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Nijenhius Tensoron Hyperbolic Hsu-Structure Manifold

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Abstract: Scope of this paper is to express the Nijenhius Tensor in various forms in Hyperbolic Hsu-Structure manifold. Firstly Hyperbolic Hsu-Structure manifold has been studied and discussed by Dr. R.S. Mishra [3], [4] and some of the great geometricians have also done work in Nijenhius Tensor in different differentiable manifold structures [5], [6], [7], [9]. In this paper, we have taken even dimensional differentiable manifold $V_n(n = 2m)$ of differentiability class C^∞**,** where we have definedthe Nijenhius Tensor in Hyperbolic Hsu-Structure manifold and the decomposition of the Nijenhius Tensor in Hyperbolic Hsu-Structure has been done. And some of its properties have also been discussed. Similarly the decomposition of the associate Nijenhius Tensor and its properties in Hyperbolic Hsu-structure manifold has been discussed.

Keywords:Hyperbolic- Hsu structure manifold, Nijenhius Tensor, HGF-structure.

I. INTRODUCTION

If on an even dimensional manifold V_n, n = 2m of differentiability class C^∞, there exists a vector valued real linear function ϕ , satisfying

$$\phi^2 = -a^r I_n , \qquad (1.1)a$$

$$\overline{\overline{X}} = -a^r X \text{ , for arbitrary vector field X.} \qquad (1.1)b$$

Where $\overline{X} = \phi X$, $0 \le r \le n$ and 'r' is an integer and 'a' is a real or imaginary number.

Then { ϕ } is said to give to V_n a Hyperbolic- Hsu structure defined by the equations (1.1) and the manifold V_n is called a Hyperbolic- Hsu structure manifold.

The equation (1.1)a gives different structure for different values of 'a' and 'r'.

If $r = 0$, it is an almost complex structure.

If $a = 0$, it is an almost tangent structure.

If $r = \pm 1$ and $a = +1$, it is an almost complex structure.

If $r = \pm 1$ and $a = -1$, it is an almost product structure.

If r =2 then it is a HGF-structure which includes

π-structure for $a \ne 0$,

an almost product structure for $a = \pm i$,

an almost complex structure for $a = \pm 1$,

an almost tangent structure for $a = 0$.

Let the Hyperbolic- Hsu structure be endowed with a metric tensor g, such that

$$g(\overline{X}, \overline{Y}) - a^r g(X, Y) = 0 \qquad (1.2)$$

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Then $\{\phi, g\}$ is said to give to V_n - metric Hyperbolic- Hsu structure and V_n is called a metric Hyperbolic-Hsu structure manifold.

The curvature tensor K, a vector -valued tri-linear function w.r.t. the connexion D is given by

$$K(X,Y,Z) = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z. \tag{1.3}$$

where

$$[X,Y] = D_X Y - D_Y X.$$

II. The Nijenhius Tensor

Definition (2.1): The Nijenhius tensor with respect to ϕ is a vector valued bilinear function N given by

$$N(X,Y) = [\phi X, \phi Y] + \phi^2[X,Y] - \phi[\phi X, Y] - \phi[X, \phi Y]. \tag{2.1a}$$

Or equivalent

$$N(X,Y) = D_{\phi X}\phi Y - D_{\phi Y}\phi X - a^r[X,Y] - \phi D_{\phi X} Y + \phi D_Y \phi X - \phi D_X \phi Y + \phi D_{\phi Y} X,$$

$$N(X,Y) = D_{\phi X}\phi Y - D_{\phi Y}\phi X - a^r D_X Y + a^r D_Y X - \phi D_{\phi X} Y + \phi D_Y \phi X - \phi D_X \phi Y + \phi D_{\phi Y} X. \tag{2.1b}$$

where D is Riemannian connexion.

Theorem (2.1): In a hyperbolic Hsu-structure manifold, we have

$$N(X,Y) = -N(Y,X)$$
$$= D_{\phi X}\phi Y - D_{\phi Y}\phi X - a^r D_X Y + a^r D_Y X - \phi D_{\phi X} Y + \phi D_Y \phi X - \phi D_X \phi Y + \phi D_{\phi Y} X, \tag{2.2a}$$

$$N(\phi X, Y) = D_{\phi^2 X}\phi Y - D_{\phi Y}\phi^2 X - a^r D_{\phi X} Y + a^r D_Y \phi X - \phi D_{\phi^2 X} Y + \phi D_Y \phi^2 X - \phi D_X \phi Y + \phi D_{\phi Y}\phi X,$$

$$N(\phi X, Y) = -a^r D_X \phi Y + a^r D_{\phi Y} X - a^r D_{\phi X} Y + a^r D_Y \phi X + a^r \phi D_X Y - a^r \phi D_Y X - \phi D_{\phi X}\phi Y + \phi D_{\phi Y}\phi X, \tag{A}$$

$$N(X, \phi Y) = D_{\phi X}\phi^2 Y - D_{\phi^2 Y}\phi X - a^r D_X \phi Y + a^r D_{\phi Y} X - \phi D_{\phi X}\phi Y + \phi D_{\phi Y}\phi X - \phi D_X \phi^2 Y + \phi D_{\phi^2 Y} X,$$

$$N(X, \phi Y) = -a^r D_{\phi X} Y + a^r D_Y \phi X - a^r D_X \phi Y + a^r D_{\phi Y} X - \phi D_{\phi X}\phi Y + \phi D_{\phi Y}\phi X + a^r \phi D_X Y - a^r \phi D_Y X, \tag{B}$$

