



# MATHEMATICAL MODELING AND SIMULATION OF PERMANENT MAGNET SYNCHRONOUS MOTOR

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**Abstract** : Introduction of permanent magnets to replace the electromagnetic poles with windings requiring less electric energy supply source resulted in compact dc machines. Likewise in synchronous machines, the conventional electromagnetic field poles in the rotor are replaced by the PM poles and by doing so the slip rings and brush assembly are dispensed. With the advent of power semiconductor devices the replacement of the mechanical commutator with an electronic commutator in the form of an inverter was achieved. These two developments contributed to the development of PMSMs and Brushless dc machines. Due to many applications of PMSM like Sensor less speed control, appropriate position control, Servo motor, etc. Mathematical modelling of Permanent Magnet Synchronous Motor is carried out and simulated using MATLAB. The most important features of PMSM is its high efficiency given with the ratio of input power after deduction of loss to the input power. There is no field current or rotor current in the PMSM.

**Keywords** : Permanent Magnets, Electronic Commutator, Synchronous motor, MATLAB, Permanent Magnet Synchronous Motor, Sensor less Speed control.

## I. INTRODUCTION

The Permanent Magnet Synchronous motor is a rotating electric machine where stator is a classic three-phase Induction Motor and rotor has permanent magnets. In this respect Permanent Magnet synchronous motor is similar to induction motor except the rotor magnet field in case of PMSM is produced by permanent magnets. The use of permanent magnet to generate a substantial air gap magnetic flux makes it possible to design highly efficient PM motors. The permanent magnet motors are classified based on type of back emf induced. Permanent magnet synchronous motor has sinusoidal back emf and Brushless DC motors have trapezoidal back emf. The features of PMSM motor are:

- Medium construction complexity, multiple fields. High reliability (no brush wear), even at very high achievable speeds.
- High efficiency.
- Low EMI.
- Driven by multi-phase inverter controllers. Sensor less speed control possible.
- Appropriate for position control.

## II. DYNAMIC MODELLING OF PMSM

The dynamic model of the permanent magnet synchronous machine (PMSM) is derived using a two-phase motor in direct and quadrature axes. This approach is done because of the conceptual simplicity obtained with only one set of two windings on the stator. The rotor has no windings, only magnets. The magnets are modelled as a current source or a flux linkage source, concentrating all its flux linkages along only one axis. The flux linkages of the stator q and d windings are derived from first principles. The physical modelling of the machine is developed from which the circuit

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model is derived. Constant inductance for windings is obtained by a transformation to the rotor by replacing the stator windings with a fictitious set of d-q windings rotating at the electrical speed of the rotor. The equivalence between the three-phase machine and its model using a set of two-phase windings is derived and this approach is suitable for extending it to model an  $n$  phase machine where  $n$  is greater than 2, with a two-phase machine. The transformation from the two-phase to the three-phase variables of voltages, currents, or flux linkages is derived in a generalized way. Derivations for electromagnetic torque involving the currents and flux linkages are obtained. The differential equations describing the PMSM are nonlinear.

The following assumptions are made to derive the dynamic model:

1. The stator windings are balanced with sinusoidally distributed magnetomotive force (mmf).
2. The inductance versus rotor position is sinusoidal.
3. The saturation and parameter changes are neglected.

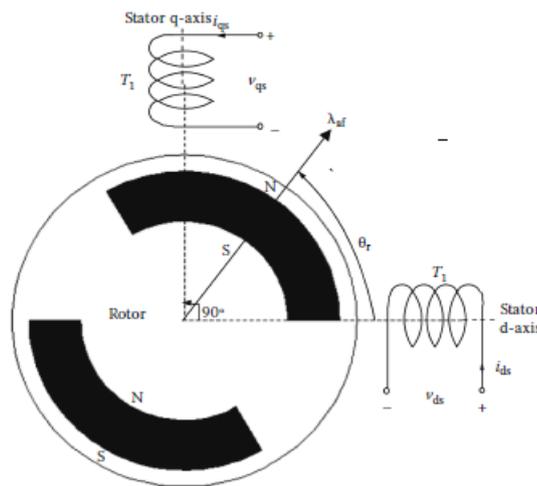


Fig1. A two-phase PMSM

The windings are displaced in space by 90 electrical degrees and the rotor winding is at an angle  $\theta_r$  from the stator d-axis winding. It is assumed that the q-axis leads the d-axis to a counter clockwise direction of rotation of the rotor. A pair of poles is assumed for this figure, but it is applicable with slight modification for any number of pairs of poles. Note that  $\theta_r$  is the electrical rotor position at any instant obtained by multiplying the mechanical rotor position by pairs of electrical poles. The d- and q-axes stator voltages are derived as the sum of the resistive voltage drops and the derivative of the flux linkages in the respective windings as

