

# Low Complexity CFO Compensation Scheme for SC-FDMA Uplink System

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**Abstract**— Single Carrier Frequency Division Multiple Access (SC-FDMA) has been adopted for fourth generation (4G) mobile uplink transmission schemes because of its low Peak to Average Power Ratio (PAPR) as compared to that of Orthogonal Frequency-Division Multiple Access (OFDMA). Since SC-FDMA has low PAPR, it is sensitive to Carrier Frequency Offsets (CFOs) similar to that of OFDMA. To minimize and compensate CFO effects on SC-FDMA, the uplink system with Zero-Forcing (ZF) equalization can be used. In this, the method involves is an inverse of an  $N \times N$  CFO induced inter-carrier interference (ICI) matrix, where  $N$  denotes the number of subcarriers. When  $N$  is large, it will leads to large number computational complexity of the system and the performance of the system will degrade. To overcome this complexity, we propose a low- complexity ZF method and Newton's method with fast Fourier transforms which is used to solve the matrix inversion problem. This reduces the complexity from  $O(N^3)$  to  $O(2N \log_2 N)$ . The proposed compensation scheme can able to improve the system performance.

**Keywords**— Single Carrier Frequency Division Multiple Access (SC-FDMA), Carrier Frequency Offsets (CFO), Fast Fourier Transform (FFT), Zero-Forcing (ZF), Newton's method.

## I. INTRODUCTION

The WiMAX cellular system uses Orthogonal Frequency-Division Multiple Access (OFDMA) for signal transmission from the base station and also from the mobile terminals. Third Generation Partnership Project (3GPP) prescribes OFDMA for downlink transmission and Single Carrier Frequency Division Multiple Access (SC-FDMA) for uplink transmission. Since OFDMA has the advantages of multiuser diversity, good BER ratio and

simpler receiver design, one disadvantage is that it has high peak-to-average power ratio (PAPR) compared to that SC-FDMA system which raises the cost and lowers the power efficiency of a transmitter's power amplifier. SC-FDMA also involves in the transmission of high data rates in the uplink communication [1]. With a low PAPR, SC-FDMA provides simpler and more power-efficient amplifiers at mobile station compared with OFDMA transmission [2], [3].

Two subcarrier allocation schemes are available in SC-FDMA uplink system. In the localized subcarrier mapping mode, the modulation symbols are assigned to adjacent subcarriers. In Interleaved FDMA (IFDMA), the symbols are equally spaced across the entire channel bandwidth. IFDMA is also sensitive to Interferences.

## II. RELATED WORK

SC-FDMA system is also sensitive to interferences as like OFDMA due to Doppler shifts and misalignment between transmitter and receiver oscillator which will affect the orthogonality among the subcarriers. This results in Carrier Frequency Offsets (CFO) which will degrade the system performance. Carrier Frequency Offsets (CFO) also caused due to interferences such as Inter-Carrier Interference (ICI) and Multi-Access Interference (MAI).

To suppress ICI and MAI many methods have been reported in the literature [5]-[7]. Semi-blind CFO based estimation scheme tolerate CFOs on uplink IFDMA [12], [13]. In [14], [10] the performance of IFDMA with CFOs was exactly analyzed. In IFDMA, multiple CFOs may occur because different users experience different CFOs [9]. CFO effects on multiuser OFDM-FDMA has been reported [8], [11]. To Compensate Carrier Frequency Offsets (CFOs), one way is to use Minimum Mean Square Error (MMSE) equalizer combined with Parallel

Interference Cancellation (PIC) [16] and the another approach is the combined Minimum Mean Square Error Frequency Domain Equalization (MMSE-FDE) [15]. In

