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On Non-Extendable Special Dio-3-Tuples

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ABSTRACT: In this paper, we present three non-extendable special Diophantine triples (a,b,c) such that the product of any two elements of the set minus with their sum and increased by a polynomial with integer coefficients is a Perfect square.

KEYWORDS: Diophantine Triples, Integer sequences

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I.INTRODUCTION

A Set of positive integers $(a_1, a_2, a_3, ... a_m)$ is said to have the property D(n), $n \in z - \{0\}$, if $a_i a_j + n$ is a perfect square for all $1 \le i \lhd j \le m$ and such a set is called a Diophantine –m-tuple with property D(n).Many mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any arbitrary integer n[1] and also for any linear polynomial in n. Further, various authors considered the connections of the problems of Diophantus, Davenport and Fibonacci numbers in(2-25).

In this paper, we exhibit three non-extendable special Diophantine triples (a,b,c) such that the product of any two elements of the set minus with their sum and increased by a polynomial with integer coefficients is a Perfect square.That is in section II.1,non-extendable D(17) dio-triple is considered. In section II.2, D(-5n) special dio-triple is considered and it is shown that it cannot be extended to special dio-quadruple. In section II.3, non-extendable D(5-6n) special dio-3-tuple is considered.

II. METHOD OF ANALYSIS

II.1: Non-extendable $D(17)$ Dio-triple:

Let $a = 4n + 17$ and $b = n + 1$ be two integers such that ab-(a+b)+17is a perfect square

Let ‘c’ be any non-zero integer such that

$$(4n+16)c - 4n = \alpha^2 \quad (1)$$

$$(n)c - n + 16 = \beta^2 \quad (2)$$

Eliminating ‘c’ from (1) and (2), we obtain

$$(n)(\alpha^2) - (4n+16)(\beta^2) = -16(3n+16) \quad (3)$$

Using the linear transformations

$$\alpha = X + (4n+16)T \quad (4)$$

$$\beta = X + nT \quad (5)$$

in (3), it leads to the pell equation

$$X^2 = (4n+16)(n)T^2 + 16 \quad (6)$$

Let $T_0 = 1$ and $X_0 = (2n+4)$ be the initial solution of (6). Thus (4) yields $\alpha_0 = 6n + 20$

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And using (1), we get $c = 9n + 25$

Hence (a,b,c)= $(4n+1, n+1, 9n+25)$ is the Diophantine triple with property $D(17)$

Some numerical examples are presented below

n	(a,b,c) with property $D(17)$
1	(21,2,34)
2	(25,3,43)
3	(29,4,52)
4	(33,5,61)
5	(37,6,70)

We show that the above triple cannot be extended to quadruple

Let 'd' be any non-zero integer such that

$$(4n+16)d - 4n = p^2 \quad (7)$$

$$nd - n + 16 = q^2 \quad (8)$$

$$(9n+24)d - (9n+8) = r^2 \quad (9)$$

Eliminating 'd' from (8) and (9), we obtain

$$(9n+24)(q^2) - (n)(r^2) = 128n + 384 \quad (10)$$

Using the linear transformations

$$q = X + nT \quad (11)$$

$$r = X + (9n+24)T \quad (12)$$

in (10), it leads to the pell equation

$$X^2 = n(9n+24)T^2 + 16 \quad (13)$$

Let $T_0 = 1$ and $X_0 = 3n + 4$ be the initial solution of (13). Thus (11) yields $q_0 = 4n + 4$

And using (8), we get $d = 16n + 33$

<u>Verify Quadruple:</u>

Substituting the above value of 'd' in L.H.S of (7), we have

$$(4n+16)(16n+33) - 4n = (8n+24)^2 - 48$$

Note that the R.H.S is not a perfect square

Hence the triple $(4n+17, n+1, 9n+25)$ with property $D(17)$ cannot be extended to a quadruple.

Note:

The triple $(4n+17, n+1, 9n+25)$ is a strong Diophantine triple and the quadruple $(4n+17, n+1, 9n+25, 16n+33)$ is almost strong Diophantine quadruple.

II.2: Non-extendable $D(-5n)$ Dio-triple:

Let $a = 9n + 27$ and $b = n + 2$ be two integers such that ab-(a+b)-5n is a perfect square

Let 'c' be any non-zero integer such that

$$(n+1)c - 6n - 2 = \alpha^2 \quad (14)$$

$$(9n+26)c - (14n+27) = \beta^2 \quad (15)$$

Eliminating 'c' from (14) and (15), we obtain

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$$(9n+26)\alpha^2-(n+1)(\beta^2)=-(8n+25)(5n+1) \quad (16)$$

Using the linear transformations

$$\alpha = X+(n+1)T \quad (17)$$

$$\beta = X+(9n+26)T \quad (18)$$

in (16), it leads to the pell equation

$$X^2=(9n+26)(n+1)T^2-(5n+1) \quad (19)$$

Let $T_0=1$ and $X_0=(3n+5)$ be the initial solution of (19). Thus (17) yields $\alpha_0=4n+6$

And using (14), we get $c=16n+38$

Hence (a,b,c)=($(9n+27, n+2, 16n+38)$) is the Diophantine triple with property $D(-5n)$