$$V_{qs} = R_q i_{qs} + p \lambda_{qs} \quad (1)$$

$$V_{ds} = R_d i_{ds} + p \lambda_{ds} \quad (2)$$

where  $p$  is the differential operator,  $d/dt$

$v_{qs}$  and  $v_{ds}$  are the voltages in the q- and d-axes windings

$i_{qs}$  and  $i_{ds}$  are the q- and d-axes stator currents

$R_q$  and  $R_d$  are the stator q- and d-axes resistances

$\lambda_{qs}$  and  $\lambda_{ds}$  are the stator q- and d-axes stator flux linkages

The stator winding flux linkages can be written as the sum of the flux linkages due to their own excitation and mutual flux linkages resulting from other winding current and magnet sources. The q and d stator flux linkages are written as

$$\lambda_{qs} = L_{qq} i_{qs} + L_{qd} i_{ds} + \lambda_{af} \sin \theta_r \quad (3)$$



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$$Y_{ds} = L_{dq}i_{qs} + L_{qd}i_{ds} + \lambda_{af}\cos\theta_r \quad (4)$$

Where  $\theta_r$  is the instantaneous rotor position. The windings are balanced and therefore their resistances are equal and denoted as  $R_s = R_q = R_d$ . The d and q stator voltages can then be written in terms of the flux linkages and resistive voltage drops as

$$V_{qs} = R_s i_{qs} + i_{qs} p L_{qq} + L_{qq} p i_{qs} + L_{qd} p i_{ds} + i_{ds} p L_{qd} + \lambda_{af} p \sin\theta_r \quad (5)$$

$$V_{ds} = R_s i_{ds} + i_{qs} p L_{qd} + L_{qd} p i_{qs} + L_{dd} p i_{ds} + i_{ds} p L_{dd} + \lambda_{af} p \cos\theta_r \quad (6)$$

$L_{qq}$  and  $L_{dd}$  are the self-inductances of the q- and d-axes windings, respectively. The mutual inductances between any two windings are denoted by  $L$  with two subscripts where the first subscript denotes the winding at which the emf is measured due to the current in the other winding indicated by the second subscript. The symmetry of the q- and d-axes windings ensures that  $L_{qd}$  and  $L_{dq}$  are equal.

Substituting the self- and mutual inductances in terms of the rotor position into the stator voltage equations will result in a large number of terms that are rotor position dependent. The final machine equations then are

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = R_s \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} L1 + L2\cos\theta_r & -L2\sin\theta_r \\ -L2\sin\theta_r & L1 - L2\cos\theta_r \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + 2w_r L2 \begin{bmatrix} -\sin\theta_r & -\cos\theta_r \\ -\cos\theta_r & \sin\theta_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_{af} w_r \begin{bmatrix} \cos\theta_r \\ -\sin\theta_r \end{bmatrix} \quad (7)$$

The third term exists because of saliency, i.e., when  $L_q \neq L_d$ . In surface mount magnet machines, the inductances are equal and, therefore,  $L2$  is zero and the third term in the above equation vanishes. Also disappearing in the matrix's second term are the position-dependent terms, resulting in a simple expression for surface mounted magnet machines in stator reference frames. It is then given by

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = R_s \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} L1 & 0 \\ 0 & L2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_{af} w_r \begin{bmatrix} \cos\theta_r \\ -\sin\theta_r \end{bmatrix} \quad (8)$$

In the salient pole PMSMs, the inductances are rotor position dependent. If the rotor position dependency is eliminated by transformation. The relationship between the flux linkages and currents is uniquely determined by the matrix containing the rotor position terms. From this relationship, given the injected currents, the rotor position can be evaluated if the flux linkages are known. This aspect of operating with injected signals and on the corresponding flux linkages equations to extract rotor position from inductance measurements makes the motor drive system independent of rotor position sensors that are expensive, less reliable.

