planes of intersection. On the basis of their hybrid or non-local approach, an abstraction was made of reducing mixed transmission problem to an equivalently functional variational equation having a sesquililinear form idealized to be living at the plane of a boundary point in a colliding and transmission state that is coercive in nature.

Further, the paper introduce the boundary operators generated by the single and double layer potentials. For the boundary integrals (pseudo-differential) operators generated by the layer potentials, the paper employed the following notations:

$$(Hg)(x) := \int_s \Gamma(x-y,\,\omega)\,g(y)\,dS_{y,}\ \mathrm{x} \in S,$$

$$(Kg)(x) := \int_s \left[\mathrm{T}(\partial_x,\,n(x))\,\Gamma(x-y,\,\omega)\right] g(y)\,dS_{y,}\ \mathrm{x} \in S,$$

$$(\tilde{K}h)(x) := \int_s [\mathrm{T}(\partial_y,\,n(y))\,\Gamma(x-y,\,\omega)]^{\mathrm{T}}\, h(y)\,dS_{y,}\ \mathrm{x} \in S,$$

$$(Lh)(x) := \{T(\partial_x,\,n(x))\,W(h)(x)\}^{\pm},\ \mathrm{x} \in S_k$$

The boundary operators *H* and *L* are pseudo-differential operators or order -1 and 1, respectively, while the operators *K* and $\tilde{K}$ are mutually adjoint singular integral operators – pseudo-differential operators of order 0.

The paper employed the same notation for the electrostatic potentials constructed by the matrix $\tilde{\mathrm{A}}(x-y)$ and the corresponding boundary operators. The main ideas for the generalization to the scale of Bessel potential and Besov spaces are based on the duality and interpolation technique and are described using the theory of pseudo-differential operators on smooth manifold without boundary.

Theorem 1: Let S be $\mathcal{C}^{\infty}$ - smooth and 1 < *p* < ∞ $\leq t \leq$ ∞ and *s* ∈

The operators $V : \left[B^s_{p.p}(S)\right]^3 \to \left[H_p^{s+1+\frac{1}{p}}(\Omega^+)\right]^3 \left[\left[B^s_{p.p}(S)\right]^3 \to \left[H_{p.loc}^{s+1+\frac{1}{p}}(\Omega^-)\right]^3 \bigcap SK(\Omega^-)\right],$

$$:\left[B^s_{p.t}(S)\right]^3 \to \left[H_{p.t}^{s+1+\frac{1}{p}}(\Omega^+)\right]^3 \left[\left[B^s_{p.t}(S)\right]^3 \to \left[H_{p,t,loc}^{s+1+\frac{1}{p}}(\Omega^-)\right]^3 \bigcap SK(\Omega^-)\right]$$

$$W\ :\left[B^s_{p.p}(S)\right]^3 \to \left[H_p^{s+\frac{1}{p}}(\Omega^+)\right]^3 \left[\left[B^s_{p.p}(S)\right]^3 \to \left[H_{p.loc}^{s+\frac{1}{p}}(\Omega^-)\right]^3 \cap SK(\Omega^-)\right],$$

$$:\left[B^s_{p.t}(S)\right]^3 \to \left[H_{p.t}^{s+\frac{1}{p}}(\Omega^+)\right]^3 \left[\left[B^s_{p.t}(S)\right]^3 \to \left[H_{p,t,loc}^{s+\frac{1}{p}}(\Omega^-)\right]^3 \cap SK(\Omega^-)\right]$$

are continuous.

