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Influence of Side Walls on Unsteady MHD Flow of a Newtonian Fluid through a Porous Medium

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Abstract: In this paper, the unsteady MHD flow of viscous fluid between two side walls is studied with the help of integral transforms technique. The plate provides an oscillating shear stress to the fluid at a time $t = 0^+$. Exact solutions have been established and given as a combination of transient and steady-state solutions. The governing equation satisfy the appropriate boundary and initial conditions. The obtained results are reduced to those solutions which are corresponding for the flow over an infinite plate. Keeping in view the measure value of velocity and shear stress in the middle of the channel, being not affected due to the side walls, the required time to attain the steady-state and the distance between the sides wall are calculated graphically.

Keywords: MHD, Porous medium, Side walls, Viscous fluid, Exact solutions, Oscillating shear stress

I. INTRODUCTION

Navier-Stokes equations are widely used to describe the Newtonian fluids behavior. It is because of the fact that they are relatively simple and their solutions are convenient. Newtonian fluid flow over an oscillating plate is not only of fundamental interest but it is used in various applied problems. These types of motions are called Rayliegh or Stokes' second problem in literature [1]. Those fluids which are conducting electrically, have great importance in the various fields of technology and engineering like magneto hydrodynamic (MHD) power generators and pumps. MHD flow has also various applications in electronics, metrology, aeronautics, solar physics, motion of earth cores and chemical engineering. The effect of magnetic field and heat transfer on the fluid flow are used in generator pump, magneto hydrodynamic accelerator, nuclear reactor, geothermal energy, extractions, plasma study and the boundary layer controlling in the field of aerodynamics [2]. In numerous industrial processes the cooling filaments or continues strips by drawn them through a quiescent fluid. Sometime these strips are stretched during this processes. When the fluid is electrically conducting due to strong magnetic field it controlled the rate of cooling and the final product with its required characteristics [3]. Penton [4] obtained starting solutions of the transient component for the flow of viscous fluid owing to an oscillating plate. Tokunda [5] studied about the impulsive motion of unsteady laminar boundary-layer flow over a semi-infinite flat plate. Puri et al. [6] obtained the exact solutions for the unsteady flow of a viscous fluid in the presence of magnetic field in rotating medium by using Laplace transform. Sulochana [7] obtained the starting solutions for MHD three dimensional rotating flow of viscous fluid in a porous medium. The flow of viscous fluid over an infinite plate with different initial and boundary conditions have been studied by many researchers [8-11]. Erdogan [9] recently established exact solutions for the motion of Newtonian viscous fluid due to sinusoidal oscillations of a plate, however he did not manage to express them as a sum of transient and steady state solutions. Bearing in mind this problem, Fetecau et al. [12] presented the starting solutions of the Stokes second problem for Newtonian fluids which is presented as a combination of steady- state and transient solution. Soundalgekar et al. [13] studied the effect of MHD on impulsively vertical infinite plate with different temperatures in the presence of magnetic field. Sami ul Haq et al.

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[14] obtained exact solution of magnetohydrodynamic flow passed an impulsively vertical plate in the presence of thermal diffusion and ramped wall temperature of the plate embedded in a medium which is porous. Fetecau et al. [15] studied magnetohydrodynamic free convection flow of an incompressible Newtonian viscous fluid past over vertical oscillating infinite plate with uniform heat flux in a medium which is porous. In another paper Fetecau et al. [16] discussed the solutions of viscous fluid between two side walls perpendicular to the plate. According to the literature, the exact solutions related to the MHD flow of a viscous fluid between two side walls through a porous medium generated by a flat plate that exerts an oscillating shear stress to the fluid has not been investigated therefore, we extended the work of Fatecau et al. [16].

