



MODELING AND SEGMENTATION OF ECG SIGNALS THROUGH CHEBYSHEV POLYNOMIALS

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ABSTRACT: ECG (Electrocardiogram) signals originating from heart muscles, generate massive volume of digital data. They need to be compressed or approximated for efficient transmission and storage. ECG signal compression is traditionally performed in three ways: direct, transform and parameter extraction. Polynomial approximation which is a form of parameter extraction method, is employed here. This paper proposes a new technique of ECG modeling using Chebyshev interpolation. The technique is implemented on a standard ECG signal with a Chebyshev polynomial of order 100 with a maximum error less than 0.1. Segmentation of ECG signal further decreases the order of the Chebyshev polynomial and thereby the maximum absolute error.

Keywords: ECG Signals, Polynomial Approximation, Chebyshev Polynomials, Chebyshev Interpolation.

I. INTRODUCTION

Heart muscles produces electrical variations on the body of skin and the variations are measured by electrodes placed on specific positions on the body. These variations are referred to as ECG (Electrocardiogram) signals. From engineering point of view, an ECG signal is periodic. Figure 1 shows one cardiac cycle of the signal which is also known as the R-R interval with three indicated parts: P wave, QRS complex, and the T wave. These waves are the result of contraction and expansion of the heart muscles. The P wave is due to the depolarization of the atria whereas QRS complex reflects the rapid depolarization of the right and left ventricles. The T-wave represents the repolarization (or recovery) of the ventricles [1].

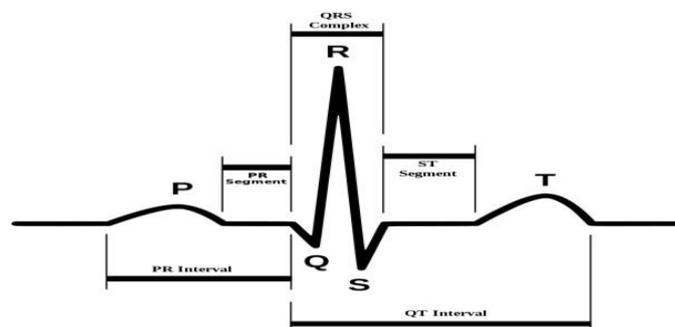


Figure.1 ECG Signal

The amount of data involved in storage and transmission of digital ECG signals is quite large. So it needs to be adequately compressed. The compression must be done in a way so that it can be accurately reconstructed.

The ECG compression techniques are broadly classified as: direct methods, transform-based methods and parameter extraction methods. In direct methods, the original ECG signal samples are compressed directly, and in transformation



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method the original samples are first transformed and then compressed. In parameter extraction methods, the features of the processed signal are extracted and then these features are used for the reconstruction of the signal [2].

Various time domain compression algorithms for ECG signals can be found in the literature. These methods are based on the idea of extracting few significant signal samples to represent the signal and then decoding the same set of samples. The time domain algorithms are based on fast heuristics in the sample selection process. These techniques are faster but suffer from suboptimality. Examples of such techniques are the FAN algorithm [3] and the AZTEC algorithm [4]. Time domain algorithms were further improved by SLOPE [5] and AZTDIS [6] techniques. The Cardinality Constrained Shortest Path (CCSP) algorithm presented in [7] is based on a mathematical model of the entire sample selection process. Signal samples are modeled as nodes in a graph and then optimization techniques are applied for achieving higher compression ratio.

For ECG data compression, several compression algorithms including polynomial approximations and polynomial interpolation have been proposed. The advantage of polynomial approximation is that it requires only polynomial coefficients and it is able to approximate the original ECG signal quite efficiently.

Third order polynomials including splines have been used for ECG interpolation in [8] and [9]. ECG signals represented by second order quadratic polynomials were studied by Nygaard et. al. in [10]. Legendre polynomials of high degree were used for ECG data compression in [11], [12] and [13]. Generalized Jacobi polynomials were employed for ECG compression in [14]. Although Chebychev polynomials are widely used in mathematical interpolation and approximation, ECG signal compression through Chebychev polynomials are hardly found in the literature. In [15] ECG data compression is done using Discrete Chebyshev Transform by segmenting the signal into blocks which consist of multiple cardiac cycles. In [16] ECG signal data compression is done in the same way as in [15] but only significant Chebyshev coefficients are selected, transmitted and stored.

