

N-Dimensional Bianchi Type-I Cosmological Models in $f(R,T)$ Theory of Gravity

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ABSTRACT : In this paper, we have obtained two n-dimensional exact solutions of Bianchi type-I space-time in $f(R,T)$ theory of gravity using assumption of constant deceleration parameter and variation law of Hubble's parameter which correspond to two different cosmological models with suitable choice of the function $f(R,T)$. The first solution yields a singular model for $r \neq 0$ while the second gives a non-singular model for $r = 0$. The physical behavior for both models have been discussed using some physical quantities..

Keywords : $f(R,T)$ theory of gravity, Bianchi type-I space-time, N-dimensional Cosmological Models.

I. INTRODUCTION

In last few years, interest among researcher have been renewed to study the nature of universe. Einstein's general relativity is a successful theory to explain most of the known gravitational phenomena but it fails to explain the accelerating expansion of the universe. It is now proved from observational and theoretical fact that the universe is not only expanding but also accelerating. In order to explain the accelerated expansion, number of cosmological models have been proposed by different authors. Justification of the current expansion of the universe comes from modified or alternative theories of gravity. The $f(R)$ gravity, $f(T)$ gravity and $f(R,T)$ theory of gravity are such examples of modified gravity theories. $f(T)$ theory of gravity where T is the scalar torsion has been proposed to explain current accelerated expansion without involving dark energy. In $f(T)$ theory of gravity, Weitzenbock connection is used instead of Levi-Civita connection. Yang R. J.[1] discussed what constraint of the coupling term may be put in $f(T)$ theories from observations of the solar systems. Ratbay M. [2] has shown that the acceleration of the universe can be understood by $f(T)$ gravity models. M. Sharif et al [3] considered spatially homogeneous and anisotropic Bianchi type-I universe in $f(T)$ gravity theory. Bamba K. et al [4] studied the cosmological evolution of the equation of state for dark energy with the combination of exponential, logarithmic and $f(T)$ theories. Wei H. et al [5] tried to constrain $f(T)$ theories with the fine structure constant.

Another modified theory is the $f(R)$ theory of gravity. $f(R)$ theory of gravity is the modification of general theory of relativity proposed by Einstein. This theory plays an important role in describing the evolution of the universe. Nojiri and Odintsov [6,7] proved that the $f(R)$ theory of gravity provides very natural unification of the early time inflation and late time acceleration. Carroll et al [8] explained the presence of late time cosmic acceleration of the universe in $f(R)$ gravity. Bertolami et al [9] have proposed a generalization of $f(R)$ modified theory of gravity, by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m . Multamaki and Vilja [10,11] investigated vacuum and perfect fluid solutions of spherically symmetric space time in $f(R)$ gravity. Capozziello et al [12] used Noether symmetries to study spherically symmetric solutions in $f(R)$ theory of gravity.

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Adhav K. S. [13] discussed the Kantowski- Sachs string cosmological model in $f(R)$ theory of gravity. Ladke L. S. [14] studied the Bianchi type –I (Kasner form) cosmological model in $f(R)$ theory of gravity

II. RELATED WORK

Harko et al [15] proposed a new generalized theory know as $f(R,T)$ theory of gravity. Gravitational Lagrangian involves the arbitrary function of the scalar Curvature R and the trace of the energy momentum tensor T . Farasat Shamir M et al [16] obtained the exact solutions of Bianchi types-I & V models in $f(R,T)$ by using the assumption of constant deceleration parameter and variation of law of Hubble parameter. Adhav K.S. [17] studied the exact solution of $f(R,T)$ field equations for locally rotationally symmetric Bianchi type-I space time. Houndjo [18] reconstructed $f(R,T)$ gravity by taking $f(R,T) = f_1(R,T) + f_2(R,T)$. Reddy D. R. K. [19] discussed the LRS Bianchi type-II universe in $f(R,T)$ theory.

The study of higher dimensional physics is important because of several results obtained in the development of super string theory. Kaluza and Klein [20, 21] have done remarkable work by introducing an idea of higher dimension space-time. Wesson [22 ,23] and D.R. K. Reddy [24] have studied several aspects of five dimensional space-time in variable mass theory and bi-metric theory of relativity respectively. Adhav K. S.et al [25] have studied the multidimensional cosmological models in general relativity and in other alternative theories of gravitations. Recently G. C. Samanta and S. N. Dhal [26] discussed higher dimensional cosmological model filled with perfect fluid in $f(R,T)$ theory of gravity. Ladke L. S. et al [27] studied five dimensional exact solutions of Bianchi type- I space-time in $f(R,T)$ theory of gravity. Motivating with the above research work we have obtained N-dimensional exact solutions of Bianchi type – I space-time in $f(R,T)$ theory of gravity using constant deceleration parameter and variation law of Hubble parameter [28] which corresponds to two different cosmological models for $r \neq 0$ and $r = 0$. The paper is organized as follows. Section-II & III is related with related work & n-dimensional field equations in $f(R,T)$ theory of gravity respectively. In section -IV, exact solutions of Bianchi type-I space –time in V_n is obtained. In section V, a discussion on some important physical quantities. Section VI & VII deal with n-dimensional models of the universe when $r \neq 0$ and $r = 0$. In last section conclusion have been drawn.

