

Multi-Objective Resource Constrained Project Scheduling Using Critical Chain Project Management Approach

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ABSTRACT: This paper presents a goal programming approach for determining project scheduling for the network by the application of Critical Chain Project Management methodology under multiple resource constraints for achieving twin objectives- minimization of time and minimization of cost.

KEYWORDS: Critical Chain Project Management, Heuristic methods, Multiple Objectives, Goal Programming.

Notation

C	: overhead cost/unit time
C(r)	: unit cost of resource r
D	: latest completion time of the project
K(r)	: maximum amount of resource r employed
K(r,i)	: amount of resource r consumed per period by the activity i
n	: number of activities scheduled in period j
R	: return on investment/period
r	: 1,2,...,m resources
T	: duration which minimizes Z
t(i)	: duration of the activity i

I. INTRODUCTION

Two primary parameters against which resource constrained project schedule [2] can be evaluated are--- a) project duration and b) schedule related resource costs. If the project is completed ahead of schedule, profit can be generated in two ways--- one is direct cost savings and the other, early income from the project due to early commissioning. But it is possible that the cost of increasing resource limits above the initial set of levels would be more than offset by resulting decreasing in overhead charges and due date penalty and the reverse situation may also be true. Hence a search procedure should be employed to seek some optimum combination of resource levels and result completion date [7]. This is indeed, a multi-objective project scheduling problem situation.

II. LITERATURE REVIEW

Broadly, there are approaches [5] to solve multi-objective scheduling problems. The first approach involves finding out a number of efficient schedules generated heuristically, for consideration of decision-makers. A feasible schedule S, i.e. a schedule satisfying both precedence and resource constraints dominates if the outcome of S in terms of each criterion is as good as that of other with at least one better. The second approach in multi-objective scheduling seeks working out an optimum schedule with mathematical programming like goal programming. The present paper intends to find out such set of efficient schedules by using goal programming. Goal programming [4] provides an answer to determine optimal value of performance of multiple objectives of project scheduling by taking into account of the various conflicting, incompatible and incommensurable performance objectives. In an exercise of this type, various objectives

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in project scheduling are required to be ranked in terms of their importance. The solution algorithm then satisfies each objective sequentially, that is the objective in the next sequential order is taken up for satisfying only when the previous one is satisfied.

III. MODEL FORMULATION

The very nature of the precedence relation between an activity and its successors brings into play ‘either-or’ nature of the problem: either the activity is completed, hence its successors may start or it is not completed, hence its successor cannot start. This, in turn, leads to integer linear programming models [1]; in particular, 0, 1 integer programming model. The main assumption made in this model is that the resources levels are maintained at all times and are paid for, whether active or idle [6].

Total cost of the project is given by

$$Z = CT + \sum_{r=1}^m K(r)C(r)T + R(T - D) \tag{1}$$

0-1 ILP Formulation:

The general mathematical form of project scheduling model is to determine $X(i, j)$ subjected to following constraints.

- i) Each activity must start only once within the EST (earliest start time) and LST (latest start time).

$$\sum_{j=EST}^{LST} X(i, j) = 1 \tag{2}$$

$X(i, j) = 1$ if the activity i is started in period j ,
 $X(i, j) = 0$ otherwise.

First, an estimate of the latest completion time D of the project by scheduling all the activities serially is required by using the CPM approach. D_{min} is the earliest possible time of completing the project obtained by using the CCPM approach [3]. Obviously, project duration under no constraints is the shortest possible time of completing the project.

Lower bound (EST) is the earliest possible time at which an activity can start and is calculated by moving first event to last event in the network diagram. Upper bound (LST) is calculated by moving backward, i.e from last event to first event in the network diagram.

Latest starting time for the last activities is given by $LST(i) = D - t(i)$.