II. FORMULATION OF THE PROBLEM

Let us consider unsteady two-dimensional MHD flow of a Newtonian viscous fluid passing through a porous medium between two parallel side walls normal to the infinite plate and initially both, fluid and infinite plate are stationary. After time $t = 0^+$ the plate exerts an oscillating shear stress of the form $\{f\cos(\omega t)$ or $f\sin(\omega t)\}$to the fluid and as a result the fluid moved gradually as shown in the Fig. 1. Its velocity is given by:

$$\mathbf{V}=u(y,z,t)\mathbf{I} \quad (1)$$

Were **i** represents unit vector along the -axis of the Cartesian coordinate system x, y and z
Momentum equation is:

$$\frac{d\mathbf{V}}{dt} = div\mathbf{T} + \rho\mathbf{F} \quad (2)$$

Here Cauchy stress tensor is denoted by **T**, ρ**F** represents body forces, defined as

$$\rho\mathbf{F}=\mathbf{J}\times\mathbf{B}+\mathbf{R} \quad (3)$$

Where **R** stand for Darcy's resistance and Lorentz force is denoted by **J**×**B**.

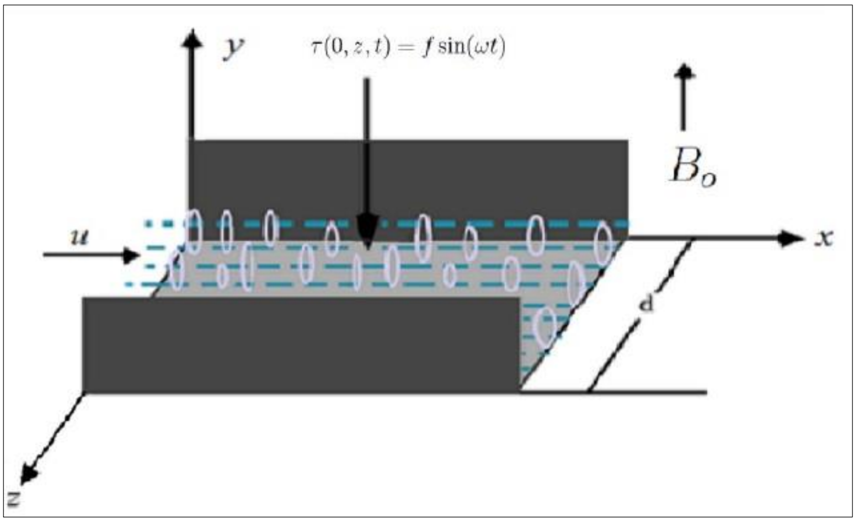

Fig. 1. Schematic diagram.

As in the present problem Newtonian fluid is taken, therefore, the constitutive equation for Cauchy stress tensor is given by:

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$$\mathbf{T}=-p\mathbf{I}+\mu\mathbf{A}_1 \qquad (4)$$

Where μ represents dynamic viscosity, p denotes pressure, **I** stands for unit tensor, the first Rivlin-Ericksen kinematic tensor is given by $\mathbf{A}_1$ define as:

$$\mathbf{A}_1=\mathbf{L}+\mathbf{L}^t \qquad (5)$$

Here **L** represents gradient of the velocity. By using Equations 1-5, we get the dimensional governing equation in the following form:

$$\frac{\partial u(y,z,t)}{\partial t}=v\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)u(y,z,t)-\left(\frac{\sigma B_0^2}{\rho}+\frac{v\varphi}{k}\right)u(y,z,t) \text{ for } z\in(0,d) \text{ and } t>0 \qquad (6)$$

where porosity of the porous medium is denoted by φ, σ is electrical conductivity, ρ stands for the density, $k>0$ is the permeability of the porous medium, B_0 is the applied magnetic field and kinematic viscosity is denoted by v. The associated dimensional boundary and initial conditions are:

$$\left.\begin{array}{c} u(y,z,0)=0 \text{ for } y>0 \text{ and } z\in[0,d] \\ u(y,0,t)=u(y,d,t)=0 \text{ for } y,t>0 \\ \tau(0,z,t)=\mu\left.\dfrac{\partial u(y,z,t)}{\partial y}\right|_{y=0}=f\sin(\omega t) \\ \text{or } f\cos(\omega t) \text{ for } z\in(0,d) \text{ and } t>0 \\ u(y,z,t),\dfrac{u(y,z,t)}{\partial y}\to 0 \text{ as } y\to\infty \end{array}\right\} \qquad (7)$$