From A and B

$$N(\phi X, Y) = N(X, \phi Y)$$
$$= -a^r D_X \phi Y + a^r D_{\phi Y} X - a^r D_{\phi X} Y - a^r D_Y \phi X + a^r \phi D_X Y - a^r \phi D_Y X - \phi D_{\phi X}\phi Y + \phi D_{\phi Y}\phi X. \tag{2.2b}$$

$$\phi N(X,Y) = -\phi N(Y,X)$$
$$= \phi D_{\phi X}\phi Y - \phi D_{\phi Y}\phi X - a^r \phi D_X Y + a^r \phi D_Y X + a^r D_{\phi X} Y - a^r D_Y \phi X + a^r D_X \phi Y + a^r D_{\phi Y} X. \tag{2.2c}$$

$$\phi N(\phi X, Y) = \phi N(X, \phi Y)$$
$$= -a^r \phi D_X \phi Y + a^r \phi D_{\phi Y} X - a^r \phi D_{\phi X} Y - a^r \phi D_Y \phi X - a^{2r} D_X Y + a^{2r} D_Y X + a^r D_{\phi X}\phi Y - a^r D_{\phi Y}\phi X, \tag{2.2d}$$

$$N(\phi X, \phi Y) = -a^r \mathrm{N}(X,Y)$$
$$= a^{2r} D_X Y - a^{2r} D_Y X - a^r D_{\phi X}\phi Y - a^r D_{\phi Y}\phi X + a^r \phi D_X \phi Y - a^r \phi D_{\phi Y} X + a^r \phi D_{\phi X} Y - a^r \phi D_Y \phi X, \tag{2.2e}$$

i.e. N is pure in X and Y.

$$\phi N(\phi X, \phi Y) = -a^r \phi \mathrm{N}(X,Y)$$
$$= a^{2r}\phi D_X Y - a^{2r}\phi D_Y X - a^r \phi D_{\phi X}\phi Y - a^r \phi D_{\phi Y}\phi X - a^{2r} D_X \phi Y + a^{2r} D_{\phi Y} X - a^{2r} D_{\phi X} Y + a^{2r} D_Y \phi X. \tag{2.2f}$$

Consequently

$$N(\phi X, \phi Y) = -a^r N(X,Y) = -\phi N(\phi X, Y) = -\phi N(X, \phi Y). \tag{2.3a}$$

$$\phi N(\phi X, \phi Y) = -a^r \phi N(X,Y) = a^r N(\phi X, Y) = a^r N(X, \phi Y). \tag{2.3b}$$

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Vol. 2, Issue 11, November 2013

Proof: Interchanging X and Y in equation (2.1b), we get (2.2a), which shows that N is Skew-Symmetric in X and Y. Barring equation (2.1b) and applying structure we get equation (2.2c). Barring X and Y separately in equation (2.1b) and using structure in the two equations and then comparing the resulting equation we get (2.2b). Barring equation (2.2b) throughout and using structure in the resulting equation, we get the equation (2.2d). Barring X and Y in equation (2.1b), using structure and comparing the resulting equation with the equation obtained by multiplying equation (2.1b) by a^r we get the equation (2.2e), which shows that N is pure in X and Y. Barring equation (2.2e)and using structure, we get the equation (2.2f).
The equation (1.3a) is obtained from the equations (2.2d) and (2.2e). The equation (2.2c) and (2.2f) yield the equation (2.3b).

Theorem (1.2): Let us put

$$P(X,Y) = D_{\phi X}\phi Y - a^r D_X Y + \phi D_Y \phi X + \phi D_{\phi Y} X. \tag{2.4}$$

Then

$$P(Y,X) = D_{\phi Y}\phi X - a^r D_Y X + \phi D_X \phi Y + \phi D_{\phi X} Y, \tag{2.5a}$$

$$P(\phi X,\phi Y) = -a^r P(X,Y) = a^{2r} D_X Y - a^r D_{\phi X}\phi Y - a^r \phi D_{\phi Y} X - a^r \phi D_Y \phi X, \tag{2.5b}$$

i.e. P is pure in X and Y.

$$P(\phi X,Y) = P(X,\phi Y) = -a^r D_X \phi Y - a^r D_{\phi X} Y - a^r \phi D_Y X + \phi D_{\phi Y}\phi X, \tag{2.5c}$$

$$\phi P(\phi X,Y) = \phi P(X,\phi Y) = -a^r \phi D_X \phi Y - a^r \phi D_{\phi X} Y + a^{2r} D_Y X - a^r D_{\phi Y}\phi X, \tag{2.5d}$$

$$\phi P(\phi X,\phi Y) = -a^r \phi P(X,Y) = a^{2r}\phi D_X Y - a^r \phi D_{\phi X}\phi Y + a^{2r} D_{\phi Y} X + a^{2r} D_Y \phi X, \tag{2.5e}$$

$$P(\phi Y,\phi X) = -a^r P(Y,X) = a^{2r} D_Y X - a^r D_{\phi Y}\phi X - a^r \phi D_{\phi X} Y - a^r \phi D_X \phi Y, \tag{2.5f}$$

$$P(\phi Y,X) = P(Y,\phi X) = -a^r D_Y \phi X - a^r D_{\phi Y} X - a^r \phi D_X Y + \phi D_{\phi X}\phi Y, \tag{2.5g}$$

$$\phi P(\phi Y,X) = \phi P(Y,\phi X) = -a^r \phi D_Y \phi X - a^r \phi D_{\phi Y} X + a^{2r} D_X Y - a^r D_{\phi X}\phi Y, \tag{2.5h}$$

$$\phi P(\phi Y,\phi X) = -a^r \phi P(Y,X) = a^{2r}\phi D_Y X - a^r \phi D_{\phi Y}\phi X + a^{2r} D_{\phi X} Y + a^{2r} D_X \phi Y. \tag{2.5i}$$

Consequently

$$P(X,Y) - P(Y,X) = N(X,Y), \tag{2.6a}$$

$$a^r P(X,Y) + \phi P(X,\phi Y) = \phi P(\phi Y,X) - P(\phi Y,\phi X) = \phi N(Y,\phi X), \tag{2.6b}$$

$$\phi P(\phi X,Y) - P(\phi X,\phi Y) = a^r P(Y,X) + \phi P(Y,\phi X) = \phi N(\phi X,Y), \tag{2.6c}$$

$$\phi P(\phi X,\phi Y) + a^r P(X,\phi Y) = \phi P(\phi Y,\phi X) + a^r P(Y,\phi X) = \phi N(\phi X,\phi Y), \tag{2.6d}$$

$$P(\phi X,Y) - \phi P(X,Y) = P(\phi Y,X) - \phi P(Y,X) = N(\phi X,Y). \tag{2.6e}$$

Proof: Interchanging X and Y in equation (2.4), we get the equation (2.5a). Barring equation (2.4) and (2.5a) throughout or different vectors in it and using structure we get the equation (2.5b), (2.5c), (2.5d), (2.5e), (2.5f), (2.5g), (2.5h) and (2.5i).
Again, subtracting equation (2.5a) from equation (2.4), we get

$$P(X,Y) - P(Y,X) = D_{\phi X}\phi Y - D_{\phi Y}\phi X - a^r D_X Y + a^r D_Y X - \phi D_{\phi X} Y + \phi D_Y \phi X - \phi D_X \phi Y + \phi D_{\phi Y} X. \tag{2.7}$$

Now using equation (1.1b) in equation (1.7), we get the equation (1.6a). Similarly, we can prove the equations (1.6b), (1.6c), (1.6d) and (1.6e).