Latest starting time for the remaining activities is given by

$$LST(i) = \text{minimum upper bound of succeeding activities of } i - t(i).$$

- ii) Constraints satisfying the precedence relationship. Completing time of activity i - start time of the succeeding activity ≤ 0

$$\sum_{j=EST}^{LST} (j + t(i)X(i, j)) - \sum_{j=EST}^{LST} jX(i, j) \leq 0 \tag{3}$$

For all succeeding activities of i

- iii) Time of the complete of each terminal activity is less than or equal to D . The optimum project duration T lies in between D and the project duration under no constraints (D_{min}).

For the last activities

$$\sum_{j=EST}^{LST} (j + t(i)X(i, j)) \leq T. \tag{4}$$

- iv) In any given period, the amount of resource r consumed on all activities that are being processed during the period shall not exceed the amount of resource available.

$$\sum_{i=1}^n X(i, j)k(r, i)V(i, j) \leq K(r) \tag{5}$$

When $r=1, 2, \dots, m$

Feasible schedules can be generated using implicit enumeration technique by varying T .

Goal Programming Formulation:

In the proposed goal programming model, the objective function represents the maximization of deviation from the minimum levels according to pre-assigned weightage W_k ($K=1, 2$).

Here, the objective function is

$$\text{Maximize } P = W_1d_1 + W_2d_2$$

Subjected to

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$$CD + \sum_{r=1}^m K(r)C(r)D - (CT + \sum_{r=1}^m K(r)C(r)T + R(T-D)) - d_1 = 0 \tag{6}$$

$$D - T - d_2 = 0 \tag{7}$$

Where d_1 and $d_2 \geq 0$

Solution Procedure:

The steps of search procedure to determine the solution which maximizes P are summarized as

- Step 1: Calculate project cost (Z) using equation (1) at T=D for the minimum amount of resource levels.
- Step 2: Varying T determine feasible schedule to the smallest value of T ($D_{min} \leq T \leq D$) using 0-1ILP formulation.
- Step 3: Find d_1 and d_2 using constraint equations (6) and (7).
- Step 4: Determine objective function value using $P = W_1 d_1 + W_2 d_2$.
- Step 5: Increase resource level and repeat steps 2 to 4.
- Step 6: A schedule which maximizes the objective function is the optimal project schedule.

Illustrative example:

The procedure is illustrated here by applying it to the project of seven activities shown in Fig 1. The pertinent data on the activities and their needs of resource are given in table 1.

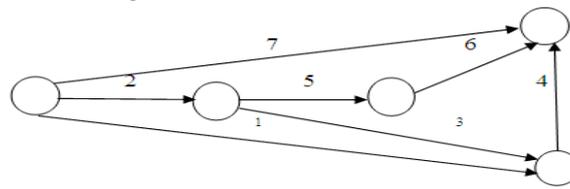


Fig. 1 example network

Table 1: the pertinent data on the activities and their needs of resource ($W_1=0.5, W_2=2000$)

Activity number	1	2	3	4	5	6	7
Duration	3	3	1	2	2	3	3
Resource required	2	3	2	3	4	2	1

Data of numerical example:

- Overhead cost/period =Rs 6000;
- Return on investment/period =Rs25000;
- Cost of unit resource/period =Rs400.

The earliest start time of all the activities has been determined first is given in Table 2. The shortest possible duration of completing the project (under no resource constraints) is 8. The possible duration of completing the project (under resource constraints) is 5 with the resource level of 4. The results are given in the feasible schedule shown in Fig 2. From this figure the following values are obtained.

$$D = 8 \text{ and } K(1) = 4$$

$$Z = 6000(8) + 400(4)8 + 2500(8-8);$$

$$Z = \text{Rs } 60\,800$$

D_{min} value is obtained from the CCPM approach.

CCPM approach: CCPM can be applied using following three steps.