Where f is constant and ω is the frequency of the shear stress. There are two non-trivial shear stresses i.e. shear stress on the bottom wall and shear stress on the parallel side walls are given as follows:

$$S_{xy}=\frac{\partial u(y,z,t)}{\partial y} \qquad (8)$$
$$S_{xz}=\frac{\partial u(y,z,t)}{\partial z}$$

In order to obtain dimensionless form of governing equation (6) and corresponding dimensionless boundary and initial conditions equation (7) and to reduce the number of essential parameters, we consider the following non-dimensional variables.

$$y^*=\frac{y}{\sqrt{vt_0}}\ ;t^*=\frac{t}{t_0}\ ;u^*=\sqrt{\frac{t}{t_0}}u$$
$$z^*=\frac{y}{\sqrt{vt_0}}\ ;\tau^*=\frac{\tau}{\mu}t_0\ ;\ f^*=\frac{f}{\mu}t_0$$

By using the above dimensionless variables into governing equation (7) and imposed initial and boundary conditions equation (8) and dropping out the star notation, we obtained dimensionless governing equation in the simplified form as:

$$\frac{\partial(y,z,t)}{\partial t}=\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)(y,z,t)-M(y,z,t)-\frac{1}{K}u(y,z,t) \qquad (9)$$

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Here $M=\dfrac{\sigma B_o^2 t_o}{\rho}$ represents Hartman number and $\dfrac{1}{K}=\dfrac{\varphi v t_o}{k_1}$ denotes porosity parameter. The corresponding non-dimensional boundary and initial conditions are:

$$\left.\begin{aligned}
&u(y,z,0)=0 \; for \; y>0 \; and \; z\in[0,d],\\
&u(y,0,t)=u(y,d,t)=0 \; for \; y,t>0,\\
&\tau(0,z,t)=\left.\frac{\partial u(y,z,t)}{\partial y}\right|_{y=0}=f\sin(\omega t) \text{ or}\\
&f\cos(\omega t) \; for \; z\in(0,d) \; and \; t>0,\\
&u(y,z,t),\frac{\partial u(y,z,t)}{\partial y}\to 0 \; as \; y\to\infty
\end{aligned}\right\} \qquad (10)$$

III. SOLUTION OF THE PROBLEM

$\tau(0,z,t)=f\sin(\omega t)$

Consider a viscous fluid, which is initially at rest, over an infinite plate in the (x,z)-plane and between two side walls placed in the planes $z=d$ and $z=0$. At time $t=0^+$ the plate provides an oscillating shear stress to the fluid:

$$\tau(0,z,t)=\left.\frac{\partial u(y,z,t)}{\partial y}\right|_{y=0}=f\sin(\omega t) \text{ or } f\cos(\omega t) \; for \; z\in(0,d) and \; t>0$$

(11)

For the solution of the problem we use the definition of Fourier sine and cosine transforms, therefore, we multiply Equation (9) with $\sqrt{\dfrac{2}{\pi}}\cos(\zeta y)\sin(\lambda_n z)$ where $\lambda_n=\dfrac{n\pi}{d}$ and integrating the obtained result with respect to z and y from 0 to d and 0 to ∞ respectively, with the help of accompanied boundary and initial conditions we obtain the differential equation:

$$\frac{\partial u_{cn}(\zeta,t)}{\partial t}+\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)u_{cn}(\zeta,t)=\sqrt{\frac{2}{\pi}}\frac{(-1)^n-1}{\lambda_n}f\sin(\omega t) \qquad (12)$$

With

$\zeta, t>0$, n=1,2,3,..