Each user has  $M=N/K$  subcarriers. The data transmitted for  $k$ th user is

$$x^{(k)} = [x_0^{(k)}, x_1^{(k)}, \dots, x_{M-1}^{(k)}]^T \quad (1)$$

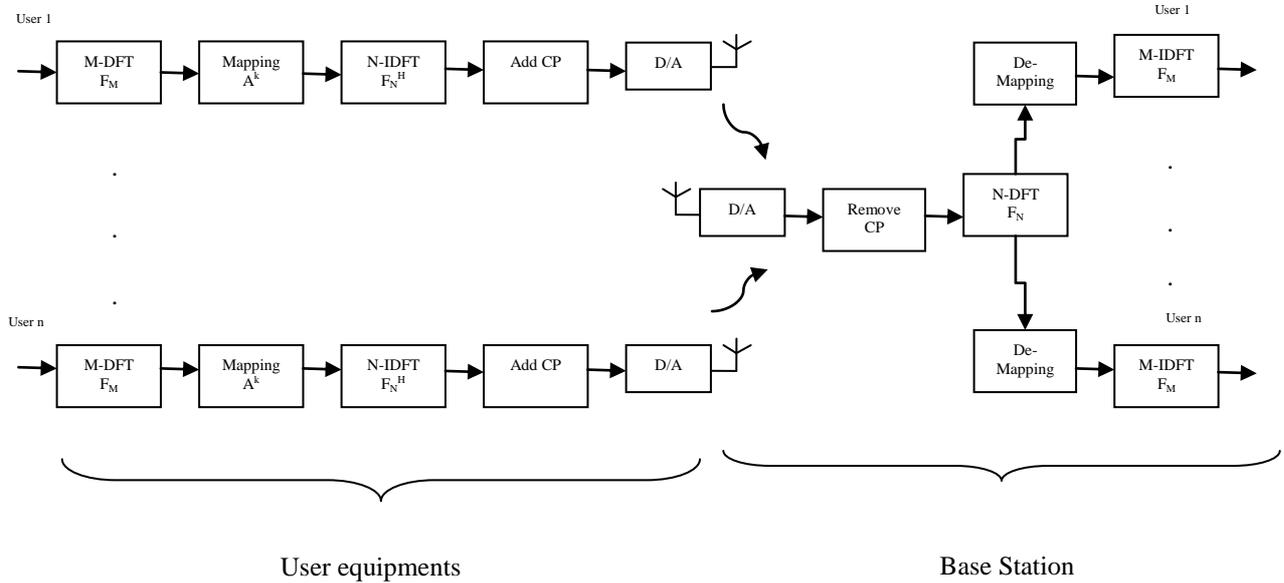


Fig.1 Uplink SC-FDMA System Model

[17], [18] the structure of equalizer based on frequency domain decision feedback method have been proposed. CFO Compensation for uplink system is very important [19], hence it can be made through MMSE+LS [21] which involves in CFO induced ICI matrix. But when the subcarrier increases, the complexity also increases. To avoid such a problem, a low-complexity ZF method is reported [20]. The proposed scheme [22] [23] provides CFO compensation for OFDMA uplink systems.

The objective of this paper is to compensate CFOs in SC-FDMA uplink system. Since Zero-Forcing method is simple and efficient approach but includes inversion of matrix method for implementation. Hence by using Newton's method for matrix iterative inversion, ICI matrix can be developed which can implement Newton's method with FFT. The performance of the proposed method is compared with Direct ZF and Banded ZF method. The performance of the proposed method is well efficient than other methods and also by using this scheme the computational complexity reduces from  $O(N^3)$  to  $O(2N \log^2 N)$ .

In Section III, the system model is introduced. Section IV presents the Proposed CFO Compensation scheme and section V presents simulation results are proposed. In section VI conclusion is presented.

### III. SYSTEM MODEL

Consider SC-FDMA uplink system Fig.1 with K users and N subcarriers. In this system, each user is communicating Base Station (BS) with its own subcarrier.

$F_M$  represents the M-point DFT,  $F_N^H$  represents N-point IDFT and  $F_N$  represents N-point DFT, sub-carrier mapping can be either interleaved or Localized to enable uplink communication.  $S_k$  represents the number of subcarriers allocated to  $k$ th user and  $A^k$  represents the subcarrier allocation matrix of order  $N \times M$  where

$$S_{i,j}^k = \begin{cases} 1, & \text{if } i \in S^k \\ 0, & \text{if } i \notin S^k \end{cases} \quad (2)$$