Table 2: Remove the safety time and reduce tasks duration by 50%

Activity	duration	Resources	EST	EFT	LST	LFT	TF
1	1.5	2	0	1.5	1.5	3	1.5
2	1.5	3	0	1.5	0	1.5	0
3	0.5	2	1.5	2	2.5	3	1
4	1	3	2	3	3	4	1
5	1	4	1.5	2.5	1.5	2.5	0
6	1.5	2	2.5	4	2.5	4	0
7	1.5	1	0	1.5	2.5	4	2.5

Activities with zero total float will form the critical path. Critical path is 2-5-6

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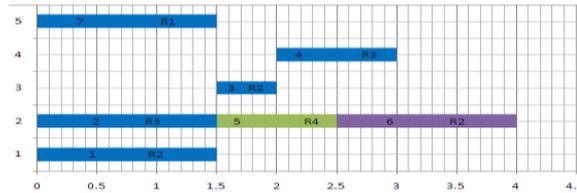


Fig. 2 project schedule by reducing the task duration by 50%

Figure 2 gives the information about project schedule after reducing the task durations to 50% and scheduling the tasks as per earliest starting time. It gives the critical path as 2-5-6.

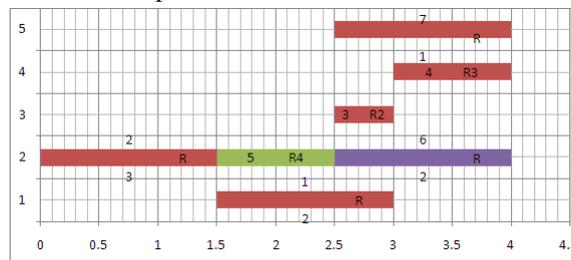


Fig. 3 creating schedule on Latest Finish dates

Figure 3 gives the information about scheduling all non critical activities from the latest finish dates and all the critical activities from earliest starting time. Here identify the resource constraints and resolve the resource constraints.

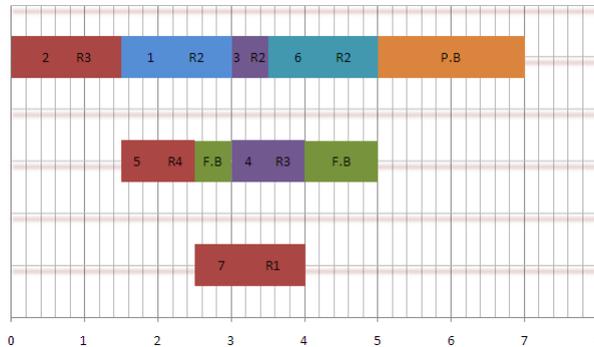


Fig. 4 project scheduling by removing the resource constraints

Figure 4 gives the information about project schedule after resolving the resource constraints. It gives the critical chain of the project schedule 2-1-3-5.

So the critical chain is 2-1-3-6 and the project duration is 7days. We are considering the D_{min} as the critical chain project duration without the buffers so the value of D_{min} is 5days.

Table 2 gives the earliest start time and the latest start time of the activities so that optimal project schedule lies between D and D_{min} .

Table 3: gives the earliest start time and the latest start time of the activities

Activity	1	2	3	4	5	6	7
EST (i)	0	0	1.5	2	1.5	2.5	0
LST (i)	5.5	4	6.5	7	5.5	6.5	6.5