In this paper, we propose a computationally efficient method to model ECG signals through Chebyshev polynomials. In order to have a better compression ratio we must have a lower order of the polynomial. So we segment one complete cycle into various portions depending upon their structures. Next, we model each segment independently using Chebyshev interpolation and combine them to reconstruct the complete signal.

The rest part of the paper is organized as follows: in the next two sections, we give a brief introduction to Chebychev polynomials and Chebyshev interpolation. In section IV we describe the proposed method along with the algorithm to achieve ECG data compression. In section V we present the implementation of our method and discuss the results obtained. In the last section we give the conclusions.

II CHEBYSHEV POLYNOMIALS

The Chebyshev polynomials of first type and degree n are defined in terms of trigonometric cosine function as:

$$T_n(x) = \cos(ncos^{-1}(x)) \text{ for } n \geq 0 \quad (1)$$

The expressions for Chebyshev polynomials are obtained from the recursive relation

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x) \text{ for } n \geq 1 \end{aligned} \quad (2)$$

The Chebyshev polynomials of degree n , $T_n(x)$, has n zeros in the interval $[-1, 1]$, which can be calculated as

$$x_j = \cos\left(\frac{2j-1}{2n}\pi\right) \quad 1 \leq j \leq n \quad (3)$$



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The roots of the Chebyshev polynomial are also known as Chebyshev points. In the same interval the $n + 1$ extrema of the polynomial $T_n(x)$ are located at

$$\tilde{x}_j = \cos\left(\frac{\pi j}{n}\right) \quad 0 \leq j \leq n \quad (4)$$

At all the maxima $T_n(x) = 1$, while at all the minima $T_n(x) = -1$. The Chebyshev polynomials are orthogonal in the interval $[-1,1]$ over the weight $w(x) = (1-x^2)^{-1/2}$. They also satisfy discrete orthogonality relationships. Other properties of Chebyshev polynomials can be found in [17].

III. POLYNOMIAL APPROXIMATION USING INTERPOLATION

A polynomial approximation problem is of finding a polynomial close to a given function and has the freedom to select the significant points. Once the significant points have been fixed, it is reduced to an interpolation problem that can be solved by polynomial interpolation. Let $y(x)$ represent ECG segment vector of length N consisting of samples of $y(x)$ such that

$$y(x) = \{y(x_0), y(x_1), \dots, \dots, \dots, y(x_N)\}, \quad x \in [a, b]$$

Given a set of $N + 1$ data points $(x_i, y(x_i))$ we want to construct a polynomial f of degree N with the property

$$f(x_i) = y(x_i) \quad i = 0, 1, \dots, N$$

Suppose the interpolation polynomial is in the form

$$f(x) = a_N x^N + a_{N-1} x^{N-1} + \dots + a_2 x^2 + a_1 x + a_0, \quad x \in [a, b] \quad (5)$$

which means that

$$f(x_i) = y(x_i) \quad \forall i \in \{0, 1, \dots, N\} \quad (6)$$

Substituting (6) in (5) we get a system of linear equations which in matrix form reads

$$\begin{bmatrix} x_0^N & x_0^{N-1} & \dots & 1 \\ x_1^N & x_1^{N-1} & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_N^N & x_N^{N-1} & \dots & 1 \end{bmatrix} \begin{bmatrix} a_N \\ a_{N-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} y(x_0) \\ y(x_1) \\ \vdots \\ y(x_N) \end{bmatrix} \quad (7)$$

The matrix on the extreme left is the Vandermonde's matrix. The system given by (7) would have a unique solution if the determinant of the Vandermonde matrix does not vanish. Solving this system for a_k we can construct the interpolating polynomial $f(x)$.

Alternatively we can write the required polynomial explicitly using Lagrange's formula as

$$f(x) = \sum_{i=0}^N f(x_i) \prod_{j=1, j \neq i}^N \frac{x-x_j}{x_i-x_j}, \quad x \in [a, b] \quad (8)$$



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Let us now construct yet another interpolating polynomial $p(x)$ by sampling $f(x)$ at n interpolation points such that $n < N$. We can estimate the difference between them, i.e., the interpolation error $E(x)$. Let Π_n denote the space of polynomials of degree $\leq n$, and let $C^{n+1}[a, b]$ denote the space of functions that have $n + 1$ continuous derivatives on the interval $[a, b]$. Then we have this theorem:

Theorem 1: Let $f(x) \in C^{n+1}[a, b]$. Let $p(x) \in \Pi_n$ such that it interpolates $f(x)$ at the $n+1$ distinct points $x_0, \dots, x_n \in [a, b]$. Then $\forall x \in [a, b]$ such that

$$E(x) = f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{j=0}^n (x - x_j)$$

where ξ is an intermediate point between x_0 and x_n .

