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Intuitionistic Fuzzy Programming Technique to Solve Multi-objective Transportation Problem

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ABSTRACT

This paper presents a solution of the multi-objective transportation problem via Fuzzy Programming Algorithm and the product is to be transported from source i to destination j. The time and cost of transportation from source i to destination j were recorded. Here we have considered MOTP with intuitionistic fuzzy numbers and completed the problem in both ways. Therefore the optimal compromise solution will remain same both the exponential and linear membership function. For the solution the membership functions are used for such a problem. LINDO/TORA statistical software was employed in the data analysis and is completed in two stages..

INTRODUCTION

Transportation Problem is a special type of Linear programming problem which arises in many practical applications.TP is one of the powerful framework which ensures efficient movement and timely availability of the raw material .It is one of the best optimization method applicable in various real life human activity. At early it was founded for determining the optimal shipping pattern, it is called transportation problem. So it deals with transportation of goods from each of m origins $i = 1,2,3, \dots .\, m$ to any of n destinations $j = 1,2,3, \dots .\, n.$ One must determine the amount x_{ij} to be transported from all the origin $i = 1,2,3, \dots .\, m,$ to all the destination $j = 1,2,3, \dots .\, n$ in such a way that the total cost is minimized. All the transportation problems are not single objective which are characterized by multiple objective functions are considered here. A special type of linear programming problem in which constraint are of equality type and all the objectives are conflicting with each other, are called MOTP. The multi-objective transportation problem is a vector minimum problem. In the case of the multi-objective fuzzy linear programming technique, only the objectives are fuzzy. The fuzzy linear programming technique for the multi- objective transportation problem gives an optimal compromise solution. Diaz [1,2] developed an algorithm for finding the solution of multi-objective transportation problem. Isermann [3] developed an algorithm for identifying all the non-dominated solution, for a linear multi-objective transportation problem. Ringuest and Rinks [4] developed two interactive algorithms for solving multi-objective transportation problem. Zimmermann [5] first applied suitable membership functions to solve linear programming problem with several objective functions. He showed that solutions obtained by fuzzy linear programming are always efficient. Bit et al. [6] applied the fuzzy programming technique with linear membership function to solve the multi-objective transportation problem. Hitchcock [7] studied and modelled basic transportation in form of standard linear programming problem. In beginning of decision making parameters of MOTP are assumed to be fixed in values. But due many uncertain situation like road conditions, traffic conditions, variation in diesel prices etc. and some other unpredicted factors like weather condition. Therefore due to these reason conventional models are not valid. Zadeh [8] introduced the concept of fuzzy sets for dealing uncertainty. Later Bellman and Zadeh [9] used it for decision making of real life problems. Verma et al. [10] applied fuzzy programming technique to solve MOTP via some non-linear membership functions. Das et al. [11] proposed solution methods for the solution of MOTP with interval cost, source and destinations parameters. Li and Lai [12] have presented a fuzzy compromise programming approach to multi-objective transportation problems. Wahed [13] initiated the optimal compromise solution of MOTP and obtained solution is tested by using degree of closeness of the compromise solution to the ideal solution using family of distances.

Leberling [14] used a special- type nonlinear membership function for the vector maximum linear programming problem. He showed that solution obtained by fuzzy linear programming with this type of nonlinear membership function is always efficient. Dhingra and H. Moskowitz [15] defined other types of the non-linear (exponential, quadratic and logarithmic) membership functions and applied them to an optimal design problem. Verma, Biswal and Biswas [16] used the fuzzy programming technique with some non-linear (hyperbolic and exponential) membership functions to solve a multi-objective transportation problem are always efficient. Recently, Antony et al [17] proposed method for the solution of transportation problem using triangular intuitionistic fuzzy numbers. Further, Singh and Yadav [18] develop a new approach for the solution of intuitionistic fuzzy transportation problem of type-2. Kumar and Hussain [19] developed a method for the fully intuitionistic fuzzy real transportation problem based ranking method. Due to storage constraints destinations are unable to receive the quantity in excess of their minimum demand. Therefore we encounter a kind of problem. Sonia and Malhotra [20] resolved such problem by using polynomial approach and solved TP into two stages. Further, Gani and Razak [21] studied two stage fuzzy transportation problems with triangular fuzzy numbers. Here, we have considered Intuitionistic Fuzzy Programming Technique to solve double Stage Multi- objective Transportation Problems (DSMOTP) in more realistic way. R. J. LI AND E. STANLEY LEE [22] uses an exponential membership function for fuzzy linear programming problem.