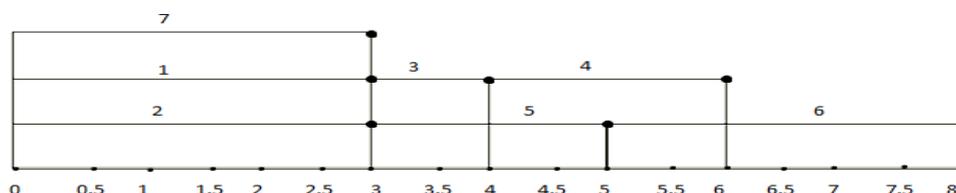
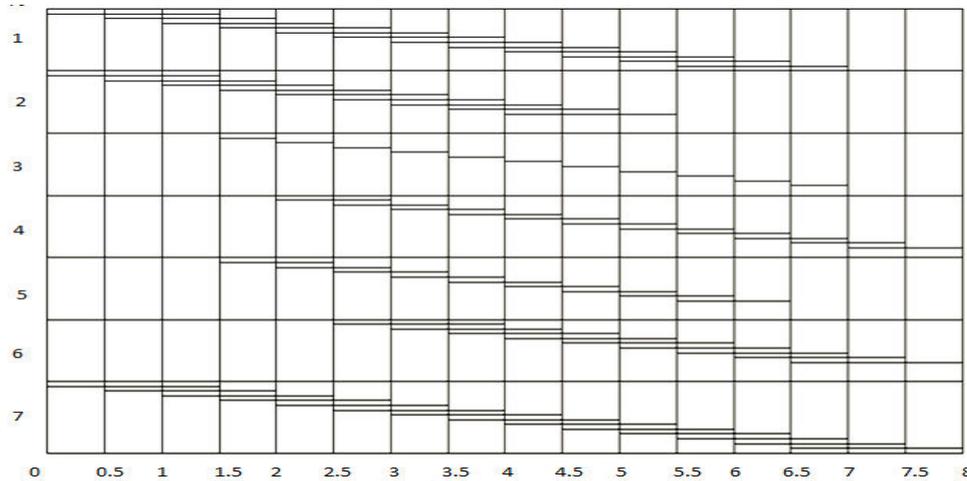


Fig. 5 Project schedule

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A "bar" represents the position of '1' values in the V (i, j) Vector

Fig.6 The bar chart

A bar chart as shown in Fig 3 is prepared for each activity to represent the position of 1 value in the V (i, j) vector. Activity 1 can be scheduled 0 and 1.5 period or 0.5 and 2 or 1 and 2.5 or 1.5 and 3 or 2 and 3.5 or 2.5 and 4 or 3 and 4.5 or 3.5 and 5 or 4 and 5.5 or 4.5 and 6 or 5 and 6.5 or 5.5 and 7. In similar fashion v value for the remaining activities can be determined.

0-1 LP Formulation: Determining X (i, j) where i= 1, 2... 7.

Subjected to the following constraints

(1) Each activity must start only once within EST and LST gives the following Constraints.

$$X(1,0)+X(1,0.5)+X(1,1)+X(1,1.5)+X(1,2)+X(1,2.5)+X(1,3)+X(1,3.5)+X(1,4)+X(1,4.5)+ X(1,5) +X(1,5.5) =1$$

$$X(2,0)+X(2,0.5)+X(2,1)+X(2,1.5)+X(2,2)+X(2,2.5)+X(2,3)+X(2,3.5)+X(2,4)=1$$

$$X(3,1.5)+X(3,2)+X(3,2.5)+X(3,3)+X(3,3.5)+X(3,4)+X(3,4.5)+X(3,5)+X(3,5.5)+X(3,6)+ X(3,6.5) =1$$

$$X(4,2)+X(4,2.5)+X(4,3)+X(4,3.5)+X(4,4)+X(4,4.5)+X(4,5)+X(4,5.5)+X(4,6)+X(4,6.5)+ X(4,7) =1$$

$$X(5,1.5)+X(5,2)+X(5,2.5)+X(5,3)+X(5,3.5)+X(5,4)+X(5,4.5)+X(5,5)+X(5,5.5)=1$$

$$X(6,2.5)+X(6,3)+X(6,3.5)+X(6,4)+X(6,4.5)+X(6,5)+X(6,5.5)+X(6,6)+X(6,6.5)=1$$

$$X(7,0)+X(7,0.5)+X(7,1)+X(7,1.5)+X(7,2)+X(7,2.5)+X(7,3)+X(7,3.5)+X(7,4)+X(7,4.5)+X(7,5)+X(7,5.5)+X(7,6)+X(7,6.5)=1$$

(2) Constraints satisfying precedence relationship

$$2 \leq 1.5X(2,0)+2X(2,0.5)+2.5X(2,1)+3X(2,1.5)+3.5X(2,2)+4X(2,2.5)+4.5X(2,3)+5X(2,3.5)+5.5X(2,4)- (1.5X(5,1.5)+2X(5,2)+2.5X(5,2.5)+3X(5,3)+3.5X(5,3.5)+4X(5,4)+ 4.5X(5,4.5)+5X(5,5)+5.5X(5,5.5)) \leq 0$$