III. N-DIMENSIONAL FIELD EQUATIONS IN $f(R,T)$ THEORY OF GRAVITY

The n-dimensional field equations in $f(R,T)$ theory of gravity are given by

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij} \square) f(R,T) = kT_{ij} - f_T(R,T)(T_{ij} + \theta_{ij}) \quad (i, j = 1,2,\dots,n), \quad (1)$$

where $f_R(R,T) \equiv \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f(R,T)}{\partial T}$, $T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\partial(\sqrt{-g} L_m)}{\partial g^{ij}}$, $\theta_{ij} = -pg_{ij} - 2T_{ij}$

$\square \equiv \nabla^i \nabla_i$, ∇_i is the covariant derivative.

The energy momentum tensor for perfect fluid yields

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad (2)$$

where ρ and p are energy density and pressure of the fluid respectively.

Contracting the above field equations (1), we have

$$f_R(R,T)R + (n-1)\square f_R(R,T) - \frac{n}{2}f(R,T) = kT - f_T(R,T)(T + \theta), \quad (3)$$

Also above field equations (1), can be written as

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$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij}\square)f_R(R,T) = kT_{ij} + f_T(R,T)(T_{ij} + pg_{ij}), \quad (4)$$

Harko et.al.[15] gives three class of models out of which we used $f(R,T) = R + 2f(T)$. For this models equation (4) can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = kT_{ij} + 2f'(T)T_{ij} + [f(T) + 2pf'(T)]g_{ij}, \quad (5)$$

where overhead prime denotes derivative w.r.to. T .

We also choose $f(T) = \lambda T$, where λ is constant. (6)

III. EXACT SOLUTIONS OF BIANCHI TYPE - I SPACE TIME IN V_n

-I space time in $f(R,T)$ in this section we find exact solutions of $n -$ dimensional Bianchi type, T theory of gravity. The line element of Bianchi type-I space-time in V_n is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 (dy^2) - C^2 \sum_{i=1}^{n-3} dx_i^2, \quad (7)$$

where A, B and C are functions of t only.

The corresponding Ricci scalar is

$$R = -2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + (n-3)\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + (n-3)\frac{\dot{B}\dot{C}}{BC} + (n-3)\frac{\dot{A}\dot{C}}{AC} + \frac{(n-3)(n-4)}{2} \frac{\dot{C}^2}{C^2} \right], \quad (8)$$

where dot denotes derivative with respect to t .

From equation (5), we obtained,

For nn -component

$$\frac{\dot{A}\dot{B}}{AB} + (n-3)\frac{\dot{B}\dot{C}}{BC} + (n-3)\frac{\dot{A}\dot{C}}{AC} + \frac{(n-3)(n-4)}{2} \frac{\dot{C}^2}{C^2} = (2n\pi + 3\lambda)\rho - (n-3)\lambda p, \quad (9)$$

For 11 -component

$$\frac{\ddot{B}}{B} + (n-3)\frac{\ddot{C}}{C} + (n-3)\frac{\dot{B}\dot{C}}{BC} + \frac{(n-3)(n-4)}{2} \frac{\dot{C}^2}{C^2} = \lambda\rho - [2n\pi + (n-1)\lambda]p, \quad (10)$$

For 22 -component

$$\frac{\ddot{A}}{A} + (n-3)\frac{\ddot{C}}{C} + (n-3)\frac{\dot{A}\dot{C}}{AC} + \frac{(n-3)(n-4)}{2} \frac{\dot{C}^2}{C^2} = \lambda\rho - [2n\pi + (n-1)\lambda]p, \quad (11)$$