Corollary (1.1): We have hyperbolic Hsu-structure manifold.

$$P(\phi Y,\phi X) = \phi P(\phi X,Y), \tag{2.8a}$$

$$-a^r P(\phi Y,X) = \phi P(\phi X,\phi Y), \tag{2.8b}$$

$$P(Y,\phi X) = \phi P(X,Y), \tag{2.8c}$$

$$-a^r P(Y,X) = \phi P(X,\phi Y), \tag{2.8d}$$

$$\phi P(Y,X) = P(X,\phi Y). \tag{2.8e}$$

Proof: The statement follows from equation (2.5).

Theorem (1.3): Let us put

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Vol. 2, Issue 11, November 2013

$Q(X,Y) = D_{\phi X}\phi Y + a^r D_Y X - \phi D_{\phi X} Y + \phi D_{\phi Y} X.$ (2.9)
Then
$Q(Y,X) = D_{\phi Y}\phi X + a^r D_X Y - \phi D_{\phi Y} X + \phi D_{\phi X} Y.$ (2.10)
$Q(\phi X,\phi Y) = -\phi Q(\phi X, Y) = a^{2r} D_X Y + a^r D_{\phi Y}\phi X + a^r \phi D_X \phi Y - a^r \phi D_Y \phi X,$ (2.11a)
$\phi Q(\phi X,\phi Y) = a^r Q(\phi X, Y) = a^{2r} \phi D_X Y + a^r \phi D_{\phi Y}\phi X - a^{2r} D_X \phi Y + a^{2r} D_Y \phi X,$ (2.11b)
$Q(X,\phi Y) = -\phi Q(X,Y) = -a^r D_{\phi X} Y + a^r D_{\phi Y} X - \phi D_{\phi X}\phi Y - a^r \phi D_Y X,$ (2.11c)
$\phi Q(X,\phi Y) = a^r Q(X,Y) = -a^r \phi D_{\phi X} Y + a^r \phi D_{\phi Y} X + a^r D_{\phi X}\phi Y + a^{2r} D_Y X,$ (2.11d)
$Q(\phi Y,\phi X) = -\phi Q(\phi Y, X) = a^{2r} D_Y X + a^r D_{\phi X}\phi Y + a^r \phi D_Y \phi X - a^r \phi D_X \phi Y,$ (2.11e)
$\phi Q(\phi Y,\phi X) = a^r Q(\phi Y, X) = a^{2r} \phi D_Y X + a^r \phi D_{\phi X}\phi Y - a^{2r} D_Y \phi X + a^{2r} D_X \phi Y,$ (2.11f)
$Q(Y,\phi X) = -\phi Q(Y,X) = -a^r D_{\phi Y} X + a^r D_{\phi X} Y - \phi D_{\phi Y}\phi X - a^r \phi D_X Y,$ (2.11g)
$\phi Q(X,\phi Y) = a^r Q(Y,X) = -a^r \phi D_{\phi Y} X + a^r \phi D_{\phi X} Y + a^r D_{\phi Y}\phi X + a^{2r} D_X Y.$ (2.11h)
Consequently
$Q(\phi X,\phi Y) - a^r Q(X,Y) = a^r Q(Y,X) - Q(\phi Y,\phi X) = N(\phi X,\phi Y) = -a^r N(X,Y),$ (2.12a)
i.e. Q is hybrid X and Y.
$Q(\phi X, Y) + Q(X,\phi Y) = -\{Q(\phi Y, X) + Q(Y,\phi X)\} = N(\phi X, Y)$ (2.12b)
$\phi Q(\phi X,\phi Y) - a^r \phi Q(X,Y) = -\{\phi Q(\phi Y,\phi X) - a^r \phi Q(Y,X)\} = \phi N(\phi X,\phi Y),$ (2.12c)
$\phi Q(\phi X, Y) + a^r Q(X,Y) = -\{\phi Q(\phi Y, X) + \phi Q(Y,\phi X)\} = \phi N(X,\phi Y).$ (2.12d)

Proof: Interchanging X and Y in equation (2.9), we get (2.10). Barring equation (2.9) and (2.10) throughout or different vector in it and using structure we get the equation (2.11a), (2.11b), (2.11c), (2.11d), (2.11e), (2.11f), (2.11g) and (2.11h).
Again, subtracting equation (2.11d) from (2.11a), we get

$$Q(\phi X,\phi Y) - a^r Q(X,Y) = a^{2r} D_X Y - a^{2r} D_Y X - a^r D_{\phi X}\phi Y + a^r D_{\phi Y}\phi X + a^r \phi D_X \phi Y - a^r \phi D_{\phi Y} X + a^r \phi D_{\phi X} Y - a^r \phi D_Y \phi X. \quad (2.13)$$

Now, using equation (2.2e) in equation (2.13), we get the equation (2.12a), using the fact that N is pure in X and Y in equation (2.12a), we see that Q is hybrid in X and Y. Similarly, we can prove the remaining equations.