$$2 \leq 3 \ 1.5X(2,0)+2X(2,0.5)+2.5X(2,1)+3X(2,1.5)+3.5X(2,2)+4X(2,2.5)+4.5X(2,3)+5X(2,3.5)+ 5.5X(2,4)-(1.5X(3,1.5)+2X(3,2)+2.5X(3,2.5)+3X(3,3)+3.5X(3,3.5)+4X(3,4)+4.5X(3,4.5)+ 5X(3,5)+5.5X(3,5.5)+6X(3,6)+6.5X(3,6.5)) \leq 0$$

$$1 \leq 4 \ 1.5X(1,0)+2X(1,0.5)+2.5X(1,1)+3X(1,1.5)+3.5X(1,2)+4X(1,2.5)+4.5X(1,3)+5X(1,3.5)+ 5.5X(1,4)+6X(1,4.5)+6.5X(1,5)+7X(1,5.5)-(2X(4,2)+2.5X(4,2.5)+3X(4,3)+$$

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$$3.5X(4,3.5)+ 4X(4,4)+4.5X(4,4.5)+5X(4,5)+5.5X(4,5.5)+6X(4,6)+6.5X(4,6.5)+7X(4,7))\leq 0$$

$$3\leq 4 \quad 2X(3,1.5)+2.5X(3,2)+3X(3,2.5)+3.5X(3,3)+4X(3,3.5)+4.5X(3,4)+5X(3,4.5)+5.5X(3,5)+6X(3,5.5)+6.5X(3,6)+7X(3,6.5)- (2X(4,2)+2.5X(4,2.5)+3X(4,3)+3.5X(4,3.5)+4X(4,4)+4.5X(4,4.5)+5X(4,5)+5.5X(4,5.5)+6X(4,6)+6.5X(4,6.5)+7X(4,7))\leq 0$$

$$5\leq 6 \quad 2.5X(5,1.5)+3X(5,2)+3.5X(5,2.5)+4X(5,3)+4.5X(5,3.5)+5X(5,4)+5.5X(5,4.5)+6X(5,5)+6.5X(5,5.5)-(2.5X(6,2.5)+3X(6,3)+3.5X(6,3.5)+4X(6,4)+4.5X(6,4.5)+5X(6,5)+5.5X(6,5.5)+6X(6,6)+6.5X(6,6.5)) \leq 0$$

(3) For the last activities

$$3X(4,2)+3.5X(4,2.5)+4X(4,3)+4.5X(4,3.5)+5X(4,4)+5.5X(4,4.5)+6X(4,5)+6.5X(4,5.5)+7X(4,6)+7.5X(4,6.5)+8X(4,7)\leq T$$

$$4X(6,2.5)+4.5X(6,3)+5X(6,3.5)+5.5X(6,4)+6X(6,4.5)+6.5X(6,5)+7X(6,5.5)+7.5X(6,6)+8X(6,6.5)\leq T$$

$$1.5X(7,0)+2X(7,0.5)+2.5X(7,1)+3X(7,1.5)+3.5X(7,2)+4X(7,2.5)+4.5X(7,3)+5X(7,3.5)+5.5X(7,4)+6X(7,4.5)+6.5X(7,5)+7X(7,5.5)+7.5X(7,6)+8X(7,6.5)\leq T$$