For $33, 44, \dots, (n-1)(n-1)$ - components

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + (n-4)\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + (n-4)\frac{\dot{B}\dot{C}}{BC} + (n-4)\frac{\dot{C}\dot{A}}{CA} + \frac{(n-4)(n-5)}{2} \frac{\dot{C}^2}{C^2} = \lambda\rho - [2n\pi + (n-1)\lambda]p, \quad (12)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + (n-4)\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + (n-4)\frac{\dot{B}\dot{C}}{BC} + (n-4)\frac{\dot{C}\dot{A}}{CA} + \frac{(n-4)(n-5)}{2} \frac{\dot{C}^2}{C^2} = \lambda\rho - [2n\pi + (n-1)\lambda]p, \quad (13)$$

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$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + (n-4)\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + (n-4)\frac{\dot{B}\dot{C}}{BC} + (n-4)\frac{\dot{C}\dot{A}}{CA} + \frac{(n-4)(n-5)}{2}\frac{\dot{C}}{C^2} = \lambda\rho - [2n\pi + (n-1)\lambda]p, \tag{14}$$

The left hand side of equation (12), (13) and (14) are identical because of the metric function C is common along x_1, x_2, \dots, x_{n-3} directions in the metric (7).

The system of these four non-linear differential equations consist of five undefined functions i.e. A, B, C, p and ρ .

Hence to find deterministic solution one more condition is necessary, so we consider well known relation between Hubble parameter H and average scale factor a [28] given as

$$H = la^{-n}, \tag{15}$$

where $l > 0$ and $n \geq 0$

Subtracting equation (10) from equation (11), equation (11) from equation (12), equation (10) from equation (12), we have

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + (n-3)\frac{\ddot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \tag{16}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{A}}{A} + (n-4)\frac{\dot{C}}{C}\right)\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0, \tag{17}$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} + (n-4)\frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) = 0. \tag{18}$$

On solving above equations, we get

$$\frac{B}{A} = d_1 \exp\left[c_1 \int \frac{dt}{a^{(n-1)}}\right], \tag{19}$$

$$\frac{C}{B} = d_2 \exp\left[c_2 \int \frac{dt}{a^{(n-1)}}\right], \tag{20}$$

$$\frac{A}{C} = d_3 \exp\left[c_3 \int \frac{dt}{a^{(n-1)}}\right], \tag{21}$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are constants of integration which satisfy the relation

$$c_1 + c_2 + c_3 = 0, \quad d_1 d_2 d_3 = 1. \tag{22}$$

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Using equation (19), (20) and (21), the metric functions are

$$A = ap_1 \exp \left[q_1 \int \frac{dt}{a^{(n-1)}} \right], \tag{23}$$

$$B = ap_2 \exp \left[q_2 \int \frac{dt}{a^{(n-1)}} \right], \tag{24}$$

$$C = ap_3 \exp \left[q_3 \int \frac{dt}{a^{(n-1)}} \right], \tag{25}$$

$$\text{where } p_1 = (d_1^{-(n-2)} d_2^{-(n-3)})^{1/(n-1)}, p_2 = (d_1 d_2^{-(n-3)})^{1/(n-1)}, p_3 = (d_1 d_2^2)^{1/(n-1)}, \tag{26}$$

$$\text{and } q_1 = -\frac{(n-2)c_1 + (n-3)c_2}{(n-1)}, q_2 = \frac{c_1 - (n-3)c_2}{(n-1)}, q_3 = \frac{c_1 + 2c_2}{(n-1)}, \tag{27}$$

satisfying the relations

$$p_1 p_2 p_3^{(n-3)} = 1, \quad q_1 + q_2 + (n-3)q_3 = 0. \tag{28}$$

V . SOME IMPORTANT PHYSICAL QUANTITIES

Here we define some important physical quantities.

The average scale factor and the volume scale factor are

$$a = (ABC^{n-3})^{\frac{1}{(n-1)}}, \quad V = a^{(n-1)} = ABC^{(n-3)}. \tag{29}$$

The generalized mean Hubble parameter H is defined by

$$H = (\ln a)_t = \frac{\dot{a}}{a} = \frac{1}{n-1} \sum_{i=1}^{n-1} H_i, \tag{30}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = H_4 = \dots = H_{n-1} = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of $x, y, x_1, x_2, \dots, x_{n-3}$ axes respectively.

The mean anisotropy parameter \bar{A} is given by

$$\bar{A} = \frac{1}{n-1} \sum_{i=1}^{n-1} \left(\frac{\Delta H_i}{H} \right)^2, \tag{31}$$

where $\Delta H_i = H_i - H$

The expansion scalar θ and shear scalar σ^2 are defined as under

$$\theta = u_{;i}^i = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{(n-3)\dot{C}}{C}, \tag{32}$$

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} \tag{33}$$

$$\text{where } \sigma_{ij} = \frac{1}{2} [\nabla_j u_i + \nabla_i u_j] - \frac{1}{(n-1)} \theta g_{ij}, \tag{34}$$

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From equation (15) and (30), we have

$$\dot{a} = la^{1-r}, \tag{35}$$

After integrating equation (35), we get

$$a = (rlt + k_1)^{1/r}, \quad r \neq 0 \tag{36}$$

And $a = k_2 \exp(lt), \quad r = 0, \tag{37}$

Where k_1 and k_2 are constants of integration.

