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On the Changing Wigner Stability of Ion Acoustic Soliton in a Four Component Incoherent Plasma and Akhmediev Breather

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ABSTRACT

We have analysed a situation on a four component incoherent plasma which shows interesting patterns in Wigner stability as the electron-ion temperature ratio (α) and the superthermal parameter (κ) is varied. The corresponding Nonlinear Schrödinger equation is deduced with the help of Krylov-Bogoliubov-Mitropolsky (KBM) approach. In the next stage we show how to take care of incoherence in the density profile with the help of dynamical equation for the correlation function <R(x,t)R(x',t')> with the help of Wigner-Moyal transform. It is observed that even this equation can be linearized to study the modulational stability which is actually Wigner Stability. An interesting and new feature is that when the ion-electron temperature ratio changes there is a change of stability. As such we get a switch-over from bright to dark soliton. A similar phenomenon is also seen to happen when the super-thermal parameter 'κ' changes. We have also seen that our NLS equation also support a very new kind of breather soliton known as Akhmediev breather.

PACS: 94.05.Fg (Solitons and solitary waves), 94.05.Pt (Wave/wave, wave/particle interactions), 94.30.Gm (Plasma instabilities), 52.35.Fp (Electrostatic waves and oscillations).

INTRODUCTION

Research on electron-ion-positron plasma has gained momentum due to observational evidence from solar atmosphere[1] and pulsar magnetosphere[2]. We are actually referring to the viking mission and Themis experiments. The physics of collective behaviour in such an nonlinear plasma has been analysed with the study of various types of acoustic waves. Sometimes in natural environment or laboratory plasma high energy particles can coexists with isothermally distributed particles and as such the resulting distribution may not be a Maxwellian one. These are actually described by kappa distribution[3], which was initially introduced by Vasylinuas[4]. These types of distributions occur as a result of external forces acting on plasma. The most general form of such a distribution is given as:

$$F_\kappa(v)=\frac{\Gamma(\kappa+1)}{(\pi\kappa\theta^2)^{3/2}\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa\theta^2}\right) \qquad (1)$$

where $\theta=[(2\kappa-3)/\kappa]V_t$ is the effective thermal speed, Γ is the standard Gamma function, T is the temperature. The most important parameter is κ which is the measure of the slope of the energy spectrum of the superthermal particles forming the tail. The investigation of the Ion-Acoustic Wave (IAW) has been carried out for oblique propagation by Panwar[4]. On the otherhand in many situation of astrophysical significance it is found that the electrons can have two distinct temperature distribution. Such situations have been analysed by Baboolal[5] and Shahmansouri[6]. On the otherhand our motivation in this paper is to study a four component plasma in presence of superthermal electrons when there is incoherence in the plasma density distribution. Such a phenomenon usually leads to a statistical description which have been taken care of with the help of Wigner-Moyal formulation[7]. The governing equation of the correlation function is analysed for the existence of modulational stability. It is seen that the type of stability changes both with respect to the parameter κ and the electron-ion temperature ratio. So that here we have a transition from the bright to dark envelope soliton. Over and above we have found that the NLS equation do support a new kind of breather, called the Akhmediev breather[8] which shows quite interesting behaviour with respect to plasma parameters.

FORMULATION

We consider an unmagnetized system comprising of warm adiabatic ions, isothermal positrons and two temperature superthermal electrons.

At equilibrium the quasineutrality condition:

$$n_{i0} + n_{p0} = n_{h0} + n_{c0} \quad (2)$$

Where n_{i0}, n_{c0}, n_{c0}, n_{h0} are respectively equilibrium densities of warm ion, positrons which are isothermal and superthermal cold electrons and hot electrons. The normalized hydrodynamics equations governing the plasma are:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \quad (3)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_c + \gamma n_h - \sigma n_i - (1-\sigma+\gamma)n_p \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_c + \gamma n_h - \sigma n_i - (1-\sigma+\gamma)n_p \quad (5)$$

For inertialess cold and hot electrons the number densities are:

$$\left.\begin{aligned} n_c &= \left[1 - \frac{\phi}{\kappa - 3/2}\right]^{(-\kappa+1/2)} = 1 + C_1\phi + C_2\phi^2 + \cdots \\ n_h &= \left[1 - \frac{\mu\phi}{\kappa - 3/2}\right]^{(-\kappa+1/2)} = 1 + C_1\mu\phi + C_2\mu^2\phi^2 + \cdots \end{aligned}\right\} \quad (6)$$

Similarly for inertialess isothermal positrons

$$n_p = \exp[-\lambda\phi] = 1 - \lambda\phi + \frac{\lambda^2\phi^2}{2} - \frac{\lambda^3\phi^3}{6} + \cdots \quad (7)$$

So, finally the Poissons's equation can be written as:

$$\frac{\partial^2 \phi}{\partial x^2} = 1 + \gamma - \eta - \sigma n_i + \gamma_1\phi + \gamma_2\phi^2 + \gamma_3\phi^3 + \cdots \quad (8)$$

Where:

$$\left.\begin{aligned} \eta &= 1 - \sigma + \gamma \\ \gamma_1 &= \gamma\mu C_1 + C_1 + \eta\lambda \\ \gamma_2 &= \gamma\mu^2 C_2 + C_2 - \eta\lambda^2/2 \\ \gamma_3 &= \gamma\mu^3 C_3 + C_3 - \eta\lambda^3/6 \end{aligned}\right\} \quad (9)$$

here, $\alpha = T_i/T_e$, $\gamma = n_{i0}/n_{e0}$, $\lambda = T_e/T_p$, $\sigma = n_{i0}/n_{e0}$, $\mu = T_e/T_h$. In the above equation, n_i is the ion number density normalized by its equilibrium value n_{i0}, T_c is the ion fluid speed normalized by the ion-acoustic wave speed, $C = (T_i/m_i)^{1/2}$ T_c, T_h, T_i and T_i are the corresponding temperatures of the cold, hot electrons, isothermal positrons and ions. The corresponding time and space variables are normalized by $\omega_{pi}^{-1} = \sqrt{\frac{m_i}{4\pi e^2 n_{e0}}}$ Instead of the usual reductive perturbation approach we proceed with the Krylov-Bogoliubov-Mitropolsky (KBM) approach[9-12]. An advantage of the KBM method is that it is conceptually more natural that the derivative expansion method where one needs to introduce artificial multiplicity of the independent variables. The basic assumptions of the KBM method is the annihilation of secular terms.