Some numerical examples are presented below

n	(a,b,c) with property $D(-5n)$
1	(36,3,54) with property D(-5)
2	(45,4,70) with property D(-10)
3	(54,5,86) with property D(-15)
4	(63,6,102) with property D(-20)
5	(72,7,118) with property D(-25)

We show that the above triple cannot be extended to quadruple

Let 'd' be any non-zero integer such that

$$(9n+26)d-(14n+27)=p^2 \quad (20)$$

$$(n+1)d-(6n+2)=q^2 \quad (21)$$

$$(16n+37)d-(21n+38)=r^2 \quad (22)$$

Eliminating 'd' from (21) and (22), we obtain

$$(16n+37)(q^2)-(n+1)(r^2)=-[75n^2+195n+36] \quad (23)$$

Using the linear transformations

$$q = X+(n+1)T \quad (24)$$

$$r = X+(16n+37)T \quad (25)$$

in (23), it leads to the pell equation

$$X^2=(16n+37)(n+1)T^2-(5n+1) \quad (26)$$

Let $T_0=1$ and $X_0=4n+6$ be the initial solution of (26). Thus (24) yields $q_0=5n+7$

And using (21), we get $d=25n+51$

<u>Verify Quadruple:</u>

Substituting the above value of 'd' in L.H.S of (20), we have

$$(9n+26)(25n+51)-(14n+27)=[15n+36]^2+(15n+3)$$

Note that the R.H.S is not a perfect square

Hence the triple $(9n+27, n+2, 16n+38)$ with property $D(-5n)$ cannot be extended to a quadruple.

Note:The triple $(9n+27, n+2, 16n+38)$ is a strong Diophantine triple and the quadruple $(9n+27, n+2, 16n+38, 25n+51)$ is almost strong Diophantine quadruple.

II.3: Non-extendable $D(5-6n)$ Dio-triple:

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Let $a = 16n + 7$ and $b = n + 3$ be two integers such that ab-(a+b)+(5-6n) is a perfect square

Let 'c' be any non-zero integer such that

$$(n+2)c - 7n + 2 = \alpha^2 \tag{27}$$

$$(16n+6)c - (22n+2) + = \beta^2 \tag{28}$$

Eliminating 'c' from (27) and (28), we obtain

$$(16n+6)(\alpha^2) - (n+2)(\beta^2) = (4-6n)(15n+4) \tag{29}$$

Using the linear transformations

$$\alpha = X + (n+2)T \tag{30}$$

$$\beta = X + (16n+6)T \tag{31}$$

in(29), it leads to the pell equation

$$X^2 = (16n+6)(n+2)T^2 + (4-6n) \tag{32}$$

Let $T_0 = 1$ and $X_0 = 4(n+1)$ be the initial solution of (32). Thus (30) yields $\alpha_0 = 5n + 6$

And using (27), we get $c = 25n + 17$

Hence (a,b,c)=($(16n + 7, n + 3, 25n + 17)$) is the Diophantine triple with property $D(5 - 6n)$

Some numerical examples are presented below

n	(a,b,c) with property $D(5-6n)$
1	(23,4,42) with property D(-1)
2	(39,5,67) with property D(-7)
3	(55,6,92) with property D(-13)
4	(71,7,117) with property D(-19)
5	(87,8,142) with property D(-25)

We show that the above triple cannot be extended to quadruple

Let 'd' be any non-zero integer such that

$$(16n+6)d - (22n+2) = p^2 \tag{33}$$

$$(n+2)d - 7n + 2 = q^2 \tag{34}$$

$$(25n+16)d - (31n+12) = r^2 \tag{35}$$

Eliminating 'd' from (34) and (35), we obtain

$$(25n+16)(q^2) - (n+2)(r^2) = (4-6n)(24n+14) \tag{36}$$

Using the linear transformations

$$q = X + (n+2)T \tag{37}$$

$$r = X + (25n+16)T \tag{38}$$

in (36), it leads to the pell equation

$$X^2 = (25n+16)(n+2)T^2 + (4-6n) \tag{39}$$

Let $T_0 = 1$ and $X_0 = 5n + 6$ be the initial solution of (39). Thus (37) yields $q_0 = 6n + 8$

And using (34), we get $d = 36n + 31$

<u>Verify Quadruple:</u>

Substituting the above value of 'd' in L.H.S of (33), we have

$$(16n+6)(36n+31) - (22n+2) = (24n+14)^2 + (18n-12)$$

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Note that the R.H.S is not a perfect square

Hence the triple $(16n+7, n+3, 25n+17)$ with property $D(5-6n)$ cannot be extended to a quadruple.

Note:

The triple $(16n+7, n+3, 25n+17)$ is a strong Diophantine triple and the quadruple $(16n+7, n+3, 25n+17, 36n+31)$ is almost strong Diophantine quadruple.

III.CONCLUSION

In this paper, considering a special dio-2-tuple with polynomial members satisfying the property D(17),D(-5n) and D(5-6n) in turn , it is shown that each cannot be extended to special dio-quadruple with corresponding property. To conclude, a dio-2-tuple with members represented by special numbers, namely, polygonal number, star number, jacobsthal number and so on, may be considered with suitable property and may attempt to extend each of them to Diophantine quadruple, Diophantine quintuple and so on.

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