The output of N-point IDFT of the  $k$ th user can be represented by,

$$s^{(k)} = F_M A^k F_N^H x^{(k)} \quad (3)$$

N-point IDFT output vector is then passed after adding Cyclic Prefix (CP). Guard interval length is  $N_g$  and the length is longer than the maximum channel delay spread. Channel is time invariant for each user and the Channel Impulse Response (CIR) for  $k$ th user is represented by vector,

$$h^{(k)} = [h_0^{(k)}, h_1^{(k)}, \dots, h_R^{(k)}]^T \quad (4)$$

After adding additive noise and removing CP, the received signal vector is represented as,

$$r = \sum_{k=1}^K e^{(k)} (s^{(k)} \otimes \hat{h}^{(k)}) + z \quad (5)$$

$e^{(k)}$  is in the order of diagonal matrix  $N \times N$  which can be expressed as,

$$e_{n,n}^{(k)} = e^{\frac{j2\pi \epsilon_k (n-1)}{N}} \quad (6)$$

$\otimes$  represents the circular convolution.  $\hat{h}^{(k)}$  represents column circular matrix (i.e.)  $\text{circ}\{[h^{(k)}; \mathbf{0}_{(N-R) \times 1}]\}$  and  $z$  represents Additive White Gaussian Noise (AWGN).

N-Point DFT output can be expressed as,

$$y = \sum_{k=1}^K H^k A^k F_M x^{(k)} + z_0 \quad (7)$$

where  $z_0$  represents output noise vector.

$H^k$  represents diagonal matrix of  $N \times N$  which can be expressed as,

$$H_{n,n}^k = \sum_{k=0}^{K-1} h_n^k e^{-j2\pi(k-1)n/N} \quad (8)$$

After FFT operation, the corresponding frequency domain signal can be represented as,

$$\tilde{r} = \sum_{k=1}^K \tilde{e}^k \tilde{y}^k + \tilde{z} \quad (9)$$

where  $\tilde{e}^k = F_M F_N^H e^{(k)}$ ,  $\tilde{y}^k = H^k x^{(k)}$ .

Finally we express the received frequency-domain signal as,

$$\tilde{r} = \tilde{M} \tilde{y} + \tilde{z} \quad (10)$$

where  $\tilde{y} = \tilde{H} \tilde{x}$ ,  $\tilde{M} = \sum_{k=1}^K \tilde{e}^k s^{(k)}$ .

#### IV. PROPOSED CFO COMPENSATION SCHEME

The straightforward method to compensate CFO is the ZF method which is given by,  $\tilde{y}_{ZF} = \tilde{M}^{-1} \tilde{r}$ . Direct ZF method can reduce the complexity of SC-FDMA, but when  $N$  increases it leads to problem. Also in most application  $N$  will be large and hence the complexity of the system will also large. To reduce this complexity, a low-complexity ZF and Newton's method with FFT have been proposed.

Let us consider  $W_k$  be the estimate of  $\tilde{M}^{-1}$  at the iteration. Newton's iteration for matrix inversion can be expressed as,

$$W_{k+1} = (2I_M - W_k \tilde{M}) W_k \quad (11)$$

Let the residual estimation be  $R_k = I_M - W_k \tilde{M}$ . Newton's iteration implies the  $\|I_M - W_k \tilde{M}\|_{I_M - W_0 \tilde{M}} \leq 2^k$  for all  $k$  and if the  $\|I_M - W_0 \tilde{M}\| \leq 1$ , then quadratic convergence can be developed. This results in computational complexity. For this, a method is developed to solve this problem. Using Newton's iteration method, a sequence of matrices can be obtained. By using this we can express  $W_k$  as,

$$W_k = \sum_{m=0}^{2^k - 1} c_k^m (W_0 \tilde{M})^m W_0 \quad (12)$$

where  $\tilde{s}_m = (W_0 \tilde{M})^m W_0 \tilde{r}$  and  $\tilde{y}_k = W_k \tilde{r}$ . Iterative step of  $\tilde{s}_m$ :  $\tilde{s}_{m+1} = (W_0 \tilde{M}) \tilde{s}_m$ . Using this method, matrix-to-matrix multiplications is transformed into matrix-to-vector multiplications.

By rewriting  $\tilde{s}_{m+1}$  as ,

$$\tilde{s}_{m+1} = W_0 F_M \left[ \sum_{k=1}^K e^k (F_N^H s^k) \tilde{s}_m \right] \quad (13)$$

In this  $\tilde{s}_{m+1}$  involves in vector multiplications rather than matrix multiplications. The computational complexity reduces by implementing DFT/IDFT with FFT/IFFT. The

computational complexity can further be reduced by using Interleaved SC-FDMA.