### III. TRANSFORMATION TO ROTOR REFERENCE FRAMES

Reference frames give a unique view of the system and dramatic simplification of the system equations. The independent rotor field position determines the induced emf and affects the dynamic system equations of both the wound rotor and the PMSMs. Therefore, looking at the entire system from the rotor, i.e., rotating reference frames, the system inductance matrix (equ.8) becomes independent of the rotor position, thus leading to the simplification and compactness of the system equations. The relationship between the stationary reference frames denoted by d- and q-axes and the rotor reference frames denoted by  $d_r$ - and  $q_r$ -axes. Transformation to obtain constant inductances is achieved by replacing the actual stator and its windings with a fictitious stator having windings on the  $q_r$  and  $d_r$ -axes. The fictitious stator will have the same number of turns for each phase as the actual stator phase windings and should produce the equivalent mmf. The actual stator mmf in any axis (say q or d) is the product of the number of turns and current in the respective axis winding. It is equated, respectively, to the mmf produced by the fictitious stator windings on the  $q_r$  and  $d_r$ -axes. Similarly, the same procedure is repeated for the d-axis of the actual stator winding. This leads to a cancellation of the number of turns on both sides of the q- and d-axes stator mmf equations, resulting in a relationship between the actual and fictitious stator currents. The relationship between the currents in the stationary reference frames and the rotor reference frames currents is written as.

$$i_{qd_s} = [T] i_{qd_r}$$

and similarly voltage relation is given as



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$$v_q d_s = [T] v_q^r d_s$$

Where T is transition matrix

$$T^t = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix}$$

The PMSM model in rotor reference frames is obtained as

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = \begin{bmatrix} R_s + L_{qp} & wrL_d \\ -wrL_q & R_s + L_{dp} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} wr\lambda_{af} \\ 0 \end{bmatrix} \quad (9)$$

Where,  $w_r$  is the rotor speed in electrical radians per second. This equation is in a form where the voltage vector is equal to the product of the impedance matrix and the current vector, with an additional component due to the motional emf of the rotor flux linkages.

## IV. ELECTROMAGNETIC TORQUE

The electromagnetic torque is the most important output variable that determines the mechanical dynamics of the machine such as the rotor position and speed. It is derived from the machine matrix equation (above) by looking at the input power and its various components such as resistive losses, mechanical power, and the rate of change of stored magnetic energy. Hence, the output power is the difference between the input power and the resistive losses in a steady state. The dynamic equations of the PMSM can be written as

$$V = [R] i + [L] \dot{i} + [G] w_r i \quad (10)$$

By premultiplying Equ.(10) by the transpose of the current vector, the instantaneous input power is

$$P_i = i^t V = i^t [R] i + i^t [L] \dot{i} + i^t [G] w_r i \quad (11)$$

Where

[R] matrix consists of resistive elements

[L] matrix consists of the coefficients of the derivative operator  $p$

[G] matrix has elements that are the coefficients of the electrical rotor speed,  $\theta_r$

The term  $i^t [R] i$  gives stator and rotor resistive losses. The term  $i^t [L] \dot{i}$  denotes the rate of change of stored magnetic energy. The air gap power, is given by the term  $i^t [G] w_r i$ . From the fundamentals, it is known that the air gap power has to be associated with the rotor speed. The air gap power is the product of the mechanical rotor speed and air gap electromagnetic torque. Hence, the air gap torque,  $T_e$ , is derived from the terms involving the rotor speed,  $w_m$ , in mechanical rad/s, as

$$w_m T_e = P_a = i^t [G] i w_r = i^t [G] i [P/2] w_m \quad (12)$$

where  $P$  is the number of poles.

Cancelling speed on both sides of the equation leads to an electromagnetic torque that is

$$T_e = (P/2) i^t [G] i \quad (13)$$

Substituting [G] in Equ.(13) with the observation from Equ.(9), the electromagnetic torque is obtained as

$$T_e = (3/2)(P/2) [y_{af} + (L_d - L_q) i_{dr}^t i_{qr}^t] \quad (\text{N.m}) \quad (14)$$

## III. IMPLEMENTING MODELED EQUATIONS BY SIMULINK

Three phase supply of  $V_{rms} = 220V$ ,  $f = 50$  Hz is provided as supply. Using Parks Transformation three phase is transformed into two phase for ease of modelling. Parks Transformation give Direct axis(d), Quadrature axis(d) and zero-sequence currents. Zero sequence currents are terminated.

### A. Current Subsystem:

The stator currents  $i_q$ ,  $i_d$  are derived by state space model and these are represented in the subsystems as shown in the fig 2.

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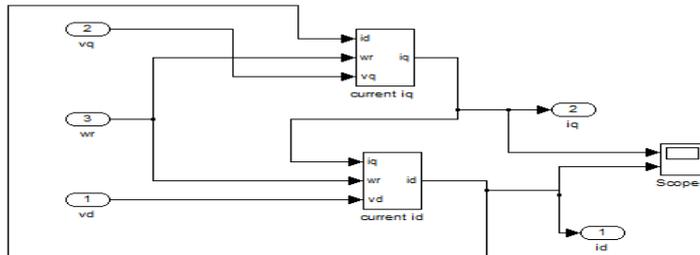


Figure 2. Current equations Block

B. *Torque Sub model* :The electromagnetic equation that is derived equ(14) is implemented in the subsystem as shown in fig 3.