If we are free to choose the interpolating points x_0, \dots, x_n within this interval, then the product $\prod_{j=0}^n (x - x_j)$ can be minimized which in turn would minimize the interpolating error $E(x)$. This can be achieved by choosing interval as $[-1, 1]$ and the interpolating points x_j as the Chebyshev points. The following theorem gives an estimate of the error for the above case.

Theorem 2: Assume that $p(x)$ interpolates $f(x)$ at x_0, x_1, \dots, x_n . Also assume that these $n+1$ interpolation points are the $(n + 1)$ roots of the Chebyshev polynomial of degree $T_{n+1}(x)$, which are given by (3). Then $\forall x \in [-1, 1]$,

$$|f(x) - p(x)| \leq \frac{1}{2^n (n+1)!} \max_{\xi \in [-1, 1]} |f^{(n+1)}(\xi)|$$

Our goal is not to approximate a function $p(x)$ on the interval $[-1, 1]$ but to approximate the values of the function $p(x)$ corresponding to the discrete set of points given by (3). An arbitrary function $p(x)$ can be approximated in the interval $[-1, 1]$ by

$$p(x) = \sum_{k=0}^n c_k T_k(x) \in [-1, 1] \tag{9}$$

where the coefficients c_j are defined as

$$c_0 = \frac{1}{n+1} \sum_{j=1}^{n+1} p(x_j)$$

$$c_k = \frac{2}{n+1} \sum_{j=1}^{n+1} p(x_j) T_k(x_j), k = 1, \dots, n \tag{10}$$

IV. PROPOSED METHOD

Since we are processing one R-R interval, our working domain is in the interval $[a, b]$. So, we first transform the interpolation interval $y \in [-1, 1]$ using

$$x = \frac{(b - a)y + (a + b)}{2} \tag{11}$$

This converts the interpolation problem for $f(x)$ on $[a, b]$ into an interpolation problem for $f(x) = g(x(y))$ in $y \in [-1, 1]$. The Chebyshev points in the interval are $y \in [-1, 1]$ the roots of the Chebyshev polynomial $T_n(x)$, i.e.,

$$y_j = \cos\left(\frac{2j-1}{2n} \pi\right), 1 \leq j \leq n$$



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The corresponding $n + 1$ interpolation points in the interval $[a, b]$ are

$$x_j = \frac{(b-a)y_j + (a+b)}{2}, \quad 1 \leq j \leq n$$

The interpolation error is given by

$$|f(x) - p(x)| \leq \frac{1}{2^{n(n+1)!}} \left| \frac{b-a}{2} \right|^{n+1} \max_{\xi \in [a, b]} |f^{(n+1)}(\xi)|$$

In our method we need to construct the function $f(x)$ using (8) with all the N ECG samples. Then we find the Chebyshev points and the Chebyshev interpolating polynomial using the nodes. Next we calculate the error and if the error is not within our tolerance we increase the order. We continue doing these operations till our error criterion is met.

Next we observe that to model the ECG signal over one R-R interval, the order of the Chebyshev polynomial may be quite high. So we segment the signal into various sections depending upon the structure of the section. Then we model each section independently using Chebyshev interpolation technique.

We present here an algorithm to show the steps of our method.

Algorithm *Chebyshev_poly_approx* = *Chebyshev_poly_approx* ($N, \varepsilon, f(x), C_k, p(x)$)

Inputs: $N, \varepsilon = 10^{-2}$

Outputs: $f(x), C_k, p(x)$

BEGIN algorithm

- 1) Construct the function $f(x)$ from N samples using (8).
- 2) Fix the order n of the Chebyshev approximation.
- 3) Transform the domain to $[-1, 1]$ using (11).
- 4) Find the zeros or the Chebyshev points x_j using equation (3).
- 5) Find Chebyshev polynomial $T_n(x_j)$ from equation (1).
- 6) Calculate coefficient c_k for $k=0$ to n using equation (10).
- 7) Construct interpolating polynomial $p(x)$ using equation (9).
- 8) Calculate error $E(x) = \max|f(x)-p(x)|$
- 9) If $E > \varepsilon$ then $n = n + 1$ and go to step 4.