In the present case KBM method dictates that n_i,u_i and ϕ are expanded as:

$$\left.\begin{aligned} \phi &= \epsilon\phi_1(a, a^*, \psi) + \epsilon^2\phi_2(a, a^*, \psi) + \cdots \\ u &= \epsilon u_1(a, a^*, \psi) + \epsilon^2 u_2(a, a^*, \psi) + \cdots \\ n &= 1 + \epsilon n_1(a, a^*, \psi) + \epsilon^2 n_2(a, a^*, \psi) + \cdots \end{aligned}\right\} \quad (10)$$

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In the above expression ϕ, u and n depend on x and t only through a, a^* and ψ, where the complex amplitude α(or α *) is assumed to be a slowly varying function of (x, t) defined by the relations

$$\left.\begin{aligned}\frac{\partial a}{\partial t} &= \epsilon A_1(a,a^*)+\epsilon^2 A_2(a,a^*)+\cdots\\ \frac{\partial a}{\partial x} &= \epsilon B_1(a,a^*)+\epsilon^2 B_2(a,a^*)+\cdots\end{aligned}\right\} \quad (11)$$

Together with the complex conjugate of equations 11. The phase $\psi = kx - \omega t$, remains unchanged. The unknowns functions B_1, B_1, A_2 , etc. should be determined so as to make the equation secularity free. Substituting equations 10 and 11 in the basic plasma equations we get in first order of ϵ ;

$$\left.\begin{aligned}n_1 &= \frac{k^2}{S}\left[a\exp[i\psi]+a^*\exp[-i\psi]\right]\\ u_1 &= \frac{k\omega}{S}\left[a\exp[i\psi]+a^*\exp[-i\psi]\right]\\ \phi &= \left[a\exp[i\psi]+a^*\exp[-i\psi]\right]\end{aligned}\right\} \quad (12)$$

along with the dispersion relation:

$$\omega^2 = \frac{\sigma k^2}{k^2+\gamma} + 3\alpha k^2 \quad (13)$$

whose graphical representation is given in **Figures 1-3.**

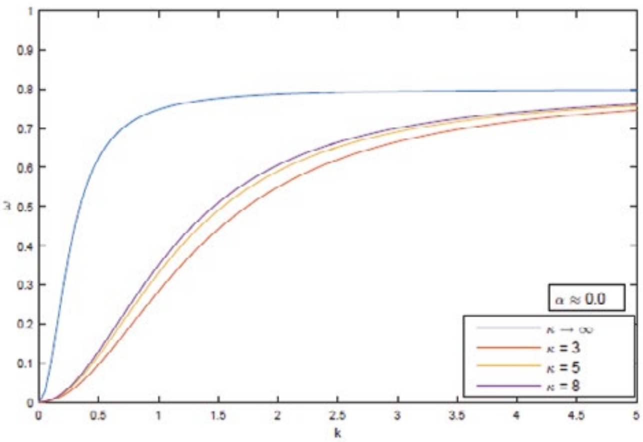

Figure 1: The plot of dispersion relation in case of a≈0 and different *k*-values.

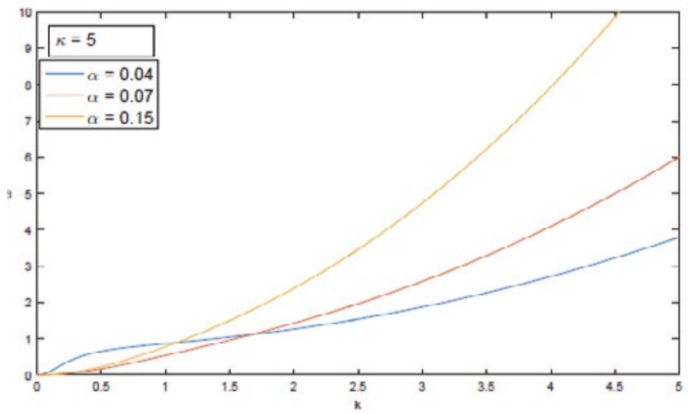

Figure 2: The plot of dispersion relation in case of α≠0.

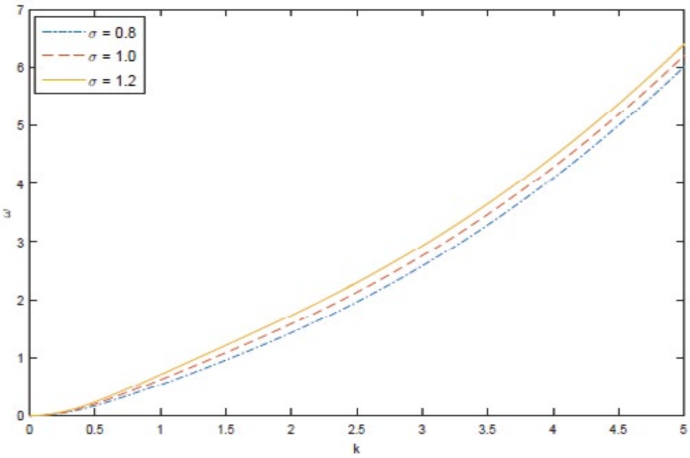

Figure 3: The plot of dispersion relation for different values of σ.

Next proceeding now to second order in ϵ, we get:

$$\left.\begin{aligned} \phi_2 &= a^2\alpha_1 \exp[2i\psi] + b_1(a,a^*)\exp[i\psi] + \delta_1(a,a^*) \\ n_2 &= a^2\alpha_2 \exp[2i\psi] + iB_1 b_2(a,a^*)\exp[i\psi] + \delta_2(a,a^*) \\ u_2 &= a^2\alpha_3 \exp[2i\psi] + iB_1 b_3(a,a^*)\exp[i\psi] + \delta_3(a,a^*) \end{aligned}\right\} \quad (14)$$

where;

$$\alpha_1 = \frac{2\gamma_2 S - \dfrac{\sigma k^4}{S^2}\left(3\omega^2 + \Omega\kappa\right)}{\kappa^2\sigma - \gamma_1 S - 4\kappa^2} \quad (15)$$

$$\alpha_2 = \alpha_1 \frac{4k^2 + \gamma_1}{\sigma} \quad (16)$$

$$\alpha_3 = \frac{\omega}{k\sigma}\alpha_1(4\kappa^2 + \gamma_1) - \frac{k^3\omega}{S^2} \quad (17)$$

$$b_1 - \frac{2k}{\sigma} \quad (18)$$

$$b_3 = -\frac{2\omega}{\sigma} - v_g \frac{k}{S} \quad (19)$$

At this point one may note that the differential equation for ϕ_2 can be written as;

$$\left\{\sigma k^2 \frac{\partial}{\partial\psi} + k^2(\omega^2 - k^2\Omega)\frac{\partial^3}{\partial\psi^3} - \gamma_1(\omega^2 - k^2\Omega)\right\}\phi_2$$

$$= \exp[i\psi]\left\{-A_1\left(\frac{2\sigma\omega k^2}{S}\right) + B_1\left(2kS - \sigma k - \frac{\sigma k\omega^2}{S} - \frac{\sigma k^3\omega}{S}\right)\right\} + a^2\exp[2i\psi]\left[\frac{3\sigma k^4\omega^2}{S^2} - \frac{\kappa^4 k\sigma\Omega}{S^2} + 2\gamma_2 S\right] \quad (20)$$