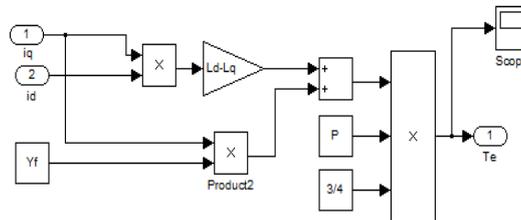


Figure 3. Electromagnetic Torque Block

C. *Speed Sub model*:

$$\text{The electromagnetic torque of motor is given by } T_e = T_l + Bw_m + Jp w_m \quad (15)$$

Where B - Friction coefficient,

J – Inertia of motor,

p – Derivative

$$\text{And from above equation speed } (w_m) \text{ is found as } w_m = (T_e - T_l - Bw_m) * 1/J \quad (16)$$

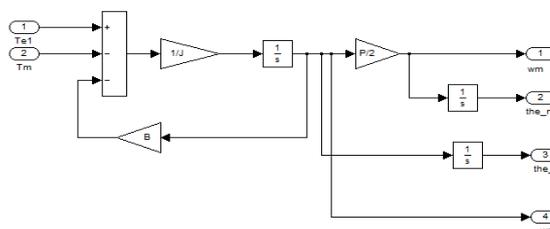


Figure 4. Motor Speed, Rotor Position

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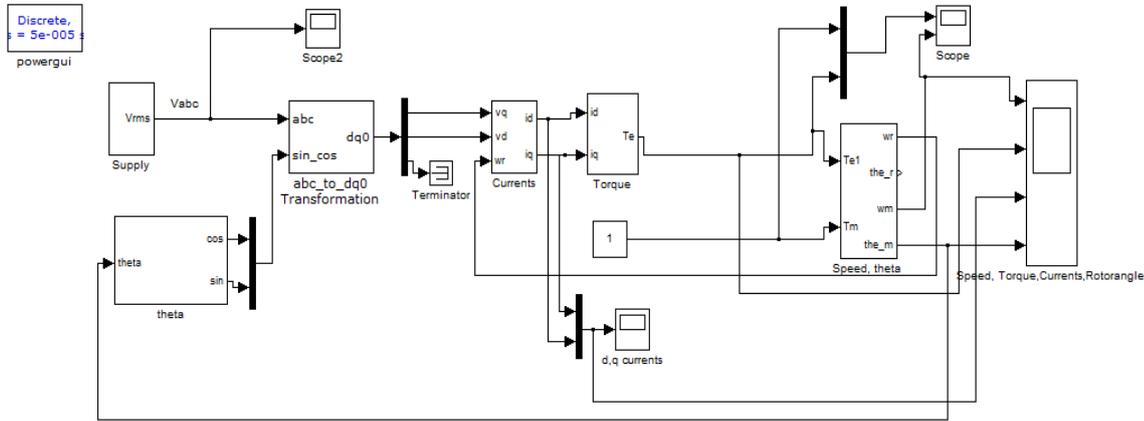


Figure 5. Simulink model of PERMANENT MAGNET SYNCHRONOUS MOTOR

## IV. SIMULATION RESULTS

The motor simulation is carried out for with values Stator Resistance  $R_s = 1.2 \Omega$ ,  $B = 4.6752e-5 \text{ kg/m}^2$ ,  $J = 2.0095e-5 \text{ N-m s}$ , Stator Resistance = 2.0357, Direct axis Inductance ( $L_d$ ) = 7.8e-3 H, Quadrature axis Inductance ( $L_q$ ) = 7.8e-3 H, flux – 0.154 V s, Poles=4,  $V_{rms} = 220\text{v}$