Theorem (1.4): Let us put
$M(X,Y) = \phi D_Y \phi X - D_{\phi Y}\phi X - a^r D_X Y - \phi D_X \phi Y.$ (2.14)
$M(Y,X) = \phi D_X \phi Y - D_{\phi X}\phi Y - a^r D_Y X - \phi D_Y \phi X.$ (2.15)
$M(\phi X,\phi Y) = -\phi M(\phi X, Y) = -a^r \phi D_{\phi Y} X - a^{2r} D_Y X - a^r D_{\phi X}\phi Y + a^r D_{\phi X}\phi Y,$ (2.16a)
$\phi M(\phi X,\phi Y) = a^r M(\phi X, Y) = a^{2r} D_{\phi Y} X - a^{2r} \phi D_Y X - a^r \phi D_{\phi X}\phi Y - a^{2r} D_{\phi X} Y,$ (2.16b)
$M(X,\phi Y) = -\phi M(X,Y) = \phi D_{\phi Y}\phi X + a^r D_Y \phi X - a^r D_X \phi Y + a^r \phi D_X Y,$ (2.16c)
$\phi M(X,\phi Y) = a^r M(X,Y) = -a^r D_{\phi Y}\phi X + a^r \phi D_Y \phi X - a^r \phi D_X \phi Y - a^{2r} D_X Y,$ (2.16d)
$M(\phi Y,\phi X) = -\phi M(\phi Y, X) = -a^r \phi D_{\phi X} Y - a^{2r} D_X Y - a^r D_{\phi Y}\phi X + a^r \phi D_{\phi Y} X,$ (2.16e)
$\phi M(\phi Y,\phi X) = a^r M(\phi Y, X) = a^{2r} D_{\phi X} Y - a^{2r} \phi D_X Y - a^r \phi D_{\phi Y}\phi X - a^{2r} D_{\phi Y} X,$ (2.16f)
$M(Y,\phi X) = -\phi M(Y,X) = \phi D_{\phi X}\phi Y + a^r D_X \phi Y - a^r D_Y \phi X + a^r \phi D_Y X,$ (2.16g)
$\phi M(Y,\phi X) = a^r M(Y,X) = -a^r D_{\phi X}\phi Y + a^r \phi D_X \phi Y - a^r \phi D_Y \phi X - a^{2r} D_Y X.$ (2.16h)
Consequently
$M(\phi X,\phi Y) - a^r M(X,Y) = a^r M(Y,X) - M(\phi Y,\phi X) = N(\phi X,\phi Y) = -a^r N(X,Y),$ (2.17a)
i.e. M is hybrid X and Y.
$M(\phi X, Y) + M(X,\phi Y) = -\{M(\phi Y, X) + M(Y,\phi X)\} = N(X,\phi Y)$ (2.17b)
$\phi M(\phi X,\phi Y) - a^r \phi M(X,Y) = a^r \phi M(Y,X) - \phi M(\phi Y,\phi X) = \phi N(\phi X,\phi Y),$ (2.17c)
$\phi M(\phi X, Y) + a^r M(X,Y) = -\{a^r M(Y,X) + \phi M(\phi Y, X)\} = \phi N(\phi X, Y).$ (2.17d)

Proof: Interchanging X and Y in equation (2.14), we get (2.15). Barring equation (2.14) and (2.15) throughout or different vector in it and using structure we get the equation (2.16a), (2.16b), (2.16c), (2.16d), (2.16e), (2.16f), (2.16g), (2.16h).

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Again, subtracting equation (2.16d) and (2.16e) from equations (2.16a) and (2.16h), we get

$$M(\phi X,\phi Y)-a^r M(X,Y)=a^r M(Y,X)-M(\phi Y,\phi X)$$
$$=a^{2r}D_X Y-a^{2r}D_Y X-a^r D_{\phi X}\phi Y+a^r D_{\phi Y}\phi X+a^r\phi D_X\phi Y-a^r\phi D_{\phi Y}X+a^r\phi D_{\phi X}Y-a^r\phi D_Y\phi X. \quad (2.18)$$

Now, using equation (1.2e) in equation (1.18), we get the equation (1.17a), using the fact that M is pure in X and Y. Similarly, we can prove the remaining equations.

Corollary (1.2): We have in hyperbolic Hsu-structure manifold.

$$M(\phi X,\phi Y)=-a^r Q(X,Y), \quad (2.19a)$$
$$M(\phi X,Y)=Q(X,\phi Y), \quad (2.19b)$$
$$M(X,\phi Y)=Q(\phi X,Y), \quad (2.19c)$$
$$a^r M(X,Y)=-Q(\phi X,\phi Y), \quad (2.19d)$$
$$a^r M(Y,X)=-Q(\phi Y,\phi X), \quad (2.19e)$$
$$M(\phi Y,\phi X)=-a^r Q(Y,X), \quad (2.19f)$$
$$M(Y,\phi X)=-\phi Q(Y,X), \quad (2.19g)$$
$$\phi M(Y,X)=-Q(Y,\phi X). \quad (2.19h)$$

Proof: The statement follows from equations (1.11) and (1.16).

Corollary (1.3): We have in hyperbolic Hsu-structure manifold

$$M(X,Y)+Q(X,Y)=-\{M(Y,X)+Q(Y,X)\}=N(X,Y), \quad (2.20a)$$
$$M(\phi X,\phi Y)+Q(\phi X,\phi Y)=-\{M(\phi Y,\phi X)+Q(\phi Y,\phi X)\}=N(\phi X,\phi Y), \quad (2.20b)$$
$$M(\phi X,Y)+Q(\phi X,Y)=-\{M(\phi Y,X)-\phi Q(\phi Y,\phi X)\}=N(X,\phi Y), \quad (2.20c)$$
$$\phi M(\phi X,Y)-Q(\phi X,\phi Y)=-\{\phi M(\phi Y,X)-Q(\phi Y,\phi X)\}=\phi N(\phi X,Y). \quad (2.20d)$$

Proof: from equation (1.9), (1.10), (1.14) and (1.15), we have

$$M(X,Y)+Q(X,Y)=-\{M(Y,X)+Q(Y,X)\}$$
$$=D_{\phi X}\phi Y-D_{\phi Y}\phi X-a^r D_X Y+a^r D_Y X-\phi D_{\phi X}Y+\phi D_Y\phi X-\phi D_X\phi Y+\phi D_{\phi Y}X. \quad (2.21)$$

Now using equation (2.1b) in equation (2.21), we get equation the equation (2.20a). Similarly, we can prove the remaining equations.