From which one can observe that the term $\exp[i\psi]$ is secular and so its coefficient should vanish and we get $A_1 + v_g B_1 = 0$, where v_g is the group velocity.

$$v_g = \frac{\sigma\omega^2 - S^2}{\sigma\omega k} \quad (21)$$

'b_1' has been undetermined and is taken to be '0'. δ_1, δ_3, δ_3 are to be fixed by secularity conditions arising at higher orders. We now proceed to the terms in third order of perturbation that is terms with coefficients 'ϵ^3'. This leads to:

$$\omega\frac{\partial n_3}{\partial\psi}-k\frac{\partial u_3}{\partial\psi}=L_1 \quad (22)$$

$$\omega\frac{\partial u_3}{\partial\psi}-k\frac{\partial\phi_3}{\partial\psi}-\Omega k\frac{\partial n_3}{\partial\psi}=L_2 \quad (23)$$

$$k^2\frac{\partial^2\phi_3}{\partial\psi^2}+\sigma n_3-\gamma_1\phi_3=L_3 \quad (24)$$

Where L_1, L_2, L_3 are very complicated expressions given in the appendix. After the elimination of unwanted terms, if we again impose the non-secularity condition then we get:

$$i(A_2+V_gB_2)+P\left(B_1\frac{\partial B_1}{\partial a}+\bar{B}_1\frac{\partial B_1}{\partial a^*}\right)+Q|a|^2a=0 \quad (25)$$

where the coefficients *P, Q,* etc. are written as:

$$P=\omega^2-k^2\Omega+\omega\sigma v_gb_2-\omega\sigma b_2+k\sigma v_gb_3-k\sigma\Omega b_2 \quad (26)$$
$$Q=(2\gamma_2-3\gamma_3)(\omega^2-k^2\Omega)-\frac{1}{4}[4\omega\sigma k^3\alpha_3-\omega^2\sigma k^2\alpha_2-k^3\sigma\omega\alpha_3+k^4\sigma\Omega\alpha_2]$$
$$S=\omega^2-k^2\Omega$$

Now following equation set (11) we set; $\xi=\epsilon(x-v_gt)$ and $\tau=\epsilon^3t$, and we obtain $A_1=\frac{1}{\epsilon}\frac{\partial a}{\partial t}$ and $A_2=\frac{1}{\epsilon^2}\frac{\partial a}{\partial t}-\frac{1}{\epsilon}A_1$ and $i\frac{\partial a}{\partial\tau}+P\frac{\partial^2a}{\partial\xi^2}+Q|a|^2a=0$. Using these in equation (24) we get;

$$i\frac{\partial a}{\partial\tau}+P\frac{\partial^2a}{\partial\xi^2}+Q|a|^2a=0 \quad (27)$$

which is the required nonlinear Schrödinger equation.

EFFECT OF INCOHERENCE AND WIGNER STABILITY

Now we consider the situation when the initial state is not fully coherent. Such a situation is governed by the correlation function $<a(x_1,t)a^*(x_2,t)>$ and analyze its evolution when satisfies equation (27).

Given a wave function $a(x,t)$, one defines the Wigner transform

$$W(x,\bar{K},t)=$$
$$W[a(t)](x,\bar{K})=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\xi\exp[-i\bar{K}\xi]\left[a\left(x+\frac{\xi}{2},t\right)a\times\left(x+\frac{\xi}{2},t\right)\right] \quad (28)$$

We now we write NLS equation at x_1 and multiply by $a^*(x_2)$ then add to the expression obtained by writing it for $a^*(x_2)$ and multiplying by $a(x_1)$, which leads to:

$$i\frac{\partial}{\partial t}<a(x_1)a^*(x_2)>+P\left(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2}\right)<a(x_1)a^*(x_2)>+Q\left[<a(x_1)a^*(x_1)a^*(x_2)a(x_1)>\right]=0 \quad (29)$$

Now it is well known that for the Gaussian distributed state the fourth commutants decompose exactly as a sum of products of second order ones. So in this approximation,

$$<a(x_1)a^*(x_1)a(x_1)a^*(x_2)>\approx 2n(x,t)W(x,X_2,t) \quad (30)$$

where finally $n(x,t)$ is the density. Finally we get the equation:

$$\frac{\partial}{\partial t}W(1,2)+P\left(\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2}\right)W(1,2)+2Q\left[n(x_1,t)-n(x_2,t)\right]=0 \quad (31)$$

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We will now use this equation to obtain the time evolution of $W(x,\bar{K},t)$ defined in equation(28). This is actually the use of Wigner-Moyal transformation for the wave envelope power spectrum density $W(x,\bar{K},t)$. We use now;

$$n(x_1,t)=n\left(x+\frac{\xi}{2},t\right)=\sum_{j=0}^{\infty}\frac{\xi^j}{2^j j!}\left(\frac{\partial^j n(x,t)}{\partial x^j}\right)\Bigg|_{x=0} \quad (32)$$

with similar expression for $n(x_2,t)$. So we get;

$$W(x_1,x_2)=\int_{-\infty}^{\infty}\exp[iK'\xi]W(K',x_1-x_2,t)dK' \quad (33)$$

whence one can have;

$$\frac{\partial}{\partial t}W(\bar{K},x,t)+2PK\frac{\partial}{\partial X}W(\bar{K},x,t)+2Qn(x,t)\sin\left(\frac{1}{2}\frac{\overleftarrow{\partial}}{\partial X}\frac{\overrightarrow{\partial}}{\partial K}\right)W(x,\bar{K},t)=0 \quad (34)$$

with x_1-x_2. The left and right handed arrows over the gradient operator with respect to *X* and *K*, denotes derivatives on the left and right hand side. The stability analysis for this equation is performed by setting:

$$\left.\begin{aligned}W(\bar{K},X,t)&=W_0(K)+\epsilon W_1(\bar{K},x,t)\\ n(x,t)&=n_0+\epsilon n_1(x,t)\end{aligned}\right\} \quad (35)$$