(4) Resource constraints

Day

$$0 \quad 2X(1,0)+3X(2,0)+1X(7,0)\leq K(1)$$

$$0.5 \quad 2X(1,0)+2X(1,0.5)+3X(2,0)+3X(2,0.5)+1X(7,0)+1X(7,0.5)\leq K(1)$$

$$1 \quad 2X(1,0)+2X(1,0.5)+2X(1,1)+3X(2,0)+3X(2,0.5)+3X(2,1)+1X(7,0)+1X(7,0.5)+1X(7,1) \leq K(1)$$

$$1.5 \quad 2X(1,0.5)+2X(1,1)+2X(1,1.5)+3X(2,0.5)+3X(2,1)+3X(2,1.5)+1X(7,0.5)+1X(7,1)+1X(7,1.5)+2X(3,1.5)+4X(5,1.5)\leq K(1)$$

$$2 \quad 2X(1,1)+2X(1,1.5)+2X(1,2)+3X(2,1)+3X(2,1.5)+3X(2,2)+2X(3,2)+3X(4,2)+4X(5,1.5)+4X(5,2)+1X(7,1)+1X(7,1.5)+1X(7,2)\leq K(1)$$

$$2.5 \quad 2X(1,1.5)+2X(1,2)+2X(1,2.5)+3X(2,1.5)+3X(2,2)+3X(2,2.5)+2X(3,2.5)+3X(4,2)+3X(4,2.5)+4X(5,2)+4X(5,2.5)+2X(6,2.5)+1X(7,1.5)+1X(7,2)+1X(7,2.5)\leq K(1)$$

$$3 \quad 2X(1,2)+2X(1,2.5)+2X(1,3)+3X(2,2)+3X(2,2.5)+3X(2,3)+2X(3,3)+3X(4,2.5)+3X(4,3)+4X(5,2.5)+4X(5,3)+2X(6,2.5)+2X(6,3)+1X(7,2)+1X(7,2.5)+1X(7,3)\leq K(1)$$

$$3.5 \quad 2X(1,2.5)+2X(1,3)+2X(1,3.5)+3X(2,2.5)+3X(2,3)+3X(2,3.5)+2X(3,3.5)+3X(4,3)+3X(4,3.5)+4X(5,3)+4X(5,3.5)+2X(6,2.5)+2X(6,3)+2X(6,3.5)+1X(7,2.5)+1X(7,3)+1X(7,3.5)\leq K(1)$$

$$4 \quad 2X(1,3)+2X(1,3.5)+2X(1,4)+3X(2,3)+3X(2,3.5)+3X(2,4)+2X(3,4)+3X(4,3.5)+3X(4,4)+4X(5,3.5)+4X(5,4)+2X(6,3)+2X(6,3.5)+2X(6,4)+1X(7,3)+1X(7,3.5)+1X(7,4)\leq K(1)$$

$$4.5 \quad 2X(1,3.5)+2X(1,4)+2X(1,4.5)+3X(2,3.5)+3X(2,4)+2X(3,4.5)+3X(4,4)+3X(4,4.5)+4X(5,4)+4X(5,4.5)+2X(6,3.5)+2X(6,4)+2X(6,4.5)+1X(7,3.5)+1X(7,4)+1X(7,4.5)\leq K(1)$$

$$5 \quad 2X(1,4)+2X(1,4.5)+2X(1,5)+3X(2,4)+2X(3,5)+3X(4,4)+3X(4,4.5)+3X(4,5)+4X(5,4.5)+4X(5,5)+2X(6,4)+2X(6,4.5)+2X(6,5)+1X(7,4)+1X(7,4.5)+1X(7,5)\leq K(1)$$

$$5.5 \quad 2X(1,4.5)+2X(1,5)+2X(1,5.5)+2X(3,5.5)+3X(4,5)+3X(4,5.5)+4X(5,5)+4X(5,5.5)+2X(6,4.5)+2X(6,5)+2X(6,5.5)+1X(7,4.5)+1X(7,5)+1X(7,5.5)\leq K(1)$$

$$6 \quad 2X(1,5)+2X(1,5.5)+2X(3,6)+3X(4,5.5)+3X(4,6)+4X(5,5.5)+2X(6,5.5)+2X(6,6)+1X(7,5)+1X(7,5.5)+1X(7,6)\leq K(1)$$