Theorem (1.5): Let us put

$$U(X,Y)=a^r D_Y X-D_{\phi Y}\phi X+\phi D_Y\phi X+\phi D_{\phi Y}X. \quad (2.22)$$

Then

$$U(Y,X)=a^r D_X Y-D_{\phi X}\phi Y+\phi D_X\phi Y+\phi D_{\phi X}Y. \quad (2.23)$$
$$U(\phi X,\phi Y)=-a^r U(X,Y)=a^r D_{\phi Y}\phi X-a^{2r}D_Y X-a^r\phi D_{\phi Y}X-a^r\phi D_Y\phi X, \quad (2.24a)$$

i.e. U is pure in X and Y.

$$U(\phi X,Y)=U(X,\phi Y)=a^r D_Y\phi X+a^r D_{\phi Y}X-a^r\phi D_Y X+\phi D_{\phi Y}\phi X, \quad (2.24b)$$
$$\phi U(\phi X,Y)=\phi U(X,\phi Y)=a^r\phi D_Y\phi X+a^r\phi D_{\phi Y}X+a^{2r}D_Y X-a^r D_{\phi Y}\phi X, \quad (2.24c)$$
$$\phi U(\phi X,\phi Y)=-a^r\phi U(X,Y)=a^r\phi D_{\phi Y}\phi X-a^{2r}\phi D_Y X+a^{2r}D_{\phi Y}X+a^{2r}D_Y\phi X, \quad (2.24d)$$
$$U(\phi Y,\phi X)=-a^r U(Y,X)=a^r D_{\phi X}\phi Y-a^{2r}D_X Y-a^r\phi D_{\phi X}Y-a^r\phi D_X\phi Y, \quad (2.24e)$$

i.e. U is pure in Y and X.

$$U(\phi Y,X)=U(Y,\phi X)=a^r D_X\phi Y+a^r D_{\phi X}Y-a^r\phi D_X Y+\phi D_{\phi X}\phi Y, \quad (2.24f)$$
$$\phi U(\phi Y,X)=\phi U(Y,\phi X)=a^r\phi D_X\phi Y+a^r\phi D_{\phi X}Y+a^{2r}D_X Y-a^r D_{\phi X}\phi Y, \quad (2.24g)$$
$$\phi U(\phi Y,\phi X)=-a^r\phi U(Y,X)=a^r\phi D_{\phi X}\phi Y-a^{2r}\phi D_X Y+a^{2r}D_{\phi X}Y+a^{2r}D_X\phi Y, \quad (2.24h)$$

Consequently

$$U(X,Y)-U(Y,X)=N(X,Y), \quad (2.25a)$$
$$U(\phi X,\phi Y)+a^r U(Y,X)=N(\phi X,\phi Y), \quad (2.25b)$$
$$U(\phi X,Y)-U(Y,\phi X)=N(\phi X,Y), \quad (2.25c)$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 11, November 2013

$U(X,\phi Y) - U(Y,\phi X) = N(X,\phi Y),$ (2.25d)
$\phi U(\phi X,\phi Y) + a^r \phi U(Y,X) = \phi N(\phi X,\phi Y),$ (2.25e)
$\phi U(\phi X,Y) - a^r \phi U(Y,\phi X) = \phi N(\phi X,Y).$ (2.25f)

Proof: Interchanging X and Y in equation (2.22), we get (2.23). Barring equation (2.22) and (2.23) throughout or different vector in it and using structure we get the equation (2.24a), (2.24b), (2.24c), (2.24d), (2.24e), (2.24f), (2.24g), (2.24h).
Again, subtracting equation (2.23) and (2.22), we get

$$U(X,Y) - U(Y,X) = D_{\phi X}\phi Y - D_{\phi Y}\phi X - a^r D_X Y + a^r D_Y X - \phi D_{\phi X} Y + \phi D_Y \phi X - \phi D_X \phi Y + \phi D_{\phi Y} X. \quad (2.26)$$

Now, using equation (2.1b) in equation (2.26), we get the equation (2.25a). Similarly, we can prove the remaining equations.

Theorem (1.6): Let us put
$J(X,Y) = D_{\phi X}\phi Y - \phi D_{\phi Y}\phi X + \phi D_Y \phi X - \phi D_X \phi Y.$ (2.27)
Then
$J(X,Y) = -J(Y,X) = D_{\phi X}\phi Y - D_{\phi Y}\phi X + \phi D_Y \phi X - \phi D_X \phi Y,$ (2.28a)
$J$is Skew-Symmetric in X and Y.
$J(\phi X,\phi Y) = -J(\phi Y,\phi X) = a^{2r} D_X Y - a^{2r} D_Y X - a^r \phi D_{\phi Y} X + a^r \phi D_{\phi X} Y,$ (2.28b)
$\phi J(\phi X,\phi Y) = -\phi J(\phi Y,\phi X) = a^{2r}\phi D_X Y - a^{2r}\phi D_Y X + a^{2r} D_{\phi Y} X - a^{2r} D_{\phi X} Y,$ (2.28c)
$J(\phi X,Y) = -J(Y,\phi X) = -a^r D_X \phi Y + a^r \phi D_{\phi Y} X - a^r \phi D_Y X - \phi D_{\phi X}\phi Y,$ (2.28d)
$\phi J(\phi X,Y) = -\phi J(Y,\phi X) = -a^r \phi D_X \phi Y - a^{2r} D_{\phi Y} X + a^{2r} D_Y X + a^r D_{\phi X}\phi Y,$ (2.28e)
$J(X,\phi Y) = -J(\phi Y,X) = -a^r D_{\phi X}\phi Y + a^r \phi D_Y \phi X + \phi D_{\phi Y}\phi X + a^r \phi D_X Y,$ (2.28f)
$\phi J(X,\phi Y) = -\phi J(\phi Y,X) = -a^r \phi D_{\phi X} Y - a^{2r} D_Y \phi X - a^r D_{\phi Y}\phi X - a^{2r} D_X Y.$ (2.28g)
$a^r J(X,Y) = -a^r J(Y,X) = a^r D_{\phi X}\phi Y - a^r \phi D_{\phi Y}\phi X + a^r \phi D_Y \phi X - a^r \phi D_X \phi Y.$ (2.28h)
Consequently
$J(\phi X,\phi Y) - a^r J(X,Y) = -\{J(\phi Y,\phi X) - a^r J(Y,X)\} = N(\phi X,\phi Y) = -a^r N(X,Y),$ (2.29a)
i.e. J is hybrid X and Y.
$J(\phi X,Y) + J(X,\phi Y) = -\{J(\phi Y,X) + J(Y,\phi X)\} = N(\phi X,Y),$ (2.29b)
$\phi J(\phi X,\phi Y) - a^r \phi J(X,Y) = -\{\phi J(\phi Y,\phi X) - a^r \phi J(Y,X)\} = a^r N(X,\phi Y),$ (2.29c)
$\phi J(\phi X,Y) + \phi J(X,\phi Y) = -\{\phi J(\phi Y,X) + \phi J(Y,\phi X)\} = \phi N(X,\phi Y).$ (2.29d)