If S is Lipschitz, then the operators

$$V\ :\left[H_2^{-\frac{1}{2}}(S)\right]^3 \to \left[H_2^1(\Omega^+)\right]^3 \left[\left[H_2^{-\frac{1}{2}}(S)\right]^3 \to \left[H_{2.loc}^1(\Omega^-)\right]^3 \bigcap SK(\Omega^-)\right],$$

$$W\ :\left[H_2^{\frac{1}{2}}(S)\right]^3 \to \left[H_2^1(\Omega^+)\right]^3 \left[\left[H_2^{\frac{1}{2}}(S)\right]^3 \to \left[H_{2.loc}^1(\Omega^-)\right]^3 \cap SK(\Omega^-)\right],$$

are continuous

Theorem 2: Let S be $\mathcal{C}^{\infty}$ - smooth and 1 < *p* < ∞ $\leq t \leq$ ∞ and g $\left[B_{p,t}^{-\frac{1}{p}}(S)\right]^3$, *h* $\in \left[B_{p,t}^{1-\frac{1}{2}}(S)\right]^3$. Then

{V(g)⁺ = {V(g)}⁻ = Hg on S,

$\{T(\partial,n)V(g)\}^{\pm} = \left[\mp 2^{-\frac{1}{3}} + K\right] g$ on S

$\{W(h)\}^{\pm} = \left[\pm 2^{-\frac{1}{3}} + \tilde{K}\right] h$ on S

$\{T(\partial,n)W(h)\}^{\pm} = \{T(\partial,n)W(h)\}^{-}$ = *Lh* on S

The same relation holds for a Lipchitz boundary S and *p* = *t* = *2*

Theorem 3: (i) Let S be C^∞ - smooth and 1 < *p* < $\in$ ≤ *t* ≤ $\in$ and s ∈ ℝ. The operators $\left[H_p^{s+1}(S)\right]^3\left[\left[B_{p,t}^s(S)\right]^3 \to \left[B_{p,t}^{s+1}(S)\right]^3\right]$,

$\pm 2^{-\frac{1}{3}} + K \pm 2^{-\frac{1}{3}} + \tilde{K} : \left[H_p^s(S)\right]^3 \to \left[H_p^s(S)\right]^3\left[\left[B_{p,t}^s(S)\right]^3 \to \left[B_{p,t}^s(S)\right]^3\right]$,

$L : \left[H_p^{s+1}(S)\right]^3 \to \left[H_p^s(S)\right]^3\left[\left[B_{p,t}^{s+1}(S)\right]^3 \to \left[B_{p,t}^s(S)\right]^3\right]$,

are continuous Fredholm operators with zero index. The principal homogenous symbol matrices of these operators are nondegenerate. Moreover, the principal homogeneous symbol matrices of the operators −H and L are positive definite

(ii) If S is Lipschitz, then the operators are continuous Fredholm operators with zero indexes.

$$H : \left[H_s^{-\frac{1}{2}}(s)\right]^3 \to \left[H_s^{\frac{1}{2}}(s)\right]^3,$$

$$\pm 2^{-1/3} + K : \left[H_s^{-\frac{1}{2}}(s)\right]^3 \to \left[H_s^{-\frac{1}{2}}(s)\right]^3,$$

$$\pm 2^{-1/3} + K : \left[H_s^{\frac{1}{2}}(s)\right]^3 \to \left[H_s^{\frac{1}{2}}(s)\right]^3,$$

$$L : \left[H_s^{\frac{1}{2}}(s)\right]^3 \to \left[H_s^{-\frac{1}{2}}(s)\right]^3,$$

Moreover, there exist positive constants C_k, *k* = 1, 2, 3, 4, such that,

$$\langle h, -Hh\rangle s \geq C_1 \left\| h \right\|^2_{\left[H_s^{-\frac{1}{2}}(s)\right]^3} - C_2 \left\| h \right\|^2_{\left[H_s^{-\frac{1}{2}}(s)\right]^3} \text{ for all } h \in \left[H_s^{-\frac{1}{2}}(s)\right]^3,$$

$$\langle Lg, -g\rangle s \geq C_3 \left\| g \right\|^2_{\left[H_s^{\frac{1}{2}}(s)\right]^3} - C_4 \left\| g \right\|^2_{\left[H_s^{0}(s)\right]^3} \text{ for all } g \in \left[H_s^{-\frac{1}{2}}(s)\right]^3,$$

where the symbol $\langle ., .\rangle s$ denotes the duality brackets between the adjoint spaces.