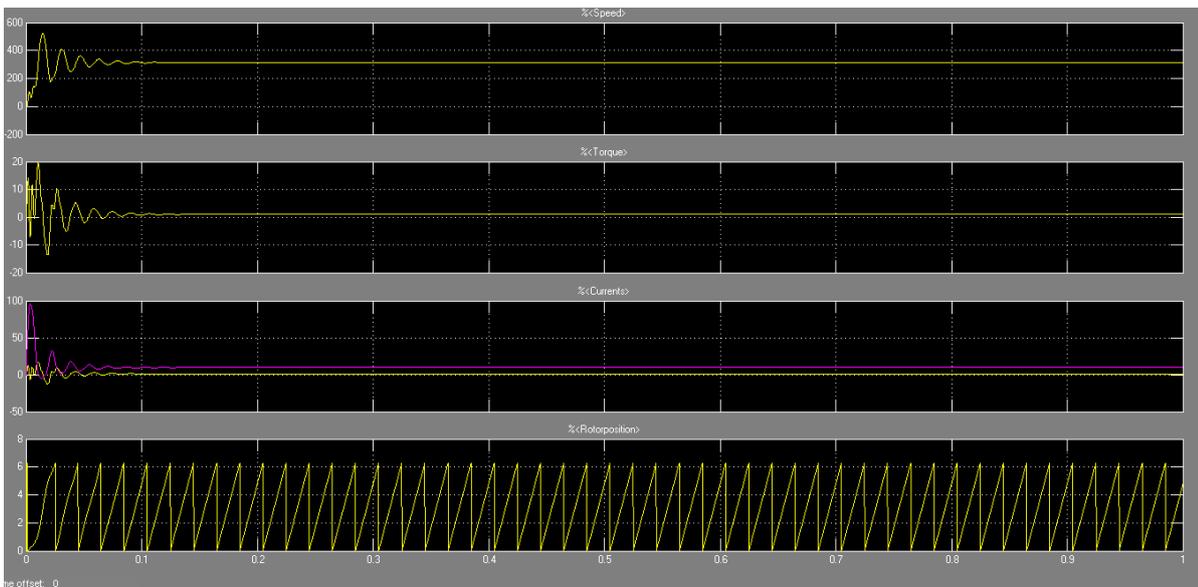


Figure 6. Motor speed ( $\omega_m$ ), Electromagnetic Torque ( $T_e$ ), Quadrature & Direct axis currents, Rotor positions

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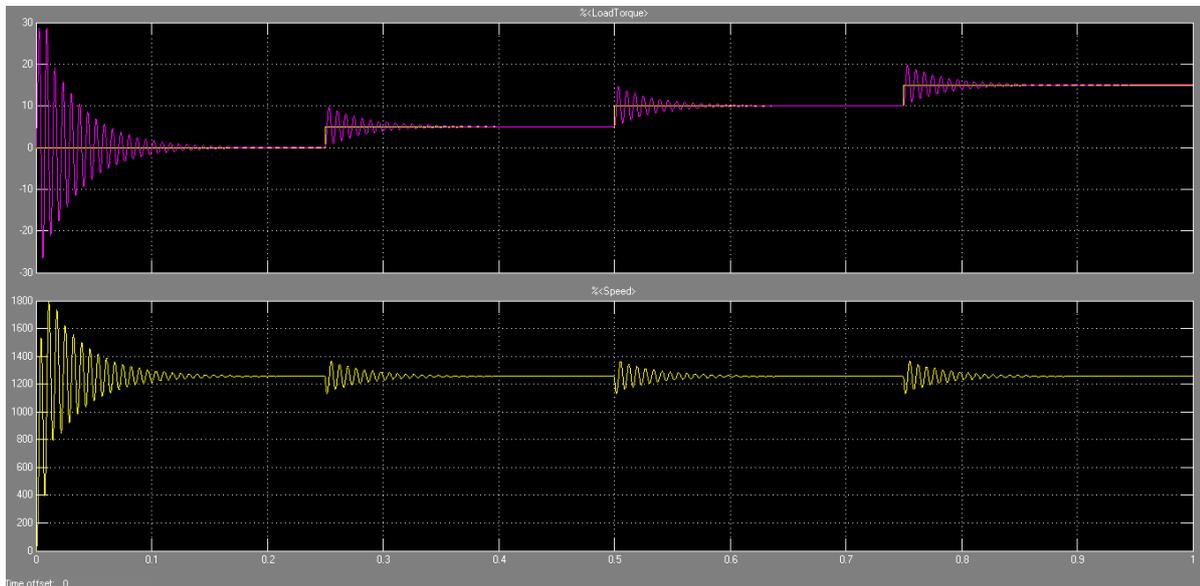


Figure 7.shows motor speed remains constant even change in the load torque

## V. CONCLUSION

In this paper simulation based mathematical model of permanent magnet synchronous motor is implemented using MATLAB. The simulation results shows the performance characteristics of permanent magnet synchronous motor, i.e. speed of motor remain constant even with variation of load torque. The stator currents and electromagnetic torque magnitudes are also obtained in the graphs. The rotor position found out at every instant and is shown in the graph.

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