Proof: Interchanging X and Y in equation (2.27), we get (2.28a). From equation (2.28a) it is clear that J is Skew-Symmetric in X and Y. Barring equation (2.28a) throughout or different vector in it and using structure we get the equation (2.28a), (2.28b), (2.28c), (2.28d), (2.28e), (2.28f), (2.28g), (2.28h).
Again, subtracting equation (2.28h) from (2.28b), we get

$$\begin{aligned} J(\phi X,\phi Y) - a^r J(X,Y) &= -\{J(\phi Y,\phi X) - a^r J(Y,X)\} \\ &= a^{2r} D_X Y - a^{2r} D_Y X - a^r D_{\phi X}\phi Y + a^r D_{\phi Y}\phi X + a^r \phi D_X \phi Y - a^r \phi D_{\phi Y} X + a^r \phi D_{\phi X} Y - a^r \phi D_Y \phi X. \end{aligned} \quad (2.30)$$

Now, using equation (2.2e) in equation (2.30), we get the equation (2.29a), using the fact that N is pure in X and Y in equation (2.29a), we see that J is hybrid in X and Y. Similarly, we can prove the remaining equations.

Theorem (1.7): Let us put
$T(X,Y) = a^r D_Y X - a^r D_X Y - \phi D_{\phi X} Y + \phi D_{\phi Y} X.$ (2.31)
Then
$T(X,Y) = -T(Y,X) = a^r D_Y X - a^r D_X Y - \phi D_{\phi X} Y + \phi D_{\phi Y} X.$ (2.32)
$T$is Skew-Symmetric in X and Y.
$T(\phi X,\phi Y) = -T(\phi Y,\phi X) = a^r D_{\phi Y}\phi X - a^r D_{\phi X}\phi Y + a^r \phi D_X \phi Y - a^r \phi D_Y \phi X,$ (2.33a)
$T(\phi X,Y) = -T(Y,\phi X) = a^r D_Y \phi X - a^r D_{\phi X} Y + a^r \phi D_X Y + \phi D_{\phi Y}\phi X,$ (2.33b)

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 11, November 2013

$T(X,\phi Y) = -T(\phi Y,X) = a^r D_{\phi Y}X - a^r D_X\phi Y - \phi D_{\phi X}\phi Y - a^r\phi D_Y X,$ (2.33c)
$a^r T(X,Y) = -a^r T(Y,X) = a^{2r}D_Y X - a^{2r}D_X Y - a^r\phi D_{\phi X}Y + a^r\phi D_{\phi Y}X,$ (2.33d)
$\phi T(\phi X,\phi Y) = -\phi T(\phi Y,\phi X) = a^r\phi D_{\phi Y}\phi X - a^r\phi D_{\phi X}\phi Y - a^{2r}D_X\phi Y + a^{2r}D_Y\phi X$ (2.33e)
$\phi T(\phi X,Y) = -\phi T(Y,\phi X) = a^r\phi D_Y\phi X - a^r\phi D_{\phi X}Y - a^{2r}D_X Y - a^r D_{\phi Y}\phi X,$ (2.33f)
$\phi T(X,\phi Y) = -\phi T(\phi Y,X) = a^r\phi D_{\phi Y}X - a^r\phi D_X\phi Y + a^r D_{\phi X}\phi Y + a^{2r}D_Y X.$ (2.33g)
$a^r\phi T(X,Y) = -a^r\phi T(Y,X) = a^{2r}\phi D_Y X - a^{2r}\phi D_X Y + a^{2r}D_{\phi X}Y - a^{2r}D_{\phi Y}X.$ (2.33h)
Consequently
$T(\phi X,\phi Y) - a^r T(X,Y) = -\{T(\phi Y,\phi X) - a^r T(Y,X)\} = N(\phi X,\phi Y) = -a^r N(X,Y),$ (2.34a)
i.e. T is hybrid X and Y.
$T(\phi X,Y) - T(\phi Y,X) = \{T(X,\phi Y) - T(Y,\phi X)\} = N(\phi X,Y),$ (2.34b)
$\phi T(\phi X,\phi Y) + a^r\phi T(Y,X) = -\{\phi T(\phi Y,\phi X) + a^r\phi T(X,Y)\} = \phi N(\phi X,\phi Y),$ (2.34c)
$\phi T(\phi X,Y) + \phi T(X,\phi Y) = -\{\phi T(Y,\phi X) + \phi T(\phi Y,X)\} = \phi N(X,\phi Y).$ (2.34d)

Proof: Interchanging X and Y in equation (2.31), we get (2.32). From equation (2.32) it is clear that T is Skew-Symmetric in X and Y. Barring equation (2.32) throughout or different vector in it and using structure we get the equation (2.33a), (2.33b), (2.33c), (2.33d), (2.33e), (2.33f), (2.33g), (2.33h).
Again, subtracting equation (2. 33d) from (2. 33a), we get

$$T(\phi X,\phi Y) - a^r T(X,Y) = -\{T(\phi Y,\phi X) - a^r T(Y,X)\}$$
$$= a^{2r}D_X Y - a^{2r}D_Y X - a^r D_{\phi X}\phi Y + a^r D_{\phi Y}\phi X + a^r\phi D_X\phi Y - a^r\phi D_{\phi Y}X + a^r\phi D_{\phi X}Y - a^r\phi D_Y\phi X. \quad (2.35)$$

Now, using equation (2.2e) in equation (2.35), we get the equation (2.34a), using the fact that N is pure in X and Y in equation (2.34a), we see that T is hybrid in X and Y. Similarly, we can prove the remaining equations.