$\left[H_s^{-\frac{1}{2}}(s)\right]^3$, and $\left[H_s^{\frac{1}{2}}(s)\right]^3, T\left[H_s^{-\frac{1}{2}}(s)\right]^3 \to \left[H_s^{-\frac{1}{2}}(s)\right]^3$, is a compact operator.

(iii) The following operator equalities hold in appropriate function spaces

$$\overset{\not{c}}{K} H = HK, LK = K L, LH = -4^{-1/3} + K^2, HL = -4^{-1/3} + \overset{\not{c}}{K}{}^2$$

Here, we recall some results from the theory of strongly elliptic pseudo-differential equations of manifolds with boundary in Besse potentials and Besov spaces, which are the main tools for providing existence theorems for mixed boundary, boundary transmission and crack problems by the potential methods.

Let $M \in \infty C$ be a compact, *n*-dimensional, non-self-intersecting manifold with boundary $\partial M \in \infty C$ and let *A* be a strongly elliptic *N* × *N* matrix pseudo-differentials operator of order *v* ∈ ℝ on *M*. Denote by $\mathfrak{S}(x, \xi)$ the principal homogenous symbol matrix of the operator A in some local coordinate system (x ∈ ∂M, ξ $\mathbb{R}^n \setminus$ {0})..

Let λ_n(x), ... λ_n (x) be the eigenvalues of the matrix $[\mathfrak{S}(x,0,\ldots,0,+1)]^{-1}\left[\mathfrak{S}(x,0,\ldots,0,-1)\right]$, x ∈ ∂M and introduce the notation $\delta_j(x) = \mathrm{Re}[(2\pi)^{-1} \ln \lambda_j(x)], j = 1,\ldots,N$. Here, the branch in the logarithmic function ln ζ is chosen with regard to the inequality $-\pi < \arg\zeta \leq \pi$. Because of the strong ellipticity of A, we have the strong inequality -1/2 < $\delta_j(x)$ < ½ for x ∈ *M, j = 1,2, ... N.* Note that the numbers do not depend on a particular choice of the local coorrodinate system at a fixed pint . In the particular case, when $\mathfrak{S}(x, \xi)$ is a positive definite matrix for every x ∈ *M* and ξ $\mathbb{R}^n \setminus \{0\}$, we have $\delta_j(x)$ = 0 for *j* =1, ... *N* because all the eigenvalues (x) λ_j(*j* = 1, ... N) are positive numbers for any x ∈ *M.*

Consequently, the Fredholm properties of strongly elliptic pseudo-differential operators on manifolds with boundary are characterized by the following theorem.

Theorem 4: Let S ∈ $\mathbb{R}, 1 < p < \infty, 1 \leq t \leq \infty$ and let *A* be a strongly elliptic pseudo-differential operator of order $v \in \mathbb{R}$ that is, there is a positive constant C_o, such that Re $\mathfrak{S}(x, \xi)\eta . \eta \geq c_0 |\eta|^2$

Then, the operators

$$A : \left[\tilde{H}_p^s(M)\right]^N \to \left[H_p^{s-v}(M)\right]^N \left[\left[\tilde{B}_{p,t}^s(M)\right]^N \to \left[B_{p,t}^s(M)\right]^N\right],$$

are Fredholm with zero index if

$$\tfrac{1}{p} - 1 + \sup_{x \in \partial M, 1 \leq j \leq N} \delta_j(x) < -\tfrac{v}{2} < \tfrac{1}{p} + \inf_{x \in \partial M, 1 \leq j \leq N} \delta_j(x)$$

Moreover, the null spaces and indicates of the operators (C1) are the same (for all values of the parameter $t \in [1, +\infty]$) provided that and satisfy the inequality (C2)

Claeys in 2016, studied quasi-local multirace boundary formulation process, also recall here well-established results on boundary formulation process wherein the paper showed that the most fundamental trace space we can introduce consist in the multi-trace space, the Cartesian product of local traces

$$\mathbb{H}\ \Sigma\ := \mathbb{H}\ \Gamma_0\ \text{x}\ldots\text{x}\ \mathbb{H}\ \Gamma_j\ \ where\ \mathbb{H}\ \Gamma_j\ := \mathbb{H}^{+\frac{1}{2}}\ \Gamma_j\ \text{x}\ \mathbb{H}^{-\frac{1}{2}}$$