Where the Fourier sine and cosine transforms

$$u_{cn}(\zeta,t)=\sqrt{\frac{2}{\pi}}\int_0^d\int_0^\infty u(y,z,t)\sin(\lambda_n z)\cos(y\zeta)dydz \qquad n=1,2,3,\ldots$$

Of $u_{cn}(\zeta,t)$ have to comply with the initial condition:

$$u_{cn}(\zeta,0)=0 \text{ for } \zeta>0 \text{ and } n=1,2,3\ldots \qquad (13)$$

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Equation (12) is the differential equation in variable t for each fixed ζ and its solution is obtained by using Equation (13) as:

$$u_{cn}(\zeta,t) = f\sqrt{\frac{2}{\pi}}\frac{(-1)^n-1}{\lambda_n}\frac{\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)\sin(\omega t)-(\omega)\cos(\omega t)}{\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)^2+(\omega)^2} + f\sqrt{\frac{2}{\pi}}\frac{(-1)^n-1}{\lambda_n}\frac{\omega}{\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

Here m=2n-1 assigning d=2h and varying the origin of the coordinate system, substituting $z = z' + h$ and ignoring the prime notation. Equation (15) implies:

$$u_s(y,z,t) = \frac{8f}{\pi d}\sum_{n=1}^{\infty}\frac{\sin(\lambda_m z)}{\lambda_m}\int_0^{\infty}\frac{(\omega)\cos(\omega t)-\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)\sin(\omega t)}{\left(\zeta^2+\lambda_m^2+M+\frac{1}{K}\right)^2+(\omega)^2}\cos(y\zeta)d\zeta + \frac{8f}{\pi d}\omega\sum_{n=1}^{\infty}\frac{\sin(\lambda_m z)}{\lambda_m}\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\lambda_m^2+M+\frac{1}{K}\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\lambda_m^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(15)

$$u_s(y,z,t) = \frac{4f}{\pi h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)\sin(\omega t)-(\omega)\cos(\omega t)}{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)^2+(\omega)^2}\cos(y\zeta)d\zeta + \frac{4f}{\pi h}\omega\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(16)

Where $\psi_n = (2n-1)\frac{\pi}{2h}$ the following identities are used in the problem:

$$\omega\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)^2+\omega^2}d\zeta = \frac{\pi}{2}\frac{C_n\cos(yC_n)+Dn\sin(yC_n)}{C_n^2+D_n^2}e^{-yD_n} \tag{17}$$

$$\int_0^{\infty}\frac{\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)\cos(y\zeta)}{\left(\zeta^2+\lambda_n^2+M+\frac{1}{K}\right)^2+\omega^2}d\zeta = \frac{\pi}{2}\frac{D_n\cos(yC_n)+Cn\sin(yC_n)}{C_n^2+D_n^2}e^{-yD_n} \tag{18}$$

Where,

$$2C_n^2 = \sqrt{\left(\psi_n^2+M+\frac{1}{K}\right)+\omega^2} - \left(\psi_n^2+M+\frac{1}{K}\right)$$

$$2D_n^2 = \sqrt{\left(\psi_n^2+M+\frac{1}{K}\right)+\omega^2} + \left(\psi_n^2+M+\frac{1}{K}\right)$$

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Vol. 6, Issue 12, December 2017

By using the above mentioned identities in the Equation (16), we obtain the simplified form of the velocity field:

$$u_s(y,z,t)=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\frac{D_n\sin(\omega t-yC_n)-C_n\cos(\omega t-yC_n)}{C_0^2+D_0^2}e^{-yD_n}+\frac{4f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(19)

Equation (19) is the starting solution which satisfy all the appropriate initial and boundary conditions, for instance, we take help from Equation (18). For some time it described the motion of fluid after its initiation. So, the transient component:

$$u_{st}(y,z,t)=\frac{4f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(20)

Vanish after that time, and starting solution coincides with the steady-state solution

$$u_{ss}(y,z,t)=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\frac{D_n\sin(\omega t-yC_n)-C_n\cos(\omega t-yC_n)}{C_n^2+D_n^2}e^{-yD_n} \quad (21)$$

We have to find two non-trivial shear stresses in the plane, one of them is along the bottom wall and the other is on the side walls. These shear stresses are denoted by $S_{xy}(y,z,t)$ and $S_{xz}(y,z,t)$ respectively. The simplified form of shear stress is follows:

$$T_s(y,z,t)=\frac{\partial u(y,z,t)}{\partial y}=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(\psi_n z)}{\psi_n}e^{-yD_n}\sin(\omega t-yC_n)+\frac{4f}{h}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(22)