Corollary (1.4): We have in hyperbolic Hsu-structure manifold.
$J(\phi Y,\phi X) = a^r T(X,Y),$ (2.36a)
$a^r J(X,Y) = T(\phi Y,\phi X),$ (2.36b)
$J(X,\phi Y) = T(\phi X,Y),$ (2.36c)
$J(\phi X,Y) = T(X,\phi Y),$ (2.36d)
$J(\phi X,\phi Y) = a^r T(Y,X),$ (2.36e)
$a^r J(Y,X) = T(\phi X,\phi Y).$ (2.36f)

Proof: The statement follows from equations (2.28) and (2.33).

Corollary (1.5): We have in hyperbolic Hsu-structure manifold
$J(X,Y) + T(X,Y) = -\{J(Y,X) + T(Y,X)\} = N(X,Y),$ (2.37a)
$J(\phi X,\phi Y) + T(\phi X,\phi Y) = -\{J(\phi Y,\phi X) + T(\phi Y,\phi X)\} = N(\phi X,\phi Y),$ (2.37b)
$J(\phi X,Y) + T(\phi X,Y) = -\{J(Y,\phi X) + T(Y,\phi X)\} = N(\phi X,Y),$ (2.37c)
$\phi J(X,\phi Y) + \phi T(X,\phi Y) = -\{\phi J(\phi Y,X) + \phi T(\phi Y,X)\} = \phi N(X,\phi Y).$ (2.37d)

Proof: Adding equation (2.28) and (2.32), we have

$$J(X,Y) + T(X,Y) = -\{J(Y,X) + T(Y,X)\}$$
$$= D_{\phi X}\phi Y - D_{\phi Y}\phi X - a^r D_X Y + a^r D_Y X - \phi D_{\phi X}Y + \phi D_Y\phi X - \phi D_X\phi Y + \phi D_{\phi Y}X. \quad (2.38)$$

Now using equation (2.1b) in equation (2.38), we get equation the equation (2.37a). Similarly, we can prove the remaining equations.
The associate Nijenhius tensor N in Hyperbolic Hsu-structure manifold is given by
$'N(X,Y,Z) = a^r g(N(X,Y),Z) = -g(N(\phi X,\phi Y),Z).$ (2.39)
which satisfies the following properties.
$'N(X,Y,Z) = -'N(Y,X,Z),$ (2.40a)
i.e. $'N$ is Skew-Symmetric in X and Y.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 11, November 2013

$'N(\phi X, Y, Z) = 'N(X, \phi Y, Z) = 'N(X, Y, \phi Z),$ (2.40b)
$'N(\phi X, \phi Y, Z) = 'N(\phi X, Y, \phi Z) = 'N(X, \phi Y, \phi Z) = -a^{r\prime} N(X, Y, Z).$ (2.40c)
i.e. $'N$ is pure in any two of three slots.

Corollary (1.6): Let us define
$'P(X, Y, Z) = g(P(X, Y), Z).$ (2.41)
Then
$'P(\phi X, \phi Y, Z) = -a^{r\prime} P(X, Y, Z),$ (2.42a)
$'P(\phi X, Y, \phi Z) = 'P(X, \phi Y, \phi Z).$ (2.42b)
and
$a^{r\prime} P(X, Y, Z) - a^{r\prime} P(Y, X, Z) = 'N(X, Y, Z),$ (2.43a)
$a^{2r\prime} P(X, Y, Z) - a^{r\prime} P(X, \phi Y, \phi Z) = a^{r\prime} N(X, Y, Z) = -'N(X, \phi Y, \phi Z).$ (2.43b)

Proof: Using equation (2.41) in equation (2.5b) and (2.5c), we get the equation (2.42a) and (2.42b). Again, using equations (2.39) and (2.41) in equations (2.6a) and (2.6b), we get the equation (2.43a) and (2.43b).

Remark (1.1): If $a \neq 0$, using the fact that $'N$ is pure in Y and Z in equation (2.43b), we get
$a^{r\prime} P(X, Y, Z) = 'P(X, \phi Y, \phi Z).$ (2.44)
i.e. $'P$ is hybrid in Y and Z.
from equations (2.42b) and (2.44), we get
$'P(\phi X, Y, \phi Z) = a^r \ 'P(X, Y, Z).$
i.e. $'P$ is hybrid in X and Z.

Corollary (1.7): Let us define
$'Q(X, Y, Z) = g(Q(X, Y), Z).$ (2.45)
Then
$'Q(\phi X, \phi Y, Z) = \ 'Q(\phi X, Y, \phi Z),$ (2.46a)
$- \ 'Q(X, \phi Y, \phi Z) = a^r \ 'Q(X, Y, Z).$ (2.46b)
i.e. $'Q$ is pure in Y and Z.
and
$a^{r\prime} Q(\phi X, \phi Y, Z) - a^{4\prime} Q(X, Y, Z) = -'N(\phi X, \phi Y, Z) = -a^{r\prime} N(X, Y, Z).$ (2.47)

Proof: Using equation (2.45) in equation (2.11a) and (2.11d) together with the fact that
$g(\phi X, Y) = -g(X, \phi Y),$
we get the equations (2.46a) and (2.46b), from equation (2.46b) it is clear that $'Q$ is pure in Y and Z. Using equations (2.39) and (2.45) in equations (2.12a), we get the equation (2.47).

Remark (1.2): If $a \neq 0$, using the fact that $'N$ is pure in X and Z in equation (2.47), we get
$'Q(\phi X, \phi Y, Z) = a^{r\prime} Q(X, Y, Z).$ (2.48)
i.e. $'Q$ is hybrid in X and Z.
from equations (2.46a) and (2.48), we get
$'Q(\phi X, Y, \phi Z) = a^r \ 'Q(X, Y, Z).$
i.e. $'Q$ is hybrid in X and Z.