We endow each $\mathbb{H}(\Gamma_j)$ with the norm $||(v,\ q)||\mathbb{H}(\Gamma_j) := \left(\|v\|^2_{H^{\frac{1}{2}}(\Gamma_j)} + \|q\|^2_{H^{-\frac{1}{2}}(\Gamma_j)}\right)^{\frac{1}{2}}$ and equip $\mathbb{H}(\Sigma)$ with the norm naturally associated with the cartesian product

$$(\|\mathrm{u}\|_{\mathbb{H}(\Sigma)} := \left(\|u_0\|^2_{\mathbb{H}(\Gamma_0)} + \ldots + \|u_n\|^2_{\mathbb{H}(\Gamma_n)}\right)^{\frac{1}{2}}$$

For u = (u_0, ..., u_n) $\in \mathbb{H}(\Sigma)^1$. We write $\langle\ ,\ \rangle_{\Gamma_n}$ for the duality paring between $\mathbb{H}^{+\frac{1}{2}}(\Gamma_j)$ and $\mathbb{H}^{-\frac{1}{2}}(\Gamma_j)$. In the sequel, we shall repeatedly refer to the continuous operator $\gamma : \Pi^n_{j=0} \mathrm{H}^1_{\mathrm{loc}}(\Delta,\Omega_j) \to \mathbb{H}(\Sigma)$ defined by $\gamma(u) := (\gamma_0), \ldots, \gamma_n(u))$ where $: \Pi^n_{j=0} \mathrm{H}^1_{\mathrm{loc}}(\Delta\Omega_j)$ should be understood as the set of u $\mathrm{L}^1_{\mathrm{loc}}$ ($\mathbb{R}^d$) such that $u|\Omega_j \in \mathrm{H}^1_{\mathrm{loc}}(\Delta\Omega_j)$ for all. *j.* We also need a bilinear duality paring for $\mathbb{H}(\Gamma_j)$and $\mathbb{H}(\Sigma)$ we opt for the skew-symmetric version

$$\left[\left[\mathfrak{u},\ \mathfrak{o}\right]\right] := \sum_{j=0}^{n}\left[\mathfrak{u}_j, \mathfrak{o}_j\right]\Gamma_j \text{ where} \left[\begin{pmatrix} u_j \\ p_j \end{pmatrix}, \begin{pmatrix} v_j \\ q_j \end{pmatrix}\right]\Gamma_j := u_j, q_j\Gamma_j - v_j, p_j\Gamma_j$$

This particular choice is well adapted to the ensuing analysis. Note that under the paring [[,]], the space $\mathbb{H}(\Sigma)$ is its own topological dual, and it is easy to show, using the duality between $\mathbb{H}^{+\frac{1}{2}}(\Gamma_j)$ and $\mathbb{H}^{-\frac{1}{2}}(\Gamma_j)$ that the pairing [[,]] induces an isometric isomorphism between $\mathbb{H}(\Sigma)$ and its dual $\mathbb{H}(\Sigma)$, equivalent to the inf-sup condition.

$$\inf_{\mathfrak{u}\in\mathbb{H}(\acute{O})} \sup_{\mathfrak{u}\in\mathbb{H}(\acute{O})} \frac{\left\|\left[\left[\mathfrak{u},\ \mathfrak{o}\right]\right]\right\|}{\|u\|\ \mathbb{H}(\Sigma)\|\mathfrak{v}\|\ \mathbb{H}(\Sigma)} = 1$$

Next we introduce the sol called single-trace space that consists in collections of traces that comply with transmission conditions. This space can be defined by

$$\mathbb{X}(\Sigma) := \mathrm{clos}_{\mathbb{H}(\Sigma)}\{\gamma(u) = (\gamma^j)(u))^n_{j=0} | u \in \ell^\infty(\mathbb{R}^d)\}$$