So that the linearized version of equation (34) becomes:

$$\frac{\partial W_1}{\partial t}+2P\bar{K}\frac{\partial W_1}{\partial X}+4Qn_1(X)\sin\left(\frac{1}{2}\frac{\overleftarrow{\partial}}{\partial X}\frac{\overrightarrow{\partial}}{\partial K}\right)W_0=0 \quad (36)$$

As per the standard procedure we set:

$$\left.\begin{aligned}W(\bar{K},X,t)&=g(\bar{K})\exp(q\bar{K}-\Omega t)\\ n_1(\bar{X},t)&=G\exp(qX-\Omega t)\end{aligned}\right\} \quad (37)$$

whence we obtain from (36)

$$1+\frac{Q}{P}\frac{1}{q}\int_{-\infty}^{\infty}\frac{W_0\left(K+\frac{q}{2}\right)-W_0\left(K-\frac{q}{2}\right)}{\bar{K}-\frac{\Omega}{2Pq}}d\bar{K}=0 \quad (38)$$

Analysis of this equation gives the idea about the stability of the system in the incoherent state. Here $W(X,\bar{K},t)$ plays the role of phase space energy density. An unstable Wigner mode generate the Benjamin-Fier side bands.

RESULTS AND DISCUSSION OF THE STABILITY ANALYSIS

The linearized equation 38 is the key ingredient for the study of the stability both in case of coherent and incoherent situation. To start with we consider the situation of v_g in different situations. For, α=0 , v_g is shown in **Figure 4** with respect to wave-vector *k* which shows that it initially increases then goes down to zero.

Here we have considered two values of σ=0.5 and σ=0.8. But if α=0.07, the situation changes drastically. This is given in **Figure 5**, which shows a completely different behaviour. On the otherhand for two different values of α, the situation again is different as seen in **Figure 5**.

So that we can infer that electron-ion temperature ration is very important for different dynamical scenario. In the next diagram we have kept α=0 , but have varied κ, the superthermal parameter. One can clearly observe that the different values of κ shows distinct variation of v_g, whereas for very large value of κ it shows an exponential decay.

Next we observed the behaviour of the dispersive coefficient 'P' for different situations, depicted in **Figures 6-8**.

The **Figure 6** clearly shows that the value of α has quite significant effect. Infact for α≈0 the value of *P* has a rectangular hyperbolic decaying nature and it ultimate reduces to zero with increasing value of wave-vector (*k*). While when α≠0 the value of p changes from positive to negative after certain value of *k*.

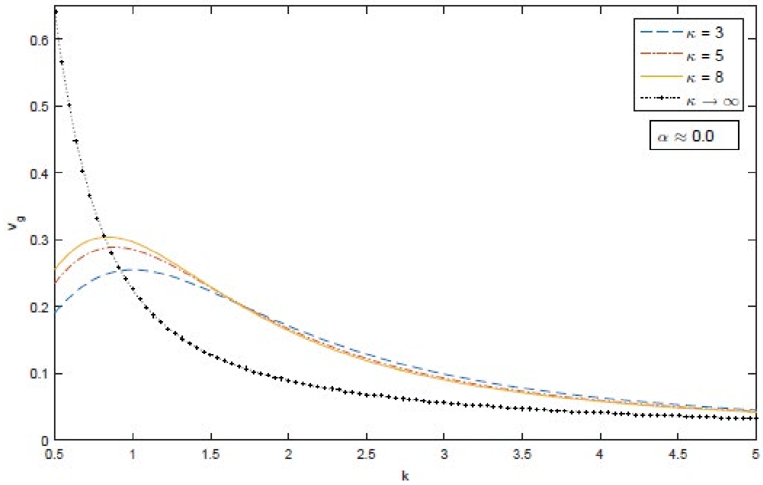

Figure 4: The plot of group velocity in case of α≈0 and different *k*-values.

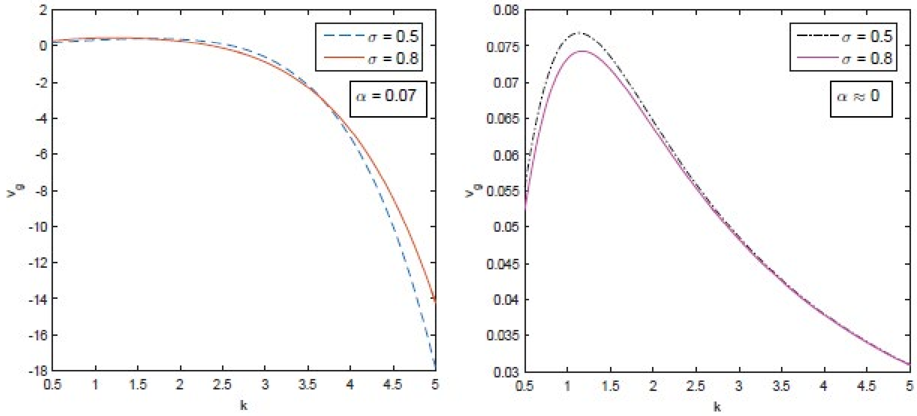

Figure 5: The plot of group velocity in case of α≈0 and different *k*-values.

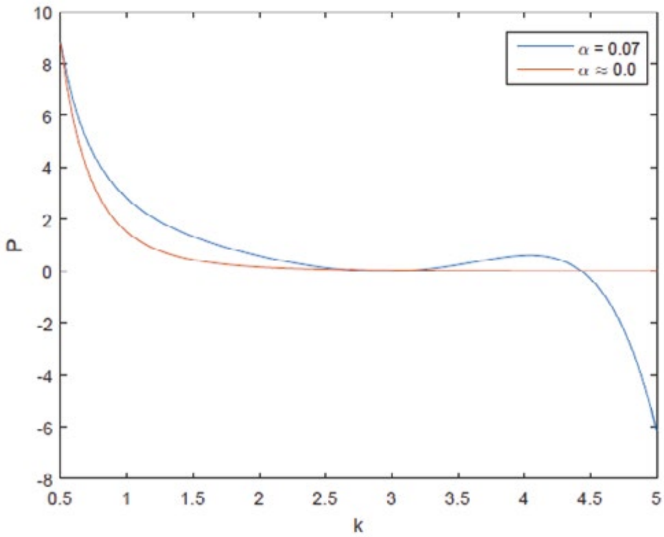

Figure 6: The plot of dispersive coefficient *P* for different values of α.