Thus we have two values of the average scale factors which correspond to two different models of the universe.

VI. N- DIMENSIONAL MODEL OF THE UNIVERSE WHEN $r \neq 0$

In this section we study the n-dimensional model of the universe for $r \neq 0$. For this singular model average scale factor a given as $a = (rlt + k_1)^{1/r}$

The metric coefficients A, B and C turn out to be

$$A = p_1((rlt + k_1)^{1/r} \exp\left[\frac{q_1(rlt + k_1)^{(r-n+1)/r}}{l(r-n+1)}\right]), \quad r \neq n-1 \tag{38}$$

$$B = p_2(rlt + k_1)^{1/r} \exp\left[\frac{q_2(rlt + k_1)^{(r-n+1)/r}}{l(r-n+1)}\right], \quad r \neq n-1 \tag{39}$$

$$C = p_3(rlt + k_1)^{1/r} \exp\left[\frac{q_3(rlt + k_1)^{(r-n+1)/r}}{l(r-n+1)}\right], \quad r \neq n-1 \tag{40}$$

The mean generalized Hubble parameter and the volume scale factor become

$$H = \frac{l}{rlt + k_1}, \quad V = (rlt + k_1)^{\frac{n-1}{r}}. \tag{41}$$

The mean anisotropy parameter \bar{A} turns out to be

$$\bar{A} = \frac{q_1^2 + q_2^2 + (n-3)q_3^2}{(n-1)l^2(rlt + k_1)^{[2(n-1)-2r]/r}}. \tag{42}$$

The deceleration parameter q in cosmology is the measure of the cosmic accelerated expansion of the universe and is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = r-1, \tag{43}$$

which is a constant.

A positive sign of q , i.e. $r > 1$ corresponds to the standard decelerating model whereas the negative sign of q , i.e. $0 < r < 1$ indicates inflation. The expansion of the universe at a constant rate corresponds to $q = 0$, i.e. $r = 1$.

The expansion θ and shear scalar σ^2 are given by

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$$\theta = \frac{(n-1)l}{rkt + k_1} \text{ and } \sigma^2 = \frac{q_1^2 + q_2^2 + (n-3)q_3^2}{2(rkt + k_1)^{2(n-1)/n}}, \tag{44}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{n(n-1)(\lambda + 2\pi)(\lambda + n\pi)} \left[[2(n-2)\lambda + n(n-1)\pi] \left\{ \frac{(n-1)(n-2)l^2}{2(rkt + k_1)^2} + \frac{q_1q_2 + (n-3)q_2q_3 + (n-3)q_3q_1 + \frac{(n-3)(n-4)}{2}q_3^2}{2(rkt + k_1)^{\frac{2(n-1)}{r}}} \right\} - \frac{(n-2)(n-3)}{2} \lambda \left\{ \frac{(n-1)l^2(1-r)}{(rkt + k_1)^2} + \frac{q_1^2 + q_2^2 + (n-3)q_3^2}{(rkt + k_1)^{2(n-1)/r}} \right\} \right], \tag{45}$$

The pressure of the universe become

$$p = \frac{-1}{n(n-1)(\lambda + 2\pi)(\lambda + n\pi)} \left[\left(\frac{(n-3)(n-4)}{2} \lambda + n(n-3)\pi \right) \left\{ \frac{(n-1)(n-2)l^2}{2(rkt + k_1)^2} + \frac{q_1q_2 + (n-3)q_2q_3 + (n-3)q_3q_1 + \frac{(n-3)(n-4)}{2}q_3^2}{2(rkt + k_1)^{2(n-1)/r}} \right\} - \frac{3}{2} (n-2)\lambda + n(n-2)\pi \right] \left\{ \frac{(n-1)l^2(1-r)}{(rkt + k_1)^2} + \frac{q_1^2 + q_2^2 + (n-3)q_3^2}{(rkt + k_1)^{2(n-1)/r}} \right\} \tag{46}$$

VII. N- DIMENSIONAL MODEL OF THE UNIVERSE WHEN $r = 0$

In this section we study the n-dimensional model of the universe for $r = 0$.