Corollary (1.8): Let us define
$'M(X, Y, Z) = g(M(X, Y), Z).$ (2.49)
Then
$'M(\phi X, \phi Y, Z) = \ 'M(\phi X, Y, \phi Z),$ (2.50a)
$- \ 'M(X, \phi Y, \phi Z) = a^r \ 'M(X, Y, Z).$ (2.50b)

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 11, November 2013

i.e. $'M$ is pure in Y and Z.
and
$a^{r}{}'M(\phi X, \phi Y, Z) - a^{2r}{}'M(X, Y, Z) = -{}'N(\phi X, \phi Y, Z) = -a^{r}{}'N(X, Y, Z).$ (2.51)

Proof: The proof follows the pattern of the proof of the corollary (1.7).

Remark (1.3): If $a \neq 0$, using the fact that $'N$ is pure in X and Y in equation (2.51), we get
$'M(\phi X, \phi Y, Z) = a^{r}{}'M(X, Y, Z).$ (2.52)
i.e. $'M$ is hybrid in X and Y.
from equations (2.50a) and (2.52), we get
$'M(\phi X, Y, \phi Z) = a^{r}\ {}'M(X, Y, Z).$
i.e. $'M$ is hybrid in X and Z.

Corollary (1.9): Let us define
$'U(X, Y, Z) = g(U(X, Y), Z).$ (2.53)
Then
$'U(\phi X, \phi Y, Z) = -a^{r}{}'U(X, Y, Z),$ (2.54a)
i.e. $'U$ is pure in X and Y.
$'U(\phi X, Y, \phi Z) = {}'U(X, \phi Y, \phi Z).$ (2.54b)
and
$a^{r}{}'U(X, Y, Z) - a^{r}{}'U(Y, X, Z) = {}'N(X, Y, Z),$ (2.55a)
$a^{r}{}'U(\phi X, \phi Y, Z) + a^{2r}{}'Q(Y, X, Z) = {}'N(\phi X, \phi Y, Z).$ (2.55b)
Proof: Using equation (2.53) in equation (2.24a) and (2.24c) together with the fact that
$g(\phi X, Y) = -g(X, \phi Y),$
we get the equations (2.54a) and (2.54b). Using equations (2.39) and (2.53) in equations (2.25a) and (2.25b), we get the equations (2.55a) and (2.55b).
Corollary (1.9): Let us define
$'J(X, Y, Z) = g(J(X, Y), Z).$ (2.56)
Then
$'J(X, Y, Z) = -\ {}'J(Y, X, Z).$ (2.57)
i.e. $'J$ is Skew-Symmetric in X and Y.
$'J(\phi X, \phi Y, Z) - a^{2r}\,{}'J(X, Y, Z) = {}'N(\phi X, \phi Y, Z) = -a^{r}\ \ {}'N(X, Y, Z),$ (2.58a)
and
$a^{r}{}'J(\phi X, Y, Z) + a^{r}{}'J(X, \phi Y, Z) = {}'N(\phi X, Y, Z),$ (2.58b)
$'J(\phi X, \phi Y, Z) - a^{r}\,{}'J(X, Y, Z) = -{}'N(X, \phi Y, Z),$ (2.58c)
$a^{r}{}'J(\phi X, Y, \phi Z) + a^{r}{}'J(X, \phi Y, \phi Z) = {}'N(X, \phi Y, \phi Z).$ (2.58d)

Proof: Using equation (2.56) in equation (2.28a), we get the equation (2.57a).From equation (2.57a), it is clear that $'J$ is Skew-structure in X and Y. Using equation (2.56) and (2.39) in equation (2.29) together with the fact that
$g(\phi X, Y) = -g(X, \phi Y),$
we get the equations (2.58).

Remark (1.4): If $a \neq 0$, using the fact that $'N$ is pure in X and Y in equation (2.58a), we get
$'J(\phi X, \phi Y, Z) = a^{r}{}'J(X, Y, Z).$
i.e. $'J$ is hybrid in X and Y.

Corollary (1.11): Let us define
$'T(X, Y, Z) = g(T(X, Y), Z).$ (2.59)
Then
$a^{r}\,{}'(\phi X, \phi Y, Z) - a^{2r}\,{}'T(X, Y, Z) = {}'N(\phi X, \phi Y, Z) = -a^{r}\ \ {}'N(X, Y, Z),$ (2.60a)

$a^{r}{}'T(\phi X, Y, Z) - a^{r}{}'T(\phi Y, X, Z) = {}'N(\phi X, Y, Z),$ (2.60b)
${}'T(\phi X, \phi Y, \phi Z) + a^{r}T(Y, X, \phi Z) = -{}'N(X, Y, Z),$ (2.60c)
$a^{r}{}'T(\phi X, Y, \phi Z) + a^{r}{}'T(X, \phi Y, \phi Z) = {}'N(X, \phi Y, \phi Z).$ (2.60d)

Proof: Using equation (2.59) and (2.39) in equation (2.34) together with the fact that
$g(\phi X, Y) = -g(X, \phi Y),$
we get the equations (2.60).

Remark (1.5): If $a \neq 0$, using the fact that ${}'N$ is pure in X and Y in equation (2.60a), we get
${}'T(\phi X, \phi Y, Z) = a^{r}{}'T(X, Y, Z).$
i.e. ${}'T$ is hybrid in X and Y.

Corollary (1.12): In hyperbolic Hsu-structure manifold the associate Nijenhius tensor ${}'N(X, Y, Z)$ can be put in the form:

$a^{r}\ {}'M(X, Y, Z) + a^{r}\ {}'Q(X, Y, Z) = {}'N(X, Y, Z),$ (2.61a)
${}'M(\phi X, \phi Y, Z) + {}'Q(\phi X, \phi Y, Z) = -{}'N(X, Y, Z),$ (2.61b)
$a^{r}\ {}'J(X, Y, Z) + a^{r}\ {}'T(X, Y, Z) = {}'N(X, Y, Z),$ (2.61c)
${}'J(\phi X, \phi Y, Z) + {}'T(\phi X, \phi Y, Z) = -{}'N(X, Y, Z),$ (2.61d)

Proof: Using equation (2.39), (2.45), (2.49), (2.56) and (2.59) in equations (2.20a), (2.20b), (2.37a), and (2.37b), we get equations (2.61).

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