The **Figure 7** shows that the parameter γ also has a good impact on the dispersive coefficient. Though the nature of the *P* has no such change but has slight variation in its value.

In figure 8 we have shown that the *P* has quite good dependence upon the superthermal parameter, *k*.

we study the variation of ‘*Q*’. For α≠0 and different ‘*k*’ the nature of *Q* is really widely different. So the combined effect of α and *k* have a strong influence on the stability. This is shown in **Figure 9**.

In **Figure 10** we have kept same values of *k* but changed α to a zero, the nature of *Q* again changes.

Another interesting feature is shown in **Figure 11**, where the variation of 'PQ' is shown.

Here for non-zero α=0.07 if we change k from 3 to 8 we get a very dramatic change of PQ. But for α=0.0, the same values of *k* shows a completely different scenario, depicted in **fFgure 12**. To exhibit the whole situation we have drawn a two parametric plot in **Figures 13 and 14**.

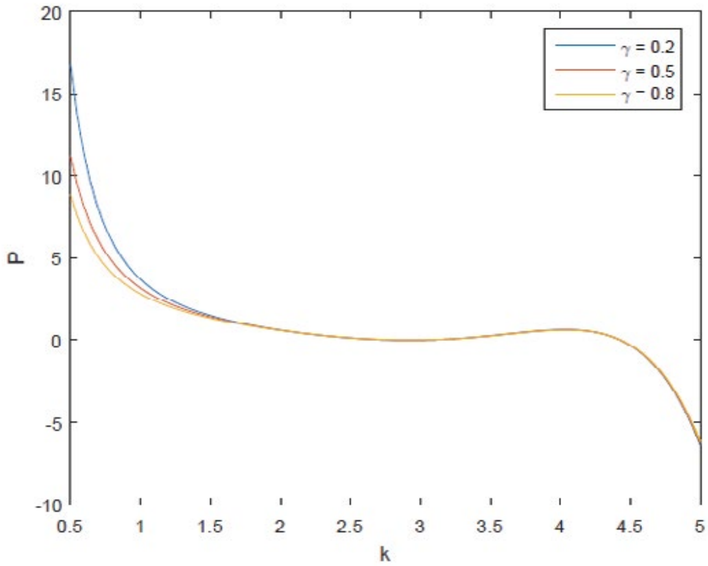

Figure 7: The plot of dispersive coefficient *P* for different values of γ.

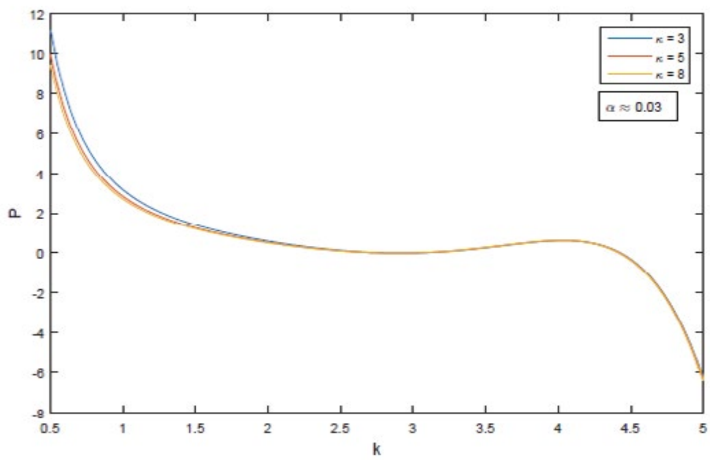

Figure 8: The plot of dispersive coefficient *P* for different values of *κ*.

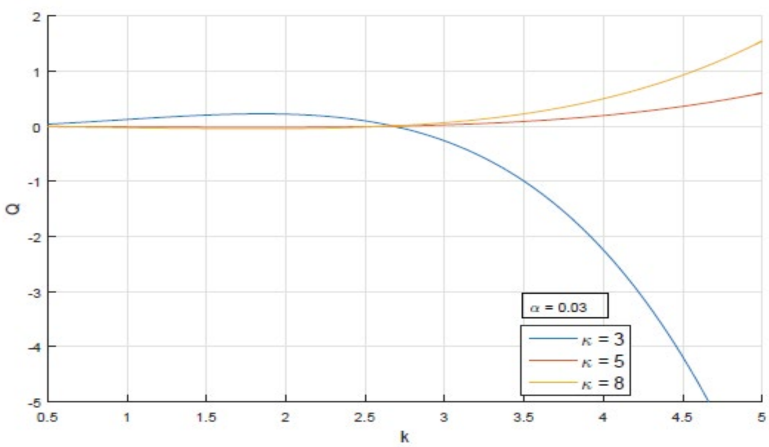

Figure 9: The variation of nonlinear coefficient with *k* for different values of *k*.

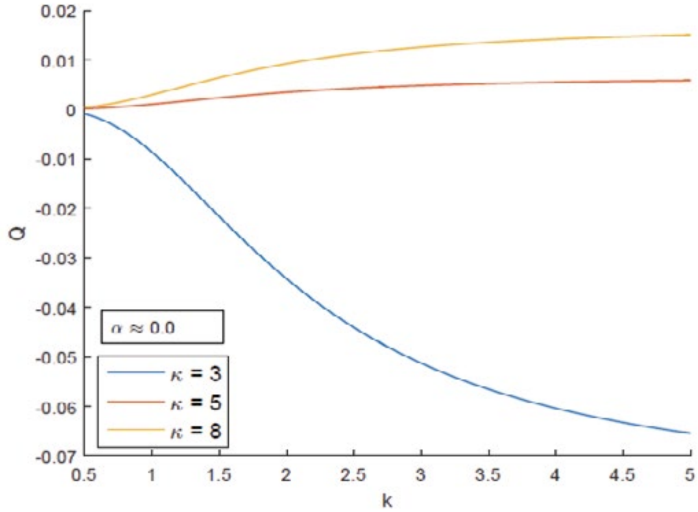

Figure 10: The variation of nonlinear coefficient with *k* for different values of *k*.