For this non-singular model average scale factor a given as $a = k_2 \exp(lt)$

Here the metric coefficients take the form

$$A = p_1 k_2 \exp(lt) \exp \left[-\frac{q_1 \exp(-(n-1)lt)}{(n-1)lk_2^{(n-1)}} \right], \tag{47}$$

$$B = p_2 k_2 \exp(lt) \exp \left[-\frac{q_2 \exp(-(n-1)lt)}{(n-1)lk_2^{(n-1)}} \right], \tag{48}$$

$$C = p_3 k_2 \exp(lt) \exp \left[-\frac{q_3 \exp(-(n-1)lt)}{(n-1)lk_2^{(n-1)}} \right]. \tag{49}$$

The mean generalized Hubble parameter becomes

$$H = l \tag{50}$$

while the volume scale factor turns out to be

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$$V = k_2^{(n-1)} \exp[(n-1)lt]. \tag{51}$$

The mean anisotropy parameter \bar{A} becomes

$$\bar{A} = \left[\frac{q_1^2 + q_2^2 + (n-3)q_3^2}{(n-1)l^2 k_2^{2(n-1)}} \right] \exp[-2(n-1)lt], \tag{52}$$

and θ and σ^2 are given by

$$\theta = (n-1)l, \sigma^2 = \left[\frac{q_1^2 + q_2^2 + (n-3)q_3^2}{2k_2^{2(n-1)}} \right] \exp[-2(n-1)lt], \tag{53}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{n(n-1)(\lambda + 2\pi)(\lambda + n\pi)} \left[[2(n-2)\lambda + n(n-1)\pi] \left\{ \frac{(n-1)(n-2)}{2} l^2 + \frac{q_1 q_2 + (n-3)q_2 q_3 + (n-3)q_3 q_1 + \frac{(n-3)(n-4)}{2} q_3^2}{k_2^{2(n-1)} \exp[2(n-1)lt]} \right\} - \frac{(n-2)(n-3)}{2} \lambda \left\{ (n-1)l^2 + \frac{q_1^2 + q_2^2 + (n-3)q_3^2}{k_2^{2(n-1)} \exp[2(n-1)lt]} \right\} \right], \tag{54}$$

The pressure of the universe becomes

$$p = \frac{-1}{n(n-1)(\lambda + 2\pi)(\lambda + n\pi)} \left[\left(\frac{(n-3)(n-4)}{2} \lambda + n(n-3)\pi \right) \left\{ \frac{(n-1)(n-2)}{2} l^2 + \frac{q_1 q_2 + (n-3)q_2 q_3 + (n-1)q_3 q_1 + \frac{(n-3)(n-4)}{2} q_3^2}{k_2^{2(n-1)} \exp[2(n-1)lt]} \right\} - \left[\frac{3}{2} (n-2)\lambda + n(n-2)\pi \right] \left\{ (n-1)l^2 + \frac{q_1^2 + q_2^2 + 3q_3^2}{k_2^{2(n-1)} \exp[2(n-1)lt]} \right\} \right] \tag{55}$$

VIII. CONCLUSION

In this paper, we have obtained two n-dimensional exact solutions of Bianchi type - I space time in f(R,T) theory of gravity using assumption of constant value of deceleration parameter and variation law of Hubble parameter. For $r \neq 0$ the solution provides a singular model with power law expansion and for $r = 0$, the solution gives a non-singular model with exponential expansion. Quantities which are of cosmological importance for both the models are also evaluated.

N- dimensional singular model of the universe for $r \neq 0$ has a singularity at $t = \frac{-k_1}{rl}$. This singularity is point type because metric coefficient vanish at this point. Average scale factor for this model is $a = (r lt + k_1)^{1/r}$. The mean generalized Hubble parameter, mean anisotropy parameter, expansion scalar θ , shear scalar σ^2 are all infinite at this point of singularity whereas volume scale factor vanish at this point. This observations suggest that universe starts its expansion with zero volume and continue to expand.

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Non-singular model of the universe with exponential expansion corresponds to $r = 0$. For this model average scale factor is $a = k_2 \exp(lt)$. Because of exponential behavior of the model, it has no singularity. The volume of universe and metric coefficients increase exponentially with the cosmic time t . Mean Hubble parameter H and expansion scalar θ are constant throughout the evaluation. Shear scalar and anisotropy parameter are finite for finite value of t . This indicates that universe expansion may take place with zero volume from infinite past. All the results obtained here are similar to the results obtained by Farasat Shamir et al [16] and results obtained by us in five dimensional space time [27]. This case is the extension of above two cases [16], [27].

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