Where $\mathrm{clos}_{\mathbb{H}(\Sigma)}$ refers to the closure with respect to the norm of $\mathbb{H}(\Sigma)$. By construction, this is a closed subspace of $\mathbb{H}(\Sigma)$. Note also that a function $u \in \mathrm{H}^1_{\mathrm{loc}}(\Delta\bar{\Omega}_j)$x ... × $\mathrm{H}^1_{\mathrm{loc}}(\Delta\bar{\Omega}_j)$ satisfy the transmission codntiiosn of (1), if and only if $\gamma(u) = (\gamma^j)(u))^n_{j=0} \in \mathbb{X}(\Sigma)$. In particular, $u \in \mathrm{H}^1(\Delta,\mathbb{R}^d)$ then $\gamma(u) = (\gamma^j)(u))^n_{j=0} \in \mathbb{X}(\Gamma)$. In the sequel, we will use this space to enforce transmission conditions.

According to Claey, the function $g_k(\chi)$ refers to the injective or ingoing Green's kernel associated to the Helmholtz operator $+\Delta + k^2$. For each subdomain, Ω_j for any (*v, q*) $\in \mathbb{H}(\Gamma_j)$ and any x $\mathbb{R}^d \backslash \Gamma_j$, define the potential operator

$$\mathrm{G}^j_k(v,q)(x) : + \int_{\Gamma_j} q(y) g_k(\chi + y) - v(u) n_j(y).(\nabla g_k(\chi + y) d\sigma(y)$$

The operator G^j_k maps continuously $\mathbb{H}(\Gamma_j)$ into H¹ $(\Delta\bar{\Omega}_j)$x H¹ $(\Delta,\mathbb{R}^d\backslash\Omega_j)$. In particular, G^j_k can be applied to a pair of traces of the form $= \gamma^j(v)$. This potential oeprator can be used to write a representation formula for solution to homogenous Helmholtz equation.

Claey paper went ahead to propose that the operator K^j maps continuously $\mathbb{H}(\Gamma_j)$into $\mathrm{H}^1(\Delta,\Omega_j)$x $\mathrm{H}^1_{\mathrm{loc}}(\Delta,\mathbb{R}^d \backslash \Omega_j)$. Moreover, for any, $\mathfrak{u}$, $\mathfrak{v} \in \mathbb{H}(\Gamma_j)$, $\mathfrak{w} \in \mathbb{H}(\Gamma_k)$ with k $\neq$ j, we have the following properties

i. $\left[\gamma^j\right].K^j(\mathfrak{u}) = \mathfrak{u}$

ii. $\left[\{\gamma^j\}.K^j(\mathfrak{u}),\ \mathfrak{v}\right]\Gamma_j = \left[\{\gamma^j\}.K^j(\mathfrak{u}),\ \mathfrak{v}\right]\Gamma_j$

iii. $\left[\gamma^j.K^k(\mathfrak{w}),\ \mathfrak{u}\right]\Gamma_j = \left[\gamma^j.K^k(\mathfrak{u}),\ \mathfrak{w}\right]\Gamma_j$

The particular treatment that was proposed relied on integral operators for the treatment of junction point. $\mathcal{K}_0$ will refer to a Green kernel of an Helmholtz equation i.e. we will assume that it satisfies $-\Delta\mathcal{K}_0 - \lambda^2\mathcal{K}_0 = \delta_0$ in $\mathbb{R}^d$ in the sense of distributions, for some $\lambda \in \mathbb{C}$. The choice of λ can be decorrelated from the wave numbers K^j coming into pay in (1). So the Green kernels $\mathcal{K}_0$ and $\mathcal{G}$ are independent