V. SECTION RESULTS AND DISCUSSION

An ECG signal is not linear, rather more curvaceous consisting of waves of various shapes. The proposed algorithm is implemented and tested over one R-R interval of the ECG signal. The performance of the Chebyshev approximation technique is measured by maximum absolute error and gridwise variation of error.

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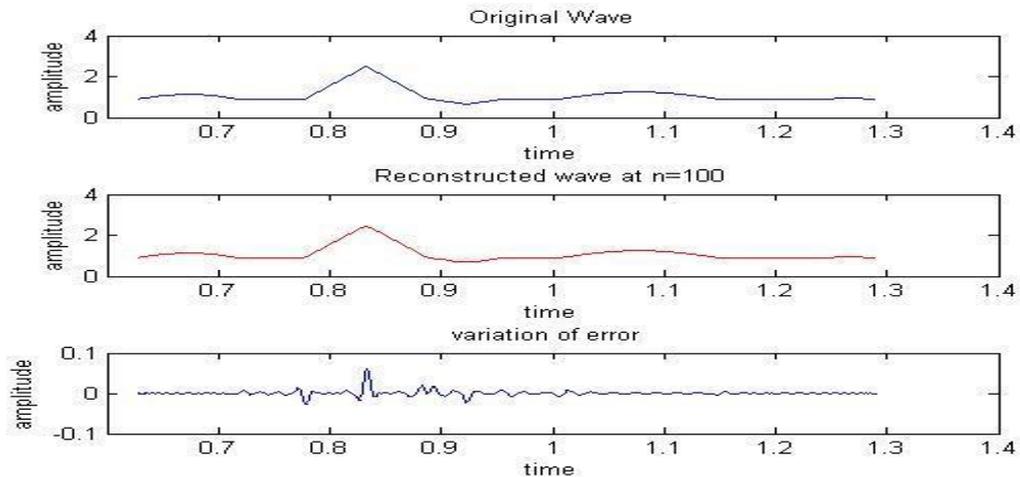


Figure.2 Original and Approximated ECG signal

For testing the performance of our algorithm we conducted our tests in MATLAB environment using a simulated standard ECG signal. The algorithm was initially applied over the R-R interval of the ECG signal. The signal consisted of 8274 samples and we fixed the error tolerance as $\epsilon = 10^{-2}$. To approximate one complete cycle we required Chebyshev polynomials upto order 100. The original and approximated signal with the error over one cardiac cycle is shown in Figure 2.

Since the order of the Chebyshev polynomials is quite high and we needed a better compression, we segment the entire cycle into various sections. The shape and duration of the various waves of ECG is considered as the basis for segmentation. Therefore the number of sample points for different segments is different. All the seven segments of the complete cycle are shown in Figure 3.

The algorithm was applied over the sections and the order of Chebyshev polynomial was increased till the error was within the specified tolerance.

The segmented original and the approximated signals along with the error over various segments is shown in Figure 4. Table I shows the variation of error with increasing order of the polynomials for different segments. It can be seen from the table that the segments with varying slopes require higher order as high as 8th order Chebyshev polynomials, whereas the segments where slope variations are less can be interpolated by Chebyshev polynomials with a maximum order of 3 within the specified error tolerance.