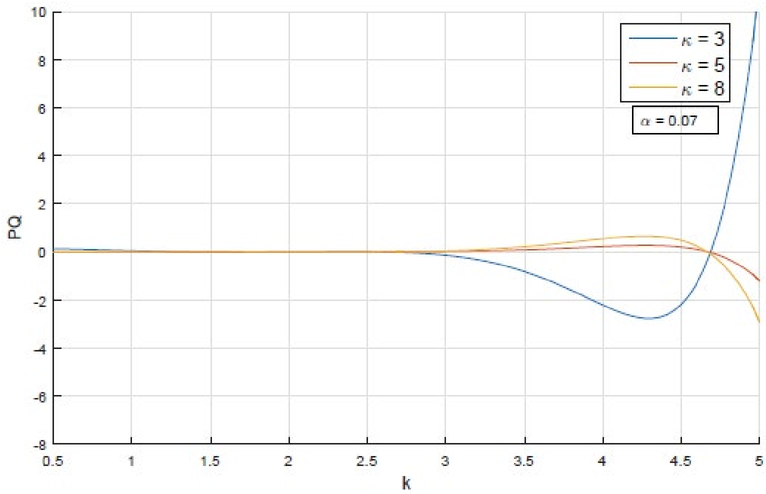

Figure 11: The Plot of the product of dispersive and nonlinear coefficient *PQ*.

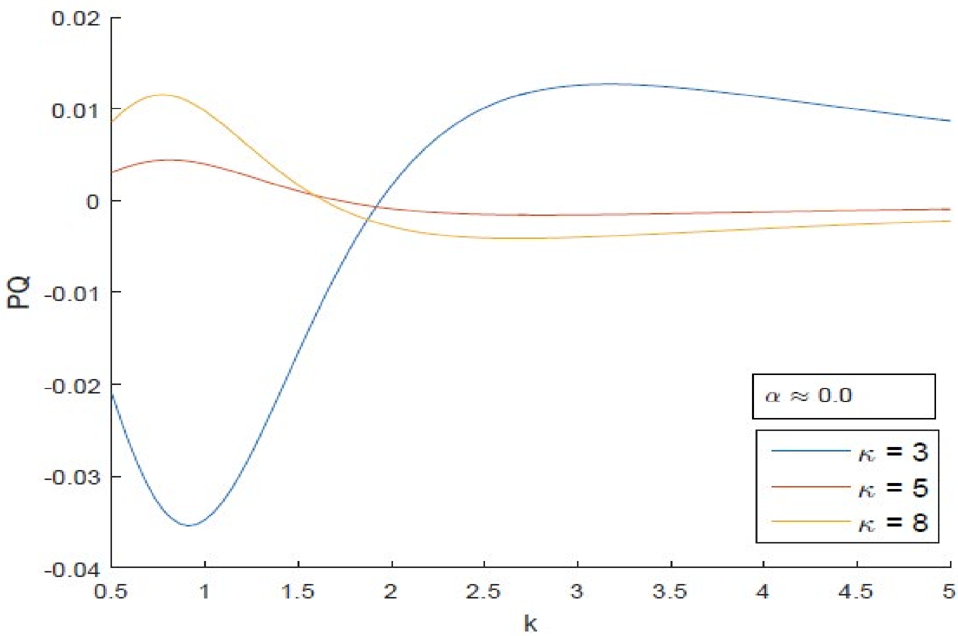

Figure 12: The Plot of the product of dispersive and nonlinear coefficient *PQ*.

Here we have shown the variation of *PQ* in (k,*κ*) plane for α≈0.0 and α≠0 situations. At last we come to the case of the growth rate ζ(ω). In **Figure 15** we have collected the variation of Ω_1 for different κ and α=0.0, but in each case the *μ* is different.

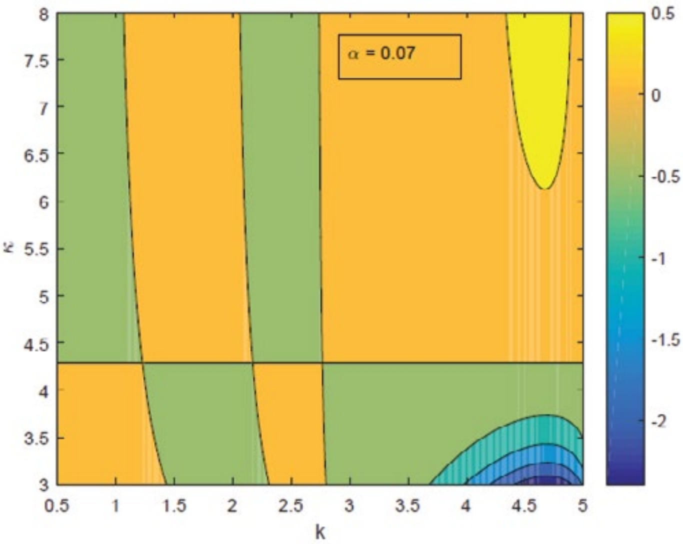

Figure 13: The Plot of the product of dispersive and nonlinear coefficient *PQ*.

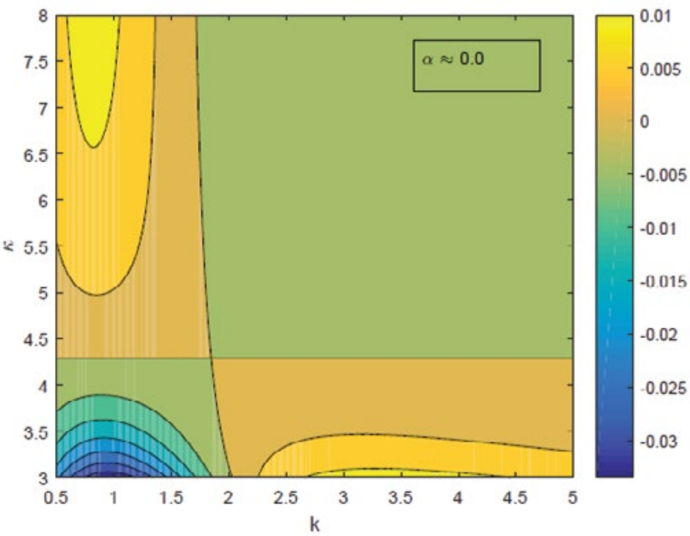

Figure 14: The Plot of the product of dispersive and nonlinear coefficient *PQ*.

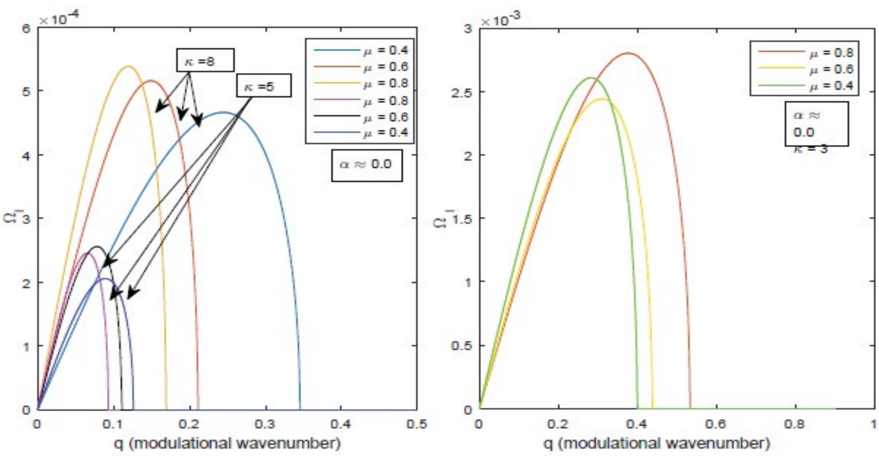

Figure 15: The variation for growth rate of instability with different plasma parameters.