Where $\tau(y,z,t)=S_{xy}(y,z,t)$

$\tau(0,z,t)=f\cos(\omega t)$(23)

Using the above procedure, we find the corresponding exact solutions for the cosine oscillation as:

$$u_c(y,z,t)=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\frac{C_n\sin(\omega t-yC_n)+D_n\cos(\omega t-yC_n)}{C_n^2+D_n^2}e^{-yD_n}+\frac{4f}{\pi h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)\cos(y\zeta)}{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(24)

Equation (23) is the starting solution which satisfy all the appropriate initial and boundary conditions, for instance, we take help from Equation (18). For some time it described the motion of fluid after its initiation. So, the transient component

$$u_{cs}(y,z,t)=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\frac{C_n\sin(\omega t-yC_n)+D_n\cos(\omega t-yC_n)}{C_n^2+D_n^2}e^{-yD_n} \quad (25)$$

We have to find two non-trivial shear stresses in the plane, one of them is along the bottom wall and the other is on the side walls. These shear stresses are denoted by $S_{xy}(y,z,t)$ and $S_{xz}(y,z,t)$. The simplified form of shear stress is follows:

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$$T_c(y,z,t)=\frac{\partial u(y,z,t)}{\partial y}=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(\psi_n z)}{\psi_n}e^{-yD_n}\cos(\omega t-yC_n)+\frac{4f}{h}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\zeta\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)\sin(y\zeta)}{\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)^2+\omega^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(26)

IV. Special cases

Flow Over an Infinite Plate

In the nonexistence of the side walls, i.e., h→∞, the solutions of Equations 19, 22, 23 and 26 reduce to their simplified forms:

$$u_s(y,t)=f\sqrt{\frac{1}{\omega}}e^{-y\sqrt{\frac{\omega}{2}}}\sin\left(\omega t-y\sqrt{\frac{\omega}{2}}+\frac{3\pi}{4}\right)-\frac{2f}{\pi}\times\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+M+\frac{1}{K}\right)^2+\omega^2}\times\left[\exp\left(-\left(\zeta^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(27)

$$T_s(y,t)=fe^{-y\sqrt{\frac{\omega}{2}}}\sin\left(\omega t-y\sqrt{\frac{\omega}{2}}\right)-\frac{2f\omega}{\pi}\times\int_0^{\infty}\frac{\sin(y\zeta)}{\left(\zeta^2+M+\frac{1}{K}\right)^2+\omega^2}\times\left[\exp\left(-\left(\zeta^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(28)

Similarly,

$$u_c(y,t)=f\sqrt{\frac{1}{\omega}}e^{-y\sqrt{\frac{\omega}{2}}}\cos\left(\omega t-y\sqrt{\frac{\omega}{2}}+\frac{3\pi}{4}\right)-\frac{2f}{\pi}\times\int_0^{\infty}\frac{\left(\zeta^2+M+\frac{1}{K}\right)\cos(y\zeta)}{\left(\zeta^2+M+\frac{1}{K}\right)^2+\omega^2}\times\left[\exp\left(-\left(\zeta^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(29)

$$T_c(y,t)=fe^{-y\sqrt{\frac{\omega}{2}}}\cos\left(\omega t-y\sqrt{\frac{\omega}{2}}\right)-\frac{2f\omega}{\pi}\times\int_0^{\infty}\frac{\zeta^3\sin(y\zeta)}{\left(\zeta^2+M+\frac{1}{K}\right)^2+\omega^2}\times\left[\exp\left(-\left(\zeta^2+M+\frac{1}{K}\right)t\right)\right]d\zeta$$

(30)

Where and z=0,ω≠0 and $C_n=D_n=\sqrt{\frac{\omega}{2}}$

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(An ISO 3297: 2007 Certified Organization)