Let  $\tilde{s}_m = [\tilde{s}_m, 0, \dots, \tilde{s}_m, K-1]^T$  and  $u_m^k = H^k \tilde{s}_m$  in which  $u_{m,i}^k$  can be expressed as,

$$u_{m,i}^k = \begin{cases} \tilde{s}_{m,i}, & \text{if } i \in \gamma_k \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

The non-zero elements in  $u_m^k$  can be obtained by circularly shifting  $u_m^k$  with  $k-1$  elements. We can express  $F_N^H s^k \tilde{s}_m$  as  $c^k a_m^k$  where  $c^k$  results from circular shift of  $u_m^k$ . Hence we can implement  $F_N^H s^k \tilde{s}_m$  by an IDFT approach with  $N/Q$  dimension rather than  $N$ .

By using this, we can reduce the complexity. Further it can be reduced with  $\tilde{s}_{m+1}$  where  $\tilde{s}_{m+1}$  can be denoted as,

$$\tilde{s}_{m+1} = W_0 F_M \left( \sum_{k=1}^K e^k c^k a_m^k \right) \quad (15)$$

where  $a_m^k$  is column vector,  $e^k, c^k$  are diagonal matrices.

This approach involves in FFT operation of size  $N$ , K. IFFT operation of size  $N/K$  and also includes vector operations. By this operation, the complexity can further be reduced.

Final approach is to find  $w_0$ . Assume  $w_0$  as a diagonal matrix. Consider  $W_0 = \text{diag}([w_0, w_1, \dots, w_{N-1}]^T)$ . For this, we have to find optimum initial value. When the initial value is good, it can reduce number of iterations significantly.

To find optimum initial values, we consider  $W_{opt,0} = \arg \min_{w_0} \|R\|_F^2$ , where  $\|R\|_F$  denotes the Frobenius form of  $R$  which can be expressed as,  $\|R\|_F = \|I_N - W_0 \tilde{M}\|_F$ . To obtain optimum values, we have to set  $\partial \{ \|I_N - W_0 \tilde{M}\|_F^2 \} / \delta w_w^* = 0$ .

We can express optimum initial value as,

$$w_{opt,p} = \tilde{m}_{p,p}^* / \sum_{j=0}^{N-1} |\tilde{m}_{p,j}|^2 \quad (16)$$

where  $\tilde{m}_{p,j} = \tilde{M}(i, j)$

By approximating  $w_{opt,p}$  values, we can further reduce the complexity. This approximation is based on the fact that ICI coming among subcarriers is mainly comes from neighboring subcarriers.

The result can be denoted as,

$$\tilde{X} = \tilde{H}^{-1} \tilde{y} \quad (17)$$

where  $\tilde{X}$  and  $\tilde{y}$  are the estimate of  $\tilde{x}$  and CFO estimate of  $\tilde{y}$  respectively.

#### V. SIMULATION RESULTS

Here we use an interleaved SC-FDMA uplink systems with  $N=2048$ ,  $k=16$ . The modulation scheme used is 16-bit QAM. Channel Length for all users can be set as 127 and the averaged bit-error-rate (BER) is adopted as the performance index, and CFOs for all users are set to  $\{0.1, -0.2, -0.05, 0.2, -0.3, 0, -0.1, 0.4, -0.3, 0.05, 0, -0.1, 0.05, -0.1, 0.3, 0.15\}$ . Fig. 2 shows BER performance comparison for Direct ZF, Banded ZF and Proposed method. From the figure it is found that when the number

of K users increases, the proposed system will not be affected by Carrier Frequency Offsets (CFO). Here the performance of proposed method is based on the condition S=2. It includes the performance of Direct ZF, banded 2F and proposed method (k=2) are same. This method performs better than banded ZF method.