The paper also consider a function $\mathcal{K} \in \mathcal{C}^\infty(\mathbb{R}^d \backslash\{0\})$ such that $\mathcal{K}_0$ coincides with $\mathcal{K}_0$ in a neighborhood of 0. It is important to note that, the kernel $\mathcal{K}(x)$ shares the same singularity at *x* = 0 as $\mathcal{K}(x)$ (*x*). Except in a neighborhood of 0, the paper does not impose that $\mathcal{K}(x)$ satisfies any particular equation. It does not a priori need to be the Green kernel of any equation. In addition, the $\mathcal{K}$ may be chosen so as to admit a bounded support located closely around x = 0. For any (*v, q*) set $\in \mathbb{H}(\Gamma_j)$.

$$K^j(v,q)(x) := \int_{\partial\Omega_j} \mathcal{K}(z-y)q(y) + \mathrm{n}_j(y).\ (\nabla\mathcal{K})(x-y)v(y)d\sigma(y)$$

Because the kernel $\mathcal{K}$ only differs from a Green's kernel up to a smooth factor, the operators K^j satisfy many remarkable properties inherited from standard potential operators. Consequently, they satisfy a symmetry property and jump formula

Accordingly, transmission operators proposed in Claey's paper recognizes a fortiori, for writing transmission conditions, that would seem desirable to use Q only at junction points, and use the operator Π anywhere else on Σ. In this section, the paper presents the construction of an operator $\tilde{\Pi}$ similar to Π, but involving the operator Q in fixed arbitrarily small neighborhoods of junction points. Define

$$\mathcal{J} = \bigcup_{0 \le i \le j \le k \le n} \partial\Omega_i \partial\Omega_j \partial\Omega_k$$

By definition, these are the points adjacent to at least three subdomains. For $\mathbb{R}^d = \mathbb{R}^2$ it consists in a finite set of points, and for $\mathbb{R}^d = \mathbb{R}^3$ it consists in a finite union of Lipschitz curves. For $\epsilon > 0$ define | dist $\mathcal{V}_\epsilon := \{x \in \mathbb{R}^d \mid dist\ (x,) < \varepsilon\}$.The paper supposes that ε is chosen small enough to guarantee that

$$\Gamma_{j,k} \setminus \mathcal{V}_{2\epsilon} \neq \varnothing \qquad \forall j,k = 0 \ldots n$$

The paper consider a $\mathcal{C}^\infty$ cut-off function $\psi : \mathbb{R}^d \to \mathbb{R}$ such that $\psi(x) = 1$ for $x \in \mathcal{V}_\epsilon$ and $x \in = 0$, and $x \in \mathbb{R}^d \setminus \mathcal{V}_{2\epsilon}$ and define $\chi := 1 - \psi^2$. In particular χ vanishes over a neighborhood of junctions points.

In the sequel, for any $\mathfrak{u} = (\, uj, pj\,)_{j=0}^n \in \mathbb{H}(\Sigma)$, the paper will denote $\chi u := \chi uj, \chi p\,)_{j=0}^n$. And the paper adopt a similar notation. This implies that $[\![\chi\mathfrak{u},\ \mathfrak{v}]\!] = [\![\mathfrak{u},\ \chi\mathfrak{v}]\!]$, and also $[\![u,\mathfrak{v}]\!] = [\![\chi\mathfrak{u},\ \mathfrak{v}]\!] + [\![\psi\mathfrak{u},\ \psi\mathfrak{v}]\!]$,. Following this, the paper defined the continuous operator $\tilde{\Pi} : \mathbb{H}(\Sigma) \to \mathbb{H}(\Sigma)$ by

$$\tilde{\Pi}(\mathfrak{u}),\mathfrak{v} := \Pi(\chi\mathfrak{u}),\mathfrak{v} + Q(\psi\mathfrak{u}),\psi\mathfrak{v} \qquad \forall \mathfrak{u},\ \mathfrak{v} \in \mathbb{H}(\Sigma)$$