Vol. 6, Issue 12, December 2017

Solution in the Absence of Magnetic and Porous Effects

By making M=0 and $\dfrac{1}{K}$=0 in Equations 19, 22, 23 and 26, we recover the following results of Fetecau et al. [17-19], Equations 14, 17-19:

$$u_s(y,z,t)=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\frac{D_n\sin(\omega t-yC_n)+C_n\cos(\omega t-yC_n)}{C_n^2+D_n^2}e^{-yD_n}+\frac{4f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\psi_n^2\right)^2+(\omega)^2}\times e^{-y(\zeta^2+\psi_n^2)d\zeta}$$

(31)

$$T_s(y,z,t)=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(\psi_n z)}{\psi_n}e^{-yD_n}\sin(\omega t-yC_n)+\frac{4f}{h}\omega\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\zeta\sin(y\zeta)}{\left(\zeta^2+\psi_n^2\right)^2+\omega^2}e^{-(\zeta^2+\psi_n^2)t}d\zeta$$

(32)

Similarly,

$$u_c(y,z,t)=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\frac{C_n\sin(\omega t-yC_n)+D_n\cos(\omega t-yC_n)}{C_n^2+D_n^2}e^{-yD_n}+\frac{4f}{h}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\cos(y\zeta)}{\left(\zeta^2+\psi_n^2\right)^2+(\omega)^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2\right)t\right)\right]d\zeta$$

(33)

$$T_c=\frac{\partial u(y,z,t)}{\partial y}=\frac{2f}{h}\sum_{n=1}^{\infty}\frac{(-1)^{p+1}\cos(\psi_n z)}{\psi_n}e^{-yD_n}\cos(\omega t-yC_n)+\frac{4f}{h}\omega\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(\psi_n z)}{\psi_n}\int_0^{\infty}\frac{\zeta\left(\zeta^2+\psi_n^2\right)\sin(y\zeta)}{\left(\zeta^2+\psi_n^2\right)^2+\omega^2}\times\left[\exp\left(-\left(\zeta^2+\psi_n^2\right)t\right)\right]d\zeta$$

(34)

V. NUMERICAL RESULTS AND DISCUSSION

In this graphical work, we studied the unsteady motion of Newtonian viscous fluid between two side walls over an infinite plate. The infinite plate provide an oscillating shear stress to the fluid and because of this shear stress the fluid moves gradually. The Fourier sine and cosine transform technique have been used to obtain the starting solution for the described motion, which are given as a combination of transient and steady state solutions and they satisfy the boundary and initial condition as well as the governing equation. The starting solution presents the motion of fluid after some time of its initiation.

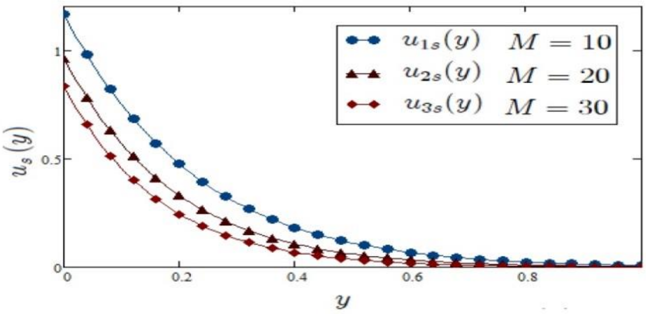

Fig. 2. Profile of velocities $u_s(y)$ given by Equation (19), for f=-5, k=0.5 and ω=2π, h=0.5, t=0.25 various values of M.

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(An ISO 3297: 2007 Certified Organization)

Vol. 6, Issue 12, December 2017

After that time, the transients vanish, and these solutions coincide with the steady state solution. Figs. 2 and 3 are plotted to show the effect of magnetic field on the fluid velocity. It is revealed from these figures that velocity is the decreasing function of magnetic parameter M Physically, larger values of M enhances the drag force called the Lorentz force, which retards the fluid flow. Figs. 4 and 5, illustrate that by increasing the permeability of the porous medium the velocity of the fluid for both sine and cosine oscillations are also increase and physically this shows that the resistive forces are decreases. In Figs. 6 and 7, we observed that the time which required to attain the steady state increases if the frequency ω of the shear stress decreases. Figs. 8 and 9 illustrate about the time which is required to attain the steady state for different values of h. It is clearly observed that there is a direct variation between time and distance of the side walls. Fig. 10 illustrates that when time $t\to 0$ the velocity $u_s(y,t)$ for sine oscillation increases due to the absence of shear stress and the velocity $u_c(y,t)$ for cosine oscillation decreases due to some constant shear stress greater than zero.