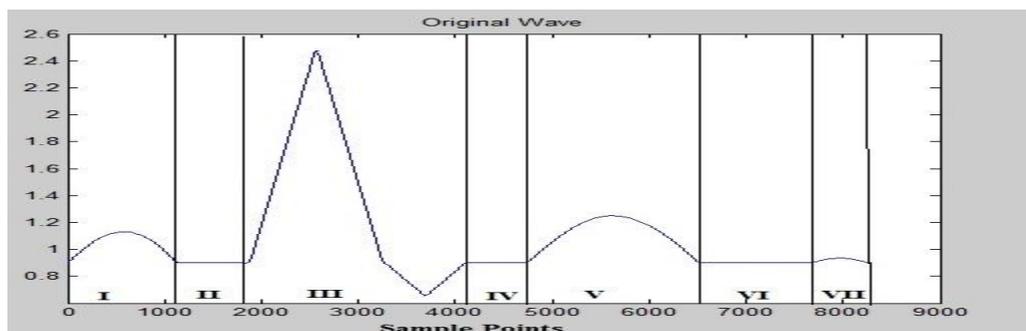


Figure.3 Segmented ECG signal

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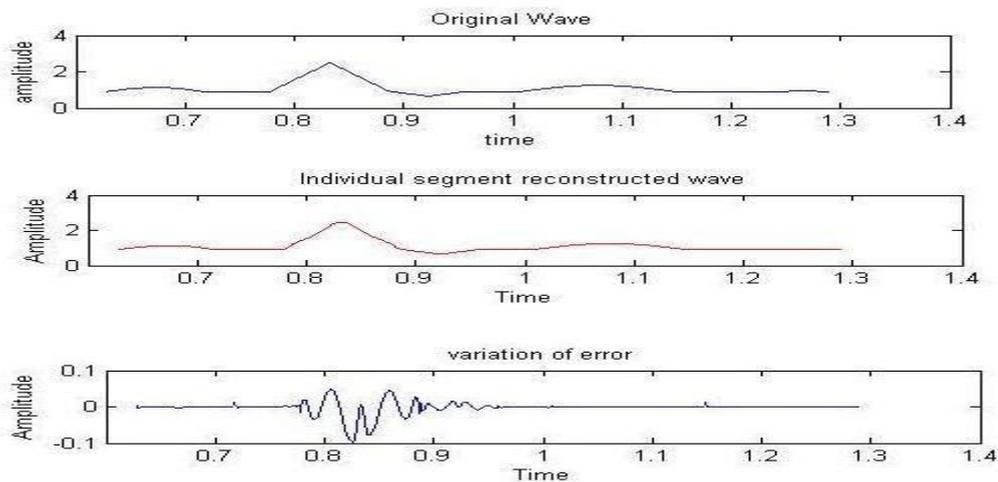


Figure.4 Original and Approximated signal

From Figure 3 and Table I it can be observed that the error for the section containing QRS complex is still quite high even for a 8 order polynomial (approximately 0.119). So this section is further segmented into three subsections. The error variation with increasing order of the approximated polynomials over different subsections are shown in Table II .

Table 1: Maximum absolute error for different sections with increasing order of the polynomials

Order Segment	2	3	4	5	6	7	8
I	0.011	0.007	0.006	0.005	0.004	0.003	0.003
II	0.015	0.012	0.008	0.007	0.007	0.005	0.005
III	0.990	0.476	0.490	0.232	0.199	0.263	0.119
IV	0.008	0.007	0.005	0.004	0.003	0.002	0.002
V	0.019	0.010	0.001	0.002	0.002	0.002	0.002
VI	0.010	0.011	0.009	0.007	0.005	0.004	0.003
VII	0.003	0.002	0.001	0.001	0.001	0.001	0.001

Table 2: Maximum absolute error between different sections of the QRS complex with increasing order of the polynomials

Order Segment	2	3	4	5	6	7	8
I	0.028	0.026	0.018	0.013	0.013	0.010	0.008
II	0.099	0.083	0.081	0.054	0.064	0.033	0.048
III	0.012	0.007	0.006	0.003	0.003	0.002	0.002

It can be seen from the table that the maximum absolute error is drastically reduced over the sections containing the QRS complex. Further it is observed that the individual subsections containing the QRS complex can be approximated with very low order polynomials. Error can also be reduced by further segmenting these subsections.



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VI.CONCLUSION

This paper proposes a technique to approximate simulated ECG signals using Chebyshev polynomials. Initially the complete signal over one cardiac cycle was approximated with higher order Chebyshev polynomials. The order of Chebyshev polynomials depends upon the shape of section ECG wave. Waves having zero or constant slope, can be approximated by lower order Chebyshev polynomials whereas waves having variable slopes require higher order Chebyshev polynomials. So various sections of simulated ECG were taken and approximated with comparatively lower Order Chebyshev polynomials. The approximation error can further be reduced by applying advanced segmentation techniques which would involve interval analysis. This work would be addressed as our future work.

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