Recalling the establishment of such boundary conditions, this paper abstracted a filtration operator from a similar Stefan-Boltzman chaotic process and idealizes transition attempts by variables to domains permissible by a transition operator filtrating from acceptance to rejection region, as a threshold gate. This paper have given concise and correlated narration of such filtration process in physical sciences (thermodynamics) as exemplified theoretically in Maxwell's demon and mathematically that it exist by the works of Spence *et. al.* (2015) on coercivity properties of boundary integral in distributive fields, Costabel (2016) on Boundary operators in Lipchultz domains, Bernadi et *al.* in 2016 studied on continuity properties of divergence in conservative fields and backward stretch to Krall in 1974, on Stieltjes differential-boundary operators. These studies in unison have corroborated and shown congruency with this paper preposition of boundary adaptation and formulation on filtration parameter. On the basis of these, this paper idealized a transition operator responsible for skewing the chaotic $\{\varphi_e \varphi_i\}$ distribution been an operator (Nmorcha) that governs the permissible filtration process that is reminiscence of Maxwell's demon. The determinant of **Table 5** matrix been the characteristic value responded to this task of an operand which is required to operate at the boundary of the critical region of a normal distribution to derive a threshold value by operating on $x + 2\sigma$ point (critical point) i.e.

det $\nabla_* \left[x + 2\sigma\right]$ = Threshold value (18)

Such value is extrapolated from the calibrated scale or fitted to show a corresponding combinatorial value of source and cause of insolvency as the limiting point of a contractor from becoming insolvent. This is in conformity with the test statistics of critical value which separates the critical (or rejection) region from the acceptance region.

This geometrically corresponds to single tail (right or left) evaluation as;

So that **Figure 3** gives Z > Zα

Figure 4 gives Z < -$-z_{\alpha\rho}$ with $|Z| > Z\alpha\}$ critical region at level of sig. α Accordingly, combining **Figure 3** and **Figure 4** gives a two (2) tail test critical region at $Z > z\frac{\alpha}{2}$ or $Z < -z\frac{\alpha}{2}$

Derivably, the critical value which corresponds to a boundary point is the threshold value, whose test was conducted at 5% significant level by Equation (18) to give 3.1025 ≅ 3.10 **(Figure 5)**.

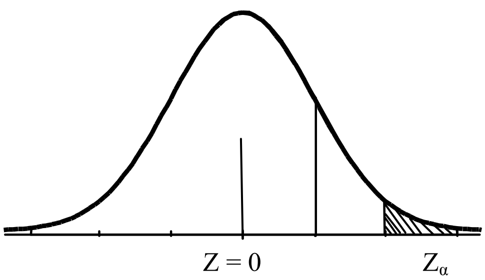


Figure 3. Right tail test (This geometrically corresponds to single tail (right or left) evaluation).

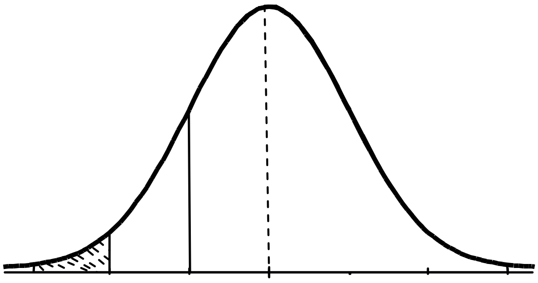


Figure 4. Left tail test (This geometrically corresponds to single tail (right or left) evaluation).