One can observe that for α≈0 situation, in compare to the instability growth rate for *κ*=5 or *κ*=8, the for κ=3 is much higher. Thus we can say that when the superthermal parameter (κ) is nearly '3', the wave is quite unstable. Thus we can conclude that the tendency of the wave to breakup into train of waveforms is higher.

One can observe that in **Figure 16** the nature of growth rate though same, but the value of maximum growth rate varies when 'κ' is varied. **Figure 16** clearly shows that the instability growth rate though higher for κ=3, but minimum for κ=5 and moderate for κ=8. Thus in this case also we can conclude the same as in **Figure 15**.

In **Figures 17 and 18** the plot of instability growth rate is shown for non-monochromatic situation. The growth rate (Ω_{11}) is plotted for three different sets of *μ* and *κ*. But in this case the behaviour is entirely different. Infact the instability growth rate has a nearly exponentially increasing nature. Thus we see that unlike monochromatic case, the instability doesn't decay or attenuate, rather it goes on increasing. Thus when the wave interaction is non-monochromatic or incoherent, the solitary wave or the soliton gets destroyed or we can say that an imbalance develops between the dispersive and nonlinear effects.

AKHMEDIEV BREATHER SOLUTION

We have observed a change in stability in the previous section there exist ample possibility for the formation of some more complex structures. One such structure is breather. Of the various kinds of these, an important one is the Akhmediev breather[14].

The NLS-equation 27 reads as:

$$i\frac{\partial\psi}{\partial t}+P\frac{\partial^2\psi}{\partial x^2}+Q|\psi|^2\psi=0$$

if we set $\psi=\rho\exp(i\nu)$ we obtain:

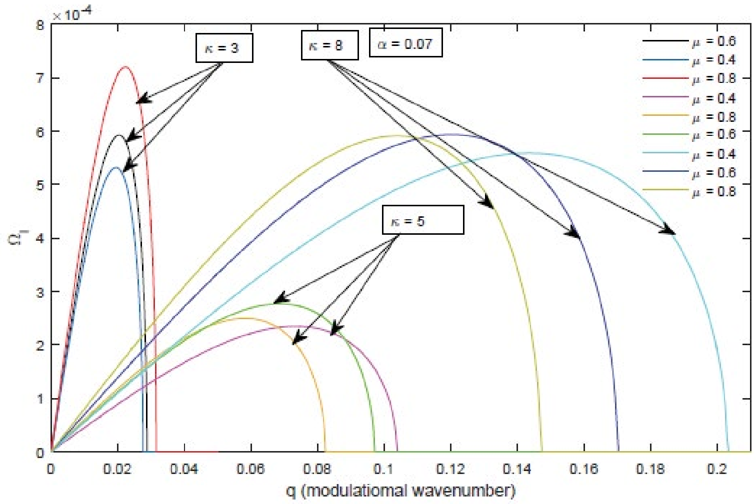

Figure 16: Another plot of instability growth rate for monochromatic case.

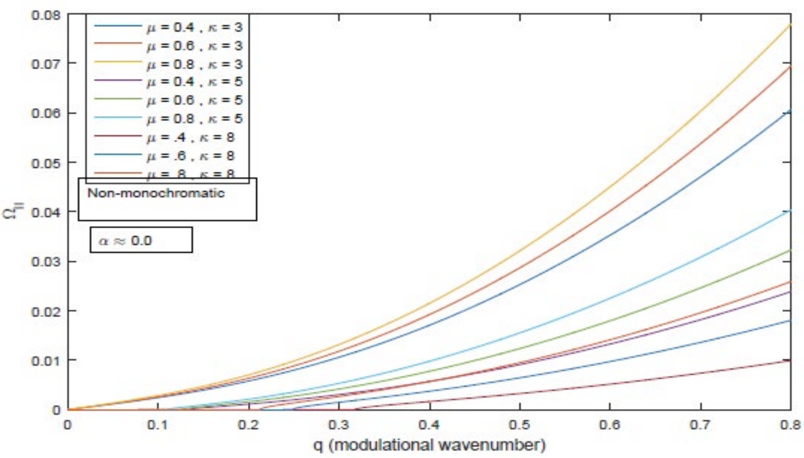

Figure 17: The instability growth rate for non-monochromatic case.

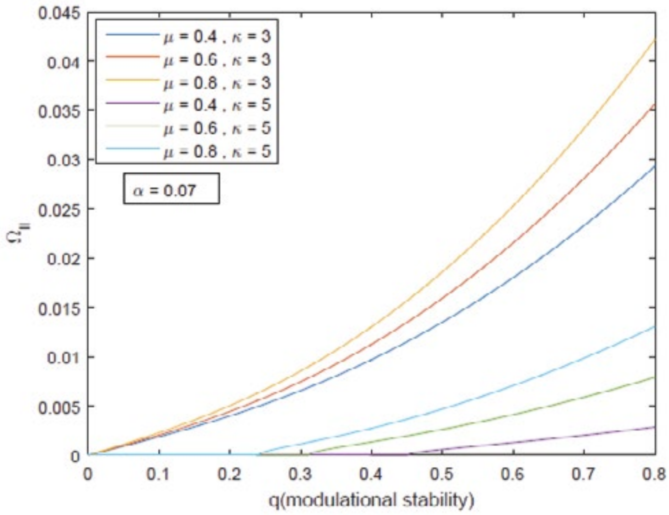

Figure 18: The instability growth rate for non-monochromatic case.

$$\left.\begin{aligned} &\rho_t + v_{xx}\rho P + 2v_x\rho_x P + (v_x)^2\rho P + Q\rho^3 = 0 \\ &v_t + P\rho v_x + 2Pv_x\rho_x - P\rho_{xx} + P\rho(v_x)^2 + Q\rho^3 = 0 \end{aligned}\right\} \quad (39)$$

whence both nonlinear are PDE and simple solution is not possible. Hence we use the Hirota method. As a result we use a bilinear derivative D_t , D_t and the NLS equation breaks up into

$$\left.\begin{aligned} &\left(iD_t + PD_x^2\right)GF = \lambda GF \\ &\left(PD_x^2 - \lambda\right)FF = QGG^* \end{aligned}\right\} \quad (40)$$