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- 6.5 $2X(1,5.5)+2X(3,6.5)+3X(4,6)+3X(4,6.5)+2X(6,5.5)+2X(6,6)+2X(6,6.5)+1X(7,5.5)+1X(7,6)+1X(7,6.5)\leq K(1)$
- 7 $3X(4,6.5)+3X(4,7)+2X(6,6)+2X(6,6.5)+1X(7,6)+1X(7,6.5)\leq K(1)$
- 7.5 $3X(4,7)+2X(6,6.5)+1X(7,6.5)\leq K(1)$

IV. RESULTS

The Above 0-1 Integer programming formulation was solve by using Mat-lab programming. It yields the following results and the optimal project schedule is shown in fig.7.

Solution: T=5.5 K (1) =4

X(1, 3) =1; X(2,0)=1; X(3,4)=1; X(4,4.5)=1; X(5,1.5)=1; X(6,2.5)=1; X(7,0)=1

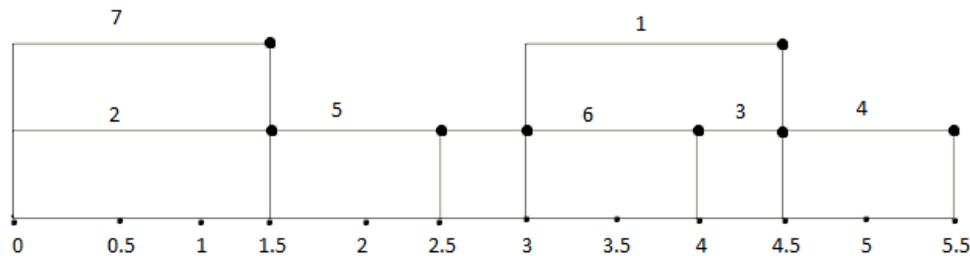


Fig.7 Optimal project schedule

Goal Programming Formulation:

Maximize $P= W_1d_1+W_2d_2 = 0.5d_1+2000d_2$

Subjected to:

$$60800-[6000T+400K(1)T+2500(T-8)]-d_1=0$$

$$8-T-d_2=0 \text{ when } d_1 \text{ and } d_2 \geq 0.$$

Substituting the optimal values of T and K (1) in eq. (6) and (7) yields the following results.

d₁= 25250; d₂= 2.5;

Substituting the values of d₁ and d₂ in eq. (5) gives the value of P.

P=17625

V. CONCLUSIONS

The goal programming model provides the best possible optimal solution. The TOC project scheduling methodology is used in this to obtain the duration which minimizes Z. The TOC project scheduling technique determines the lower bound of the project length by using the 0-1 integer programming and goal programming formulation. The lower bound is then used to establish the project buffer and feeding buffer. It is observed that the application of this approach is very much suitable for the small networks. The application of this method for large networks might not be feasible with 0-1 integer programming algorithm because the task of writing constraints is in itself a formidable one.

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BIOGRAPHY

Smt. CH. Lakshmi thulasi is having 7 years of teaching experience in both U.G and P.G levels. She pursued her M. Tech degree in the field of Industrial Engineering. Currently pursuing her Ph. D as a full time research scholar.



Dr. A. Ramakrishna Rao is a Retd. Professor in the department of mechanical Engineering, S.V.U. College of engineering, S.V. University Tirupati. He has more than 33years of experience in teaching for both U. G and P.G level and in Research also. He has published more than 55 papers in referred national and international journals. He has presented more than 45 research articles in national and international conferences. His current area of research includes Supply Chain Management, Operations Management, Lean Manufacturing, Graph Theory and TOC.