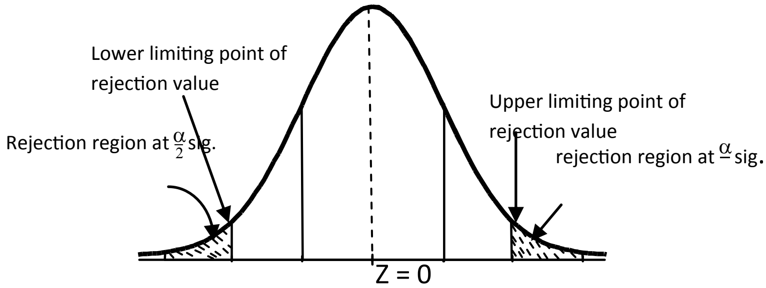
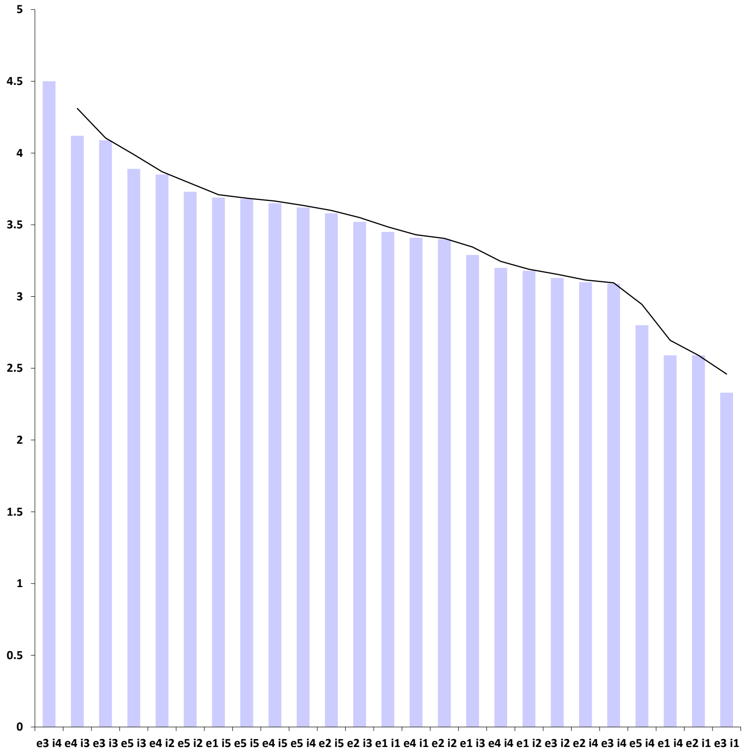


Figure 5. Rejection region.



Graph 1. Calibration scale.

When this value is fitted on the insolvency calibration scale, it corresponded to e_2i_4 which implies that with other leading factors giving rise to insolvency, **(Graph 1)** that are tolerable, a contractor will slip into becoming insolvent when he starts falsification of the projects' financial status (e_2) arising from poor project cost control (i_4).

CONCLUSION

A sectors' matrix determinant from Maxwell's theory of poles on spherical harmonic polynomial, conscripted the idea that the construction industry sector of any spherical economy requires the determination of insolvent point of a contractor in construction projects by thought experiment, mind's eye and model and model base reasoning. The result of the sectors' investigation precipitated five (5) main causes of insolvency from five (5) main causes and five (5) significant unethical practice yielded combinatorial variables of ij^{th} combinations to 'n' values. Those values were subjected to the normal distribution test for consistency. On successful normality test, the variables were conscripted into a chaos field in the normal distribution space. The chaos experienced in the space was governed by the space potential that tends to analyse variable by gradient transmission to accept or reject domain, otherwise without such potentiation entity, variables would remain in the tolerable domain. It is the conjecture of this paper that variables in the reject domain are not there by accident nor in the accept domain. Consequently, model-based reasoning, mind's eye and thought experiment led this investigation to idealize a plane at the boundary between accept or reject domain. Such boundary existence has been acknowledged greatly in the works of Athanasiadis, et al. Claey, Spence, etc. By introspecting on Maxwell, James Clark's work on thermodynamic demon, this paper moved to propose an *Nmorcha* operator at the accept or reject boundary plane of the normal distribution space responsible for the filtration of variables into either divide. The precipitated five (5) significant unethical practice and five (5) main causes of insolvency in matrix form yielded a characteristic determinant as pseudo-differential operator that operates on all the variables chaotically distressed at the boundary plane of accept or reject. This operator at this joint filters variables by gradient transmission to either divide. This process resulted in e_2i_4 variable on the insolvent point of contractors which on a metric calibration presented falsified project's financial status and poor project cost control as the point of an insolvent contractor.

Recommendations

This paper moved from a thought experiment to idealize a fitting point on a calibrated insolvency scale to identify insolvent point of contracts. Clearly project management investigation should harness the construction economics theory of this paper to understand when contractors become insolvent.

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