Mathematical model

In this section, a fuzzy programming technique to solve MOTP taking a membership and exponential functions. A variable X_{ij} represents the quantity to be transported from origin O_i to destination D_j. A multi-objective transportation problem may be stated mathematically

Maximize $Z_K = \sum m \sum n\ cij\ xij, k = 1,2,...K$

Subject to

$$\sum n\, x_{ij} = a_i, i = 1,2,3...m \tag{2}$$

$$\sum m\, x_{ij} = b_j, j = 1,2,3...n \tag{3}$$

$x_{ij} \geq 0, \text{for all}\, i, j$

Where the subscript on Zk and superscript on $Cij\ k$, denoted the Kth penalty Criterion. We assume that ai > 0 for all i, bj > 0, for all j, cij > 0 $for\ all\ i\ and\ j$.

Where the subscript on Z_k and superscript on C, denoted the K^{th} penalty Criterion. We assume that $a_i > 0\,\text{for all}\, i, b_j > 0\,\text{for all}\, j, c^{ij} > 0\,\text{for all}\, i\,\text{and}\, j$

$\sum_i^m a_i = \sum_j^n b_j$ (Equilibrium constant)

We treat equilibrium condition as necessary and sufficient condition for the existence of a feasible solution to the balanced condition linear transportation problem. A transportation problem has exactly (mn) variables and $(m + \ )$ constraints.

Fuzzy Algorithm to solve double stage multi-objective transportation problem

Step 1: Construct the Fuzzy transportation problem.

Step 2: Solve multi-objective transportation problem k times taking one objective at a time derives corresponding values for every objective at each solution and we make payoff matrix as fallows.

$$\begin{array}{c} \\ X^1 \\ X^2 \\ X^3 \end{array} \begin{array}{c} \begin{array}{ccc} Z_1 & Z_2 & Z_K \end{array} \\ \begin{bmatrix} Z_{11} & Z_{12} & Z_{1K} \\ Z_{21} & Z_{22} & Z_{2K} \\ Z_{K1} & Z_{K2} & Z_{KK} \end{bmatrix} \end{array}$$

Where, $X1, X2, \ldots, XK$ are the isolated optimal solutions of the K different transportation problems for K different objective functions $Zij = Zj\ (\ )\ (i = 1,2, \ldots k\ and\ j = 1,2, \ldots k)$ be the ith row and jth column element of the pay-off matrix.

Step 3: From step 2, set upper and lower bounds for each objective for degree of acceptance and degree of rejection corresponding to the set of solution.

For membership functions: Upper and Lower bound for membership function

$U_K^\mu = Max(Z_K(X_r))$

$L_K^\mu = Max(Z_K(X_r)),\ 0 \leq r \leq K$

Where the upper bound $UK\mu$ and the lower bound for $LK\mu$ for the Kth objective function Zk,

k=1, 2,...k, $UK\mu$ is the highest acceptable level of achievement for objective k, $LK\mu$ is the aspired level of achievement for objective k and $d\ k$=$Uk\mu$−$Lk\mu$, the degradation allowance for objective k.

Step 4: Consider the membership function as following linear functions:-

$$\mu_k\{Z_k(X)\} = \begin{cases} 1, L_k^\mu \geq Z_k(X) \\ 1 - \dfrac{U_k^\mu + Z_k(\mathrm{X})}{d_k},\ \ L_k^\mu \leq Z_k(X) \leq U_k^\mu; d_k = L_k^\mu - U_k^\mu \\ 0, Z_k(X) \geq U_k^\mu \end{cases}$$

Step 5. Find an equivalent crisp model by using a linear membership function for the initial fuzzy model

If we use a linear membership function, the crisp model can be simplified as

Minimize α

Such that

$$Z_k(X) - \alpha d_k \le L_k^{\mu}$$

$$\sum_j^n x_{ij} = a, i = 1,2,3..\text{m}$$

$$\sum_i^m x_{ij} = b_j, j = 1,2,3..n$$

$$x_{ij} \ge 0, for\,all\,i\,and\,\alpha \ge 0$$

Above linear programming problem can be solved by using LONDO/TORA statistical software.

Step 6. Solve the crisp model by an appropriate mathematical programming algorithm.

Minimize α

Subject to

$$C_{ij}^k X_{ij} - \alpha(d_k) \le L_k^{\mu},\ k = 1,2,.......K,$$

Subject to

$$\sum_j^n x_{ij} = a_i,\ \ i = 1,2,3,.....\text{m}$$

$$\sum_i^m x_{ij} = b_j,\ \ j = 1,2,3,......\text{x}_{ij} \ge 0, for\,all\,i, j$$

We use another membership function such as Hyperbolic Tangent function can be formulated as