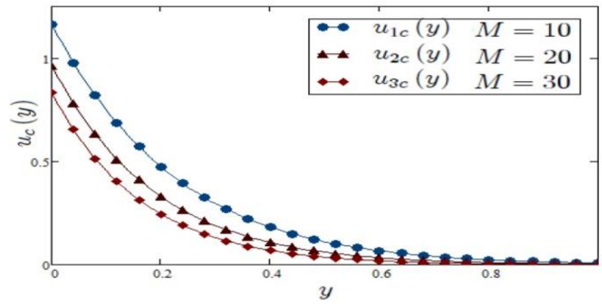

Fig. 3. Profile of velocities $u_c(y)$ given by Equation 23, for f=5, k=0.5 and ω=2π, h=0.5, t=0.25 various values of M.

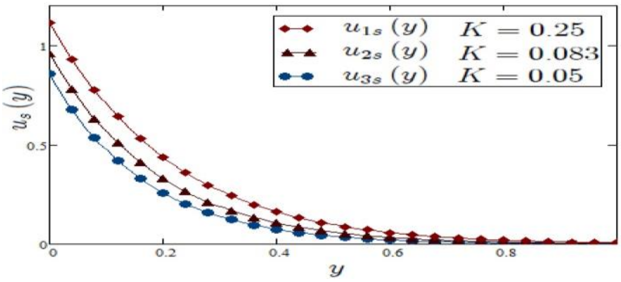

Fig. 4. Profile of velocities $u_s(y)$ given by Equation 19, for f=-5, M=10 and ω=2π, h=0.5, t=0.25 various values of K.

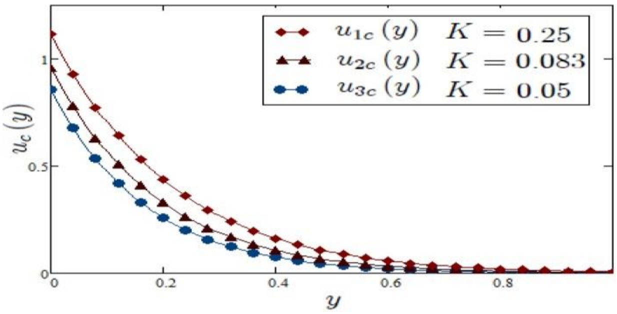

Fig. 5. Profile of velocities $u_c(y)$ given by Equation 23, for f=5, k=0.5 and ω=2π, h=0.5, t=0.25 various values of K.

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(An ISO 3297: 2007 Certified Organization)

Vol. 6, Issue 12, December 2017

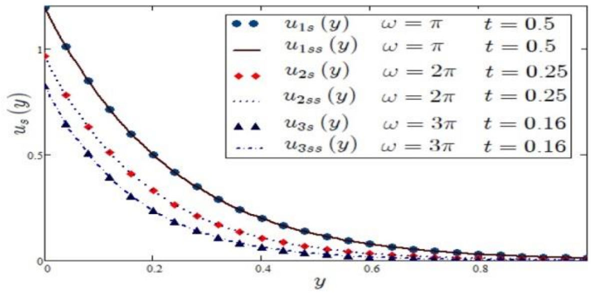

Fig. 6. Profile of velocities $u_s(y)$ and $u_{ss}(y)$ given by Equation 19 and Equation 21, for f=-5, k=0.5,M=10, h=0.5 and various values of ω and t.