Where $\psi = G/F$, 'G' is complex and 'F' is real. Thus

$$\left.\begin{aligned} &F = 1 + \epsilon f_1 + \epsilon^2 f_2 + \cdots \\ &G = g_0(1 + \epsilon g_1 + \epsilon^2 g_2 + \cdots) \\ &g_0 = \rho_0 \exp(i\tilde{K}x - \Theta t) \end{aligned}\right\} \quad (41)$$

where, $\Theta - P\tilde{K}^2 = -Q\rho_0^2$

From 1^{st} order in ϵ we get;

$$\left.\begin{aligned} &\left\{i\left(D_t + 2\tilde{K}PD_x\right) + D_x^2\right\}\left(g_1 + f_1\right) = 0 \\ &\left[PD_x^2 - 2\rho^2\right]\left(f_1 + f_1^*\right) = -Q\left(g_1 + g_1^*\right) \end{aligned}\right\} \quad (42)$$

Next we seek for solution as $f_1 = \exp(\eta)$ and $\upsilon \exp(\rho)$. Here, υ is a complex quantity and $\eta = \sigma x - \mu t$. Using the above set of equations;

$$\left.\begin{aligned} &\left\{i\left(-\mu - 2\tilde{K}P\sigma\right)\right\}\left(1 - \upsilon\right) + \sigma^2\left(1 + \upsilon\right) = 0 \\ &\left(P\sigma^2 - 2\rho_0\right) = -Q\left(\upsilon + \upsilon^*\right) \end{aligned}\right\} \quad (43)$$

So, one gets;

$$\psi = \rho_0 \exp[i(\tilde{K}x - \Theta t)]\frac{1 + \epsilon\upsilon\exp(\sigma x - \Theta t)}{1 + \exp(\sigma x - \Theta t)} \quad (44)$$

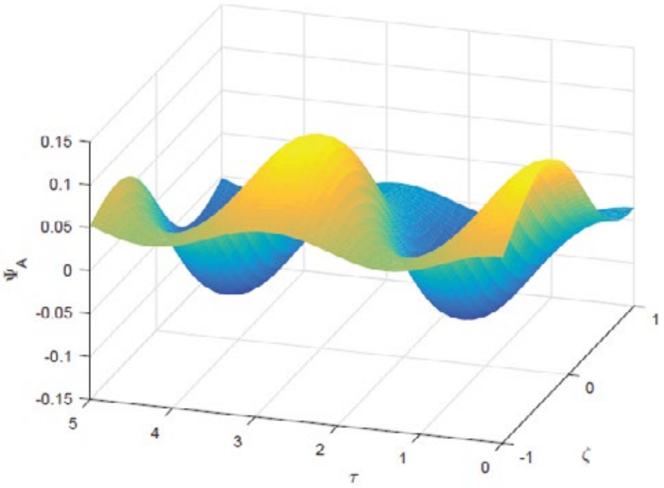

Figure 19: The Akhmediev Breather.

Now going to higher order of 'g_i' one can set 'g_i', 'f_i' to zero for $j\geqslant 2$. Thus the above equation is an exact solution and ψ decays with time.

Now if we set;

$$b_0 = A_0(\zeta)[G(\zeta,\tau)\exp(i\nu(\zeta))-1] \quad (45)$$

where, '$G(\zeta,\tau)$' is the displaced amplitude and $\nu(\zeta)$ the displaced phase. Thus we get;

$$\left.\begin{aligned}&\partial_\tau G\cos(\nu)+Qr_0^2G^2\sin(2\nu)-[G\nu'(\zeta)+P\partial_\zeta^2 G+Qr_0^2G^3]=0\\&\partial_\tau G\sin(\nu)+Qr_0^2G^2(2cos^2(\nu)+1)+[G\nu'(\zeta)+P\partial_\zeta^2G+Qr_0^2G(G^2+2)\cos(\nu)]=0\end{aligned}\right\} \quad (46)$$

where,

$$r_0=\lim_{\zeta\to\pm\infty}|A(\zeta,\tau)|$$

Next setting, $G = 1/H$, we obtain

$$\frac{\partial H}{\partial\tau}-Qr_0^2\sin(2\nu)+Qr_0^2\sin(\nu)=0 \quad (47)$$

Thus solving we get;

$$H(\tau,\zeta)=\frac{\overline{Q_0}-\nu(\zeta)}{\overline{P(\tau)}} \quad (48)$$

$\nu(\zeta)$ is the constant of integration and

$$Q_0(\tau)=-Qr_0^2\int\overline{P}(\varepsilon)\sin(\nu(\zeta))d\varepsilon$$

which finally resulted in

$$b(\zeta,\tau)=\left\{\frac{\overline{\nu}^2\cosh(\sigma\tau)-i\overline{\sigma}\sinh(\sigma\tau)}{\cosh(\sigma\tau)-\sqrt{1-\frac{\nu^2}{4}}\cos(\nu\zeta)}\right\}\exp(i\nu(\zeta)) \quad (49)$$

which is the required breather solution of NLS-equation whose pictorial representation is given by **Figure 19**.

CONCLUSION

- In this communication we have considered a four component e-p-i plasma having two temperature ions. Through the hot and cold electrons are considered to obey the Super-thermal (κ-distribution) distribution

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but the positrons are considered to obey the Maxwellian distribution. The hydrodynamics of the ions are taken into account which are warm adiabatic in nature. The effect of magnetic field is not considered here which means the model is an unmagnetized one. Next, using the reductive perturbation technique prescribed by Krylov-Bogoliubov-Mitropolsky the NLS-equation is obtained in order to study the characteristics of the IAW's in this case. The stability analysis is done in a way different from the traditional method. The method which was prescribed by E. Wigner long before[7, 13]. The application of this method took us to two different observations viz.

- (i) when the wave interaction is monochromatic or coherent.
- (ii) when the interaction is non-monochromatic or incoherent.

In the first case we observed that as usual the instability growth rate increases, reached a maximum value and ultimately drops to zero. This supports the fact that the solitons are sustained. While in the second case/ non-monochromatic case the instability growth rate has a nearly exponential increasing nature which states that the solitons are ultimately annihilated. At last, we have given the pictorial/graphical representation of Akhmediev breather as one of the solution of the Nonlinear Schrödinger equation.

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