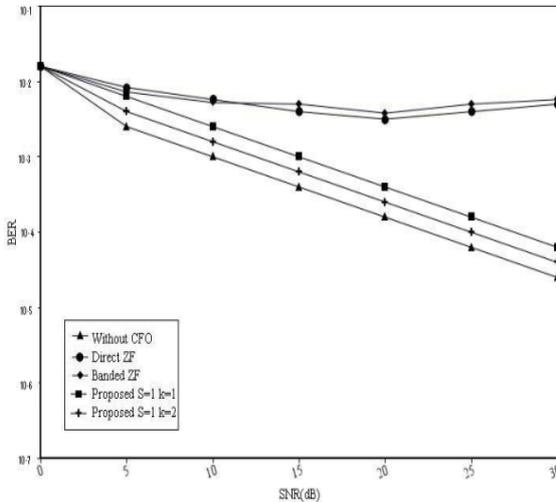


Fig. 2 BER performance comparison for Direct ZF, Banded ZF and Proposed method

Fig. 3 shows the complexity comparison for Direct ZF, Banded ZF and Proposed method. From this figure, we can conclude that the real multiplications/real divisions/real additions for the proposed method have lower complexity than Direct ZF and Banded ZF methods. Computational complexity for ZF method can be reduced from  $O(N^3)$  to  $O(2N \log_2(K^2/N))$  in which K denotes the number of subcarriers. When subcarrier increases complexity also increases. While the proposed method reduces this complexity from  $O(N^3)$  to  $O(2N \log_2 N)$ . Also it is known that the complexity for Banded ZF is also lower compared to Direct ZF. But the results show that complexity of proposed method is lower than Banded ZF. From this we concluded that the proposed method has low complexity as the number of carrier increases.

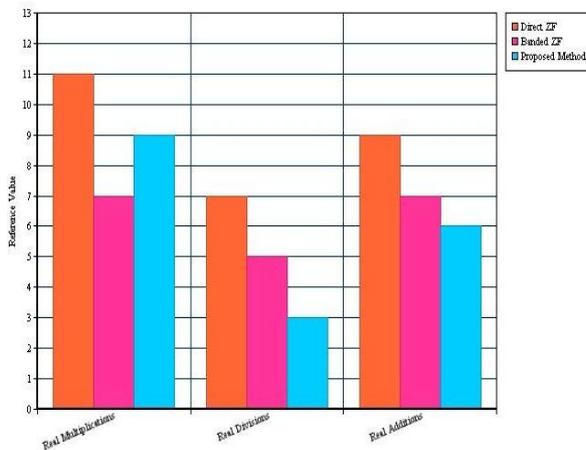


Fig. 3 Complexity comparison for Direct ZF, Banded ZF and Proposed method.

## VI. CONCLUSION

In this paper, we propose a low-complexity CFO compensation scheme for an interleaved SC-FDMA uplink system. The proposed method involves in the implementation of ZF method and also has the advantage of effective solutions to the CFO problem in the uplink system. Using Newton's method for matrix inversion, an iterative method is developed which can be implemented through FFTs. Hence by using ZF and Newton's method CFO effects can be reduced in the SC-FDMA system. This results in the reduction of complexity from  $O(N^3)$  to  $O(2N \log_2 N)$ .

## REFERENCES

- [1] H. G. Myung, Junsung Lim and D. J. Goodman, "Single carrier FDMA for uplink wireless transmission," *IEEE Vehicular Technology Magazine*, Vol. 1, Issue 3, pp. 30-38, Sep.2006.
- [2] K.Raghunath and A. Chockalingam "SC-FDMA versus OFDMA: Sensitivity to Large Carrier Frequency and Timing Offsets on the Uplink" *IEEE Global Telecommunications Conference*, pp.1-6,2009.
- [3] Cristina Ciochina and Hikmet Sari, "A Review of OFDMA and Single-Carrier FDMA and Some Recent Results" *IEEE, Advances in Electronics and Telecommunications*, Vol. 1, no. 1, April 2010.
- [4] 3rd Generation Partnership Project (3GPP) Technical Specification Group Radio Access Network; Physical layer aspects for Evolved Universal Terrestrial Radio Access (E-UTRA) (Release 10). 3GPP TS 36.211, V10.0.0, Dec.2010.
- [5] Shih-Chan Huang, Jia-Chin Lin and Kao-Peng Chou Department of Communication Engineering, National Central University, Taiwan, "Novel Channel Estimation Techniques on SC-FDMA Uplink Transmission" *IEEE Vehicular Technology Conference*, pp.1-5, May, 2010
- [6] Xiu Pei Zhang and Heung-Gyoon Ryu, "Suppression of ICI and MAI in SC-FDMA Communication System in presence of Carrier Frequency Offsets" *IEEE Transactions on Consumer Electronics*, Vol.56, no. 2, May 2010.
- [7] Alka Kalra, Rajesh Khanna, Charu Garg "Inter Carrier Interference Analysis of SC-FDMA Systems using Frequency Domain Equalization" © 2012 Published by Elsevier Ltd.
- [8] Chao Zhang, Zhaocheng Wang, Zhixing Yang, Jun Wang, and Jian Song "Frequency Domain Decision Feedback Equalization for Uplink SC-FDMA" *IEEE Transactions on Broadcasting*, Vol. 56, no. 2, June 2010.
- [9] Gang Wang, Xia Lei, Yue Xiao, and Shaoqian Li, "Multiple Carrier Frequency Offsets Compensation for the Uplink of Interleaved OFDMA," *IEEE WCSP*, pp. 1-5, Oct. 2010.
- [10] U. Sorger, I. De Broeck and M. Schnell, "Interleaved FDMA-a new spread-spectrum multiple-access scheme," *IEEE ICC*, pp. 1013-1017, June 1998
- [11] J. Choi, C. Lee, H. W. Jung, and Y. H. Lee, "Carrier frequency offset compensation for uplink of OFDM-FDMA systems," *IEEE Commun.Letter*, vol. 4, pp. 414-416, Dec. 2000.
- [12] Malik Muhammad Usman Gul, Sungeun Lee, and Xiaoli Ma "Carrier Frequency Offset Estimation for Interleaved SC-FDMA Systems" *Information Sciences and Systems*, pp.1-5, March 2012
- [13] Yu Zhuo and Khaled Ben Letaief, "CFO Estimation and Compensation in SC-IFDMA Systems," *IEEE Transactions on Wireless Communications*, vol. 9, no. 10, October 2010.
- [14] Jia Yang, Gang Wu, Xu He, Yue Xiao and Shaoqian, "Performance Analysis of Uplink IFDMA in presence of Carrier Frequency Offsets" *Wireless Communications, Networking and Mobile Computing*, pp.1-4, September 2011.

- [15] GuoliangChen,Yu Zhu and Khaled Ben Letaief, “Combined MMSE-FDE and Interference Cancellation for Uplink SC-FDMA with Carrier Frequency Offsets” *IEEE ICC* 2010.
- [16] F. S. Al-kamali M. I. Dessouky · B. M. Sallam · F. Shawki · F. E. Abd El-Samie “Equalization and Carrier Frequency Offset Compensation for the SC-FDMA Systems” © *Springer Science+Business Media, LLC. 2011*.
- [17] Chao Zhang, Zhaocheng Wang, Zhixing Yang, Jun Wang, and Jian Song “Frequency Domain Decision Feedback Equalization for Uplink SC-FDMA” *IEEE Transactions on Broadcasting*, Vol. 56, no. 2, June 2010.
- [18] Satoshi Yamazakhi, David K.Asano“SC-FDMA Transmission Frequency Domain Equalization Based on a Wiener Filter For Broadband Wireless Communications”, *IEEE Transactions On Electronics, Information and Systems*, Vol.131, No.7, Pp 1384-1392.2011
- [19] C.-Y. Hsu and W.-R. Wu, “Low-complexity CFO compensation for uplink OFDMA systems,” in *Proc. IEEE PIMRC’06*, Helsinki, Finland, Sept. 2006.
- [20] C. Y. Hsu and W. R. Wu, “A low-complexity zero-forcing CFO compensation scheme for OFDMA uplink systems,” *IEEE Trans. WirelessCommun*, vol. 7, no.10, pp. 3657–3661, Oct. 2008.
- [21] ArmanFarhang, Nicola Marchetti, and Linda E. Doyle “Low Complexity LS and MMSE Based CFO Compensation Techniques for the Uplink of OFDMA Systems” *IEEE ICC 2013 - Wireless Communication*.
- [22] Zhongren Cao, Member, IEEE, UfukTureli, Member, IEEE, and Yu-Dong Yao, Senior Member, IEEE “Low-Complexity Orthogonal Spectral Signal Construction for Generalized OFDMA Uplink With Frequency Synchronization Errors” *IEEE Transactions on Vehicular Technology*, vol. 56, no. 3, May 2007
- [23] Kilbom Lee, Student Member, IEEE and Inkyu Lee, Senior Member, IEEE, “CFO Compensation for Uplink OFDMA Systems with Conjugated Gradient” *Proc. of the IEEE ICC’11*, pp. 1-5, June 2011
- [24] A. S. Householder, “*The Theory of Matrix in Numerical Analysis*” New York: Dover Publications, 1964.
- [25] V. Y. Pan, Y. Rami, and X. Wang, “Structured matrices and Newton’s iteration: unified approach,” *Linear Algebra Appl.*, vol. 343-344, pp. 233–265, Mar. 2002.