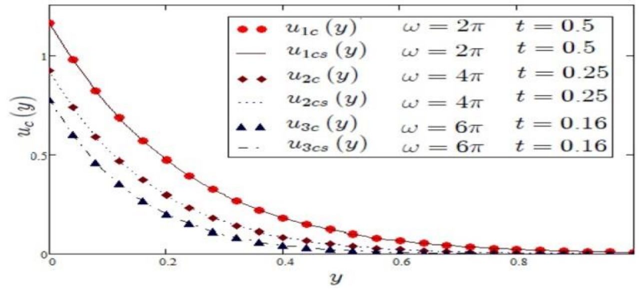

Fig. 7. Profile of velocities $u_c(y)$ and $u_{cs}(y)$ given by Equation 23 and Equation 25, for f=5, k=0.5,M=10, h=0.5 and various values of ω and t.

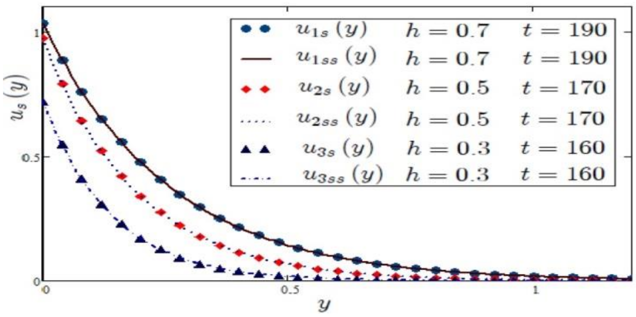

Fig. 8. Profile of velocities $u_s(y)$ and $u_{ss}(y)$ given by Equations 19 and Equation 21, for f=-5, k=0.5,M=10, ω=0.6 and various values of h and t.

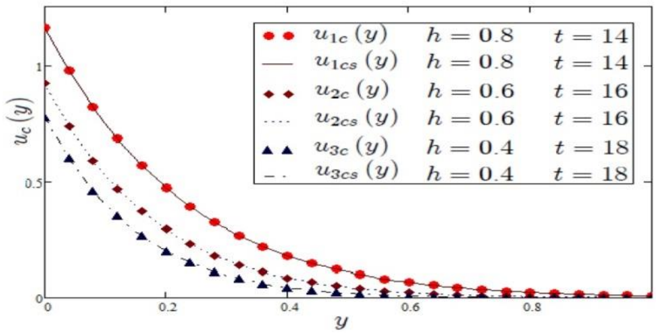

Fig. 9. Profile of velocities $u_c(y)$ and $u_{cs}(y)$ given by Equation 23 and Equation 25, for f=5, k=0.5,M=10, ω=0.2 and various values of h and t.

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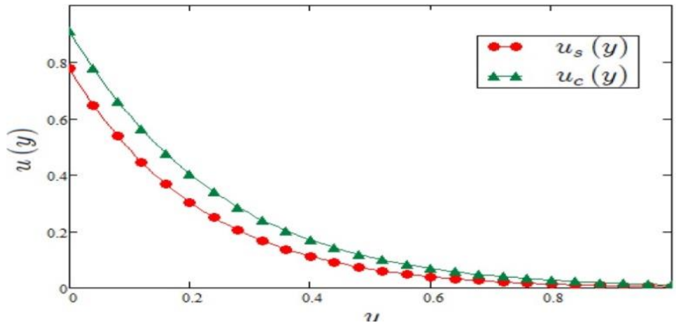

Fig. 10. The comparison of velocities $u_s(y)$ and $u_c(y)$ given by Equation 19 and Equation 23, for f=-5, k=0.5,M=10, t=0.25 $\omega = \pi$..

VI. CONCLUSION

Unsteady flow of an electrically conducting viscous fluid between two side walls of an infinite plate is examined in a porous medium. The plate exerts an oscillating shear stress to the fluid. The Fourier cosine and sine transforms have been used and exact solutions are determined in general form. Results are plotted and it is found that Magnetic parameter reduce the fluid motion and permeability parameter increase the fluid velocity. It is noted that, there is a direct variation between time and distance of walls. It is also found that the required time to attain the steady state increases if the frequency ω of the shear stress decreases. This work can be generalized to several non-Newtonian fluids. Hence, this study will provide a base for other researchers.

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