Minimize α

$$\alpha \ge \frac{1}{2} + \frac{1}{2}\tanh[\{\frac{U_k + L_k}{2} + Z_k\}\tau_k, Where\,\tau_k = \frac{s}{U_k - L_k}, \text{Where}\,s\,\text{is the no.of constraints}$$

$$\sum_j^n x_{ij} = a_i, i = 1,2,3,...m$$

$$\sum_i^m x_{ij} = b_j, j = 1,2,3,...n$$

$$x_{ij} \ge 0, for\,all\,i\,and\,\alpha \ge 0$$

Step 7. An intuitionistic fuzzy optimization for MOLP problem with such an Exponential membership function for the kth objective function is defined as

$$\mu_k^e\{Z_k(x)\} = \begin{cases} 1,\ L_k^{\mu} \ge Z_k(X) \\ e - \frac{1}{2}\left(\frac{Z_k - L_k^{\mu}}{d_k}\right),\ L_k^{\mu} \le Z_k(X) \le U_k^{\mu};\ d_k = U_k^{\mu} - L_k^{\mu} \\ 0,\ Z_2(X) \ge U_k^{\mu} \end{cases}$$

Where, k=1,2,.....k

Data Analysis

This section shall discuss how fuzzy programming technique to solve MOTP using the algorithm. This data were collected from poultry dealer located in Srinagar j&k India. Who supplies the product to different destinations after taking it from different sources? There are multiple suppliers who supplies to different destinations. We study the average total cost of transportation and time .The data for cost and time of supply of product from source to destination are written in the form of equations.

$$\text{Maximize}\,Z_1 = 1X_{11} + 2X_{12} + 7X_{13} + 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34}$$

$$\text{Maximize}\,Z_2 = 4X_{11} + 4X_{12} + 3X_{13} + 4X_{14} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34}$$

Such that

$$X_{11} + X_{12} + X_{13} + X_{14} = 18$$

$$X_{21} + X_{22} + X_{23} + X_{24} = 19$$

$$X_{31} + X_{32} + X_{33} + X_{34} = 17$$

$$X_{11} + X_{21} + X_{31} = 11$$

$$X_{12} + X_{22} + X_{32} = 3$$

$$X_{13} + X_{23} + X_{33} = 14$$

$$X_{14} + X_{24} + X_{34} = 16$$

$$X_{ij} \ge 0,\ \ i = 1,2,3\,and\ j = 1,2,3$$

The optimal compromise solution of the problem is represented as

$X^1 = \{X_{13} = 14, X_{14} = 4, X_{22} = 3, X_{24} = 6, X_{31} = 11 \, and \, X_{34} = 6$

and rest all X_{ij} are zeros

$Z_1(X_1) = 301$

The optimal compromise solution of the problem is represented as

$X^2 = \{X_{11} = 5, X_{12} = 3, X_{23} = 3, X_{24} = 16, X_{31} = 6, X_{33} = 11$

and rest all X_{ij} are zeros

$Z_2(X_2) = 310$

Similarly, $Z_1(X_2) = 176 \, and \, Z_2(X_1) = 214$

Pay -off table is

	Z_1	Z_2
X_1	301	214
X_1	176	310

From this we get $U_1^{\mu} = 301, U_2^{\mu} = 310, L_1^{\mu} = 176, and \, L_2^{\mu} = 214$

The membership functions is given by

$$\mu_1\{Z_1(x)\} = \begin{cases} 1, \; 176 \geq Z_1(X) \\ 1 - \left(\dfrac{301 + Z_1(\text{X})}{d_{k1}} \right), \; 176 \leq Z_1(X) \leq 301; \; d_{k1} = 125 \\ 0, \; Z_1(X) \geq 301 \end{cases}$$

$$\mu_2\{Z_2(x)\} = \begin{cases} 1, \; 214 \geq Z_2(X) \\ 1 - \left(\dfrac{301 + Z_2(\text{X})}{d_{k2}} \right), \; 214 \leq Z_2(X) \leq 301; \; d_{k2} = 96 \\ 0, \; Z_2(X) \geq 301 \end{cases}$$

We find an equivalent crisp model

$Minimize \, \alpha$

$Z_1(X) - 125\alpha + \leq 176 \qquad Z_2(X) - 96\alpha \leq 214$

Solve the crisp model

$1X_{11} + 2X_{12} + 7X_{13} + 7X_{14} + X_{21} + 9X_{22} + 3X_{23} + 4X_{24} + 8X_{31} + 9X_{32} + 4X_{33} + 6X_{34} - 125\alpha \leq 176$

$4X_{11} + 4X_{12} + 3X_{13} + 4X_{14} + 5X_{21} + 8X_{22} + 9X_{23} + 10X_{24} + 6X_{31} + 2X_{32} + 5X_{33} + 1X_{34} - 96\alpha \leq 214$

$X_{11} + X_{12} + X_{13} + X_{14} = 18$

$X_{21} + X_{22} + X_{23} + X_{24} = 19$

$X_{31} + X_{32} + X_{33} + X_{34} = 17$

$X_{11} + X_{21} + X_{31} = 11$

$X_{12} + X_{22} + X_{32} = 3$

$X_{13} + X_{23} + X_{33} = 14$

$X_{14} + X_{24} + X_{34} = 16$

The optimal compromise solution of the problem by using LINDO Software is represented as

$\{X_{12} = 1.098, X_{13} = 3.90, X_{14} = 13, X_{21} = 11, X_{23} = 8, X_{32} = 1.90, X_{33} = 12, X_{34} = 3$

$and \; rest \; all \; X_{ij} \; are \; zeros\}$

$Z_1^* = 238.496 \qquad Z_2^* = 261.82 \qquad \alpha = 0.50$

If we use another membership function then an equivalent crisp model for the fuzzy model can be further formulated as:

Minimize α

$\alpha \geq \frac{1}{2}+\frac{1}{2}\tanh\left[\left\{\frac{U_K+L_K}{2}+Z_K\right\}\tau_K\right]$, Where $\tau_k=\frac{7}{U_K-L_K}$

This further implies that

$$\tau_k Z_K-\tan^{-1}(2\alpha-1)\leq\frac{U_K+L_K}{2}\tau_K$$

$$Minimize\ w$$

$$\frac{7}{125}(1X_{11}+2X_{12}+7X_{13}+7X_{14}+X_{21}+9X_{22}+3X_{23}+4X_{24}+8X_{31}+9X_{32}+4X_{33}+6X_{34})-w\leq\frac{7}{125}\left(\frac{477}{2}\right)$$

$$\frac{7}{96}(4X_{11}+4X_{12}+3X_{13}+4X_{14}+5X_{21}+8X_{22}+9X_{23}+10X_{24}+6X_{31}+2X_{32}+5X_{33}+1X_{34})-w\leq\frac{7}{96}\left(\frac{524}{2}\right)$$

$$X_{11}+X_{12}+X_{13}+X_{14}=18$$
$$X_{21}+X_{22}+X_{23}+X_{24}=19$$
$$X_{31}+X_{32}+X_{33}+X_{34}=17$$
$$X_{11}+X_{21}+X_{31}=11$$
$$X_{12}+X_{22}+X_{32}=3$$
$$X_{13}+X_{23}+X_{33}=14$$
$$X_{14}+X_{24}+X_{34}=16$$

Solving above problem the optimal solution is given by

$\{X_{12}=1.098, X_{13}=3.90, X_{14}=13, X_{21}=11, X_{23}=8, X_{32}=1.90, X_{33}=12, X_{34}=3$

$and\ rest\ all\ X_{ij}\ are\ zeros\}$ $Z_1^*=238.496$ $Z_2^*=261.82$ $\alpha=0.64,$

$w=\tanh^{-1}(2\alpha-1)\ and\ w=0.29$

$$\mu_1^e\{Z_1(x)\}=\begin{cases}1,\ 176\geq Z_1(X)\\ e-\frac{1}{2}\left(\frac{Z_1-176}{d_k}\right),\ 176\leq Z_1(X)\leq 301;\ d_k=125\\ 0,\ Z_1(X)\geq U_k^{\mu}\end{cases}$$

$$\mu_2^e\{Z_2(x)\}=\begin{cases}1,\ 214\geq Z_1(X)\\ e-\frac{1}{2}\left(\frac{Z_2-214}{d_k}\right),\ 214\leq Z_2(X)\leq 301;\ d_k=96\\ 0,\ Z_2(X)\geq 310\end{cases}$$

If we use the exponential membership functions, an equivalent crisp model can be formulated as

Minimize α

Subject to

$$\alpha\leq e-\frac{1}{2}\left(\frac{Z_1-176}{d_k}\right) and\ \alpha\leq e-\frac{1}{2}\left(\frac{Z_2-214}{d_k}\right)$$

The problem is solved by LINDO

$$\{X_{12}=1.098, X_{13}=3.90, X_{14}=13, X_{21}=11, X_{23}=8, X_{32}=1.90, X_{33}=12, X_{34}=3\ and\ all\ X\ are\ zeros\}.$$

$$\alpha=0.60$$

Conclusion

In this paper three types of functions are used to solve intuitionistic Fuzzy Multi-objective transportation Problem. Problem is completed in three steps, in first step which gives optimal compromise solution of the problem. If we use the second type of membership function the crisp model become linear. We use different exponential membership function again the optimal compromise solution does not change significantly. The method is very clear and easy to handle real life transportation problems because in some cases due to limited storage capacity, minimum quantity received by the destinations. In this Situation, a single shipment is not possible. Therefore, items are shipped to destinations from the origins into different stages. Initially, the minimum quantity may be shipped from origins to the destinations. The present method is based on intuitionistic fuzzy sets. Therefore, it will be perfect to handle real